5.1 Notes: Verifying Trig Identities

- Objective: Students will be able to verify trig identities (Pythagorean and Even-Odd)

Reciprocal Identities:

$\sin x = \frac{1}{\csc x}$	$\cos x = \frac{1}{\sec x}$	$\tan x = \frac{1}{\cot x}$
$\csc x = \frac{1}{\sin x}$	$\sec x = \frac{1}{\cos x}$	$\cot x = \frac{1}{\tan x}$

Quotient Identities:

$\tan x = \frac{\sin x}{\cos x}$	$\cot x = \frac{\cos x}{\sin x}$

Pythagorean Identities:

$\sin^2 x + \cos^2 x = 1$	$1 + \tan^2 x = \sec^2 x$	$1 + \cot^2 x = \csc^2 x$

Even-Odd Identities

$\sin(-x) = -\sin x$	$\cos(-x) = \cos x$	$\tan(-x) = -\tan x$
$\csc(-x) = -\csc x$	$\sec(-x) = \sec x$	$\cot(-x) = -\cot x$

In general,

- Step 1: Choose the more complicated side, and work with only that side
- Step 2: Apply fundamental identities (p.650)
- Step 3: Write one side using the strategies below:
  - Re-write in terms of sin and cos
  - Factor
  - Separate fractions $\left(\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}\right)$
  - Combine fractions (get a common denominator)
  - Introduce expressions we need

Examples: Verify each trig identity.

1. $\cos x \csc x = \cot x$

2. $\cos^2 x - \sin^2 x = 2\cos^2 x - 1$

Examples: Verify each trig identity.

3. $\tan\theta + \cot\theta = \sec\theta\csc\theta$

4. $\csc x\tan x = \sec x$

5. $\sin x - \sin x\cos^2 x = \sin^3 x$

6. $\frac{1+\cos\theta}{\sin\theta} = \csc\theta + \cot\theta$

7. $\frac{\sin x}{1+\cos x} + \frac{1+\cos x}{\sin x} = 2\csc x$

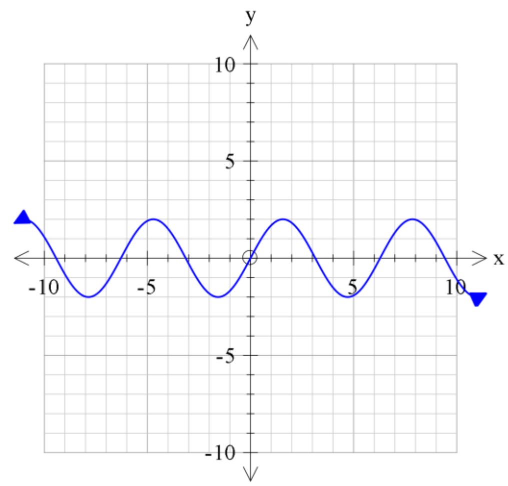
8. $\frac{\cos x}{1+\sin x} = \frac{1-\sin x}{\cos x}$

9. $\frac{\sec x+\csc(-x)}{\sec x\csc x} = \sin x - \cos x$

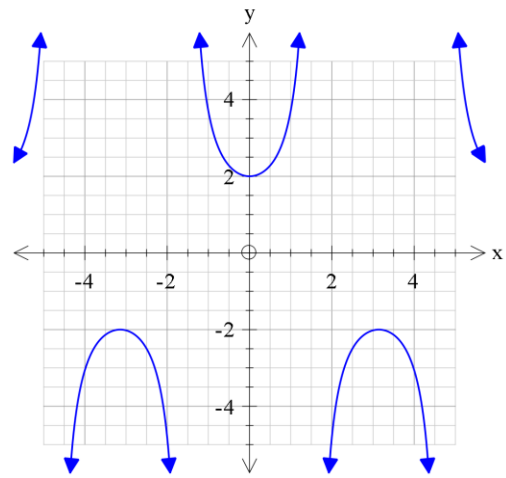
10. $\frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta} = 2 + 2\tan^2\theta$

(p. 659 #63, 65, 66) **Use the graph to complete each identity. Then verify.**

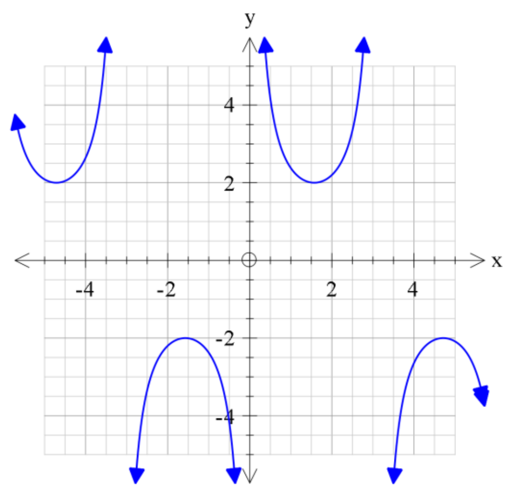
11. $\frac{\cos(x)+\cot(x)\cdot\sin(x)}{\cot(x)} = ?$



12. $\frac{1}{\sec x+\tan x}+\frac{1}{\sec x-\tan x} = ?$



13. $\frac{1+\cos x}{\sin x}+\frac{\sin x}{1+\cos x} = ?$



5.2 Day 1 Notes: Sum and Difference of Two Angles (foldable)

- Objectives:
  - Students will verify identities using sum and difference formulas
  - Students will find exact values of trig functions by using sum and difference formulas

https://www.youtube.com/watch?v=hWTXrnbaNvQ

The cosine of the Difference of Two Angles: $\cos(\alpha - \beta) = \cos\alpha \cos\beta + \sin\alpha \sin\beta$

Sum and Difference Formulas

$\cos(\alpha + \beta) = \cos\alpha \cos\beta - \sin\alpha \sin\beta$	$\cos(\alpha - \beta) = \cos\alpha \cos\beta + \sin\alpha \sin\beta$
$\sin(\alpha + \beta) = \sin\alpha \cos\beta + \cos\alpha \sin\beta$	$\sin(\alpha - \beta) = \sin\alpha \cos\beta - \cos\alpha \sin\beta$
$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha \tan\beta}$	$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha \tan\beta}$

Examples: Use Sum and Difference Formulas to find exact values below without a calculator.

1. Find the exact value of $\cos 15$ (angle is given in degrees).

2. Find the exact value of $\cos 100 \cos 55 + \sin 100 \sin 55$ (angles are given in degrees).

3. Verify the identity: $\frac{\cos(\alpha-\beta)}{\cos\alpha\cos\beta} = 1 + \tan\alpha\tan\beta$

4. Find the exact value: $\sin\frac{5\pi}{12}$

5. Suppose that $\sin\alpha = \frac{4}{5}$ and angle α is in quadrant II. Also, given that $\sin\beta = \frac{1}{2}$ with angle β in quadrant I. Find the exact value of each of the following:

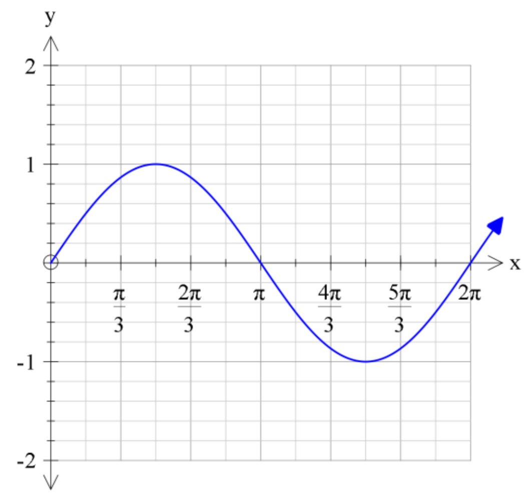
a) $\cos\alpha$

b) $\cos\beta$

c) $\cos(\alpha+\beta)$

d) $\sin(\alpha+\beta)$

6. The graph shows $y = \cos\left(x + \frac{3\pi}{2}\right)$ in a $[0, 2\pi, \frac{\pi}{2}]$ by [-2, 2, 1] viewing rectangle. Describe the graph using another equation. Verify that the two equations are equivalent.



7. Find the exact value of $\sin\left(\frac{\pi}{3} + \frac{3\pi}{4}\right)$ without using a calculator.

For #8 – 9: Simplify each expression without using a calculator.

8. $\tan\left(\frac{5\pi}{4} + \theta\right)$

9. $\tan(\pi - \beta)$

5.2 Day 2 Notes: Sum and Difference Formulas, continued.

For Examples 1 – 4: Find the exact value of each expression without using a calculator.

1. $\cos\frac{5\pi}{12}\cos\frac{\pi}{12}+\sin\frac{5\pi}{12}\sin\frac{\pi}{12}$

2. cos 75 (in degrees)

3. $\tan\left(\frac{4\pi}{3}-\frac{\pi}{4}\right)$

4. in degrees: $\frac{\tan 10+\tan 35}{1-\tan 10\tan 35}$

For #5 – 6: Verify each identity.

5. $\cos\left(x-\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}(\cos x+\sin x)$

6. $\sin\left(x+\frac{\pi}{2}\right)=\cos x$

For #7 – 8: Verify each identity.

7. $\frac{\sin(\alpha-\beta)}{\cos\alpha\cos\beta} = \tan\alpha - \tan\beta$

8. $\frac{\cos(x+h)-\cos x}{h} = \cos x\frac{\cos h-1}{h} - \sin x\frac{\sin h}{h}$

9. Find the exact value of the following if $\sin\alpha = \frac{3}{5}$, α lies in quadrant I, and $\sin\beta = \frac{5}{13}$, β lies in quadrant II.

   a) $\cos(\alpha+\beta)$

   b) $\sin(\alpha+\beta)$

   c) $\tan(\alpha+\beta)$

5.3 Day 1 Notes: Double Angle and Half-Angle Formulas

- Objectives:
  - Students will verify double angle and half-angle formulas
  - Students will find exact values of trig functions without using a calculator

Double Angle Formulas

$\sin 2\theta = 2\sin\theta\cos\theta$		
$\cos 2\theta = \cos^2\theta - \sin^2\theta$	$\cos 2\theta = 2\cos^2\theta - 1$	$\cos 2\theta = 1 - 2\sin^2\theta$
$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$		

Half-Angle Formulas

$\sin\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1-\cos\alpha}{2}}$	$\cos\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1+\cos\alpha}{2}}$	$\tan\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}$
		$\tan\left(\frac{\alpha}{2}\right) = \frac{1-\cos\theta}{\sin\theta}$
		$\tan\left(\frac{\alpha}{2}\right) = \frac{\sin\theta}{1+\cos\theta}$

Examples:

1. If $\sin\theta = \frac{4}{5}$ and θ lies in quadrant II, find the exact value of each of the following:

   a) $\sin(2\theta)$

   b) $\cos(2\theta)$

   c) $\tan(2\theta)$

2. Find the exact value of $\frac{2\tan 15}{1-\tan^2 15}$, where each angle is in degrees.

3. Verify the identity: $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

4. Given that $\cos 210^{o} = -\frac{\sqrt{3}}{2}$, find the exact value of $\cos 105^{o}$

5. Verify the identity: $\tan\theta = \frac{\sin 2\theta}{1+\cos 2\theta}$

6. Use a half-angle formula to find the exact value of $\tan \frac{5\pi}{12}$

7. Verify the identity: $\tan\left(\frac{\alpha}{2}\right) = \frac{\sec\alpha}{\sec\alpha\csc\alpha + \csc\alpha}$

8. Given that $\csc\alpha = \frac{-25}{24}$, and α is in the 4^{th} quadrant. Find the requested values without a calculator.

   a) $\sin\frac{\alpha}{2}$ b) $\cos\frac{\alpha}{2}$ c) $\tan\frac{\alpha}{2}$

9. Use a half-angle formula to find the exact value of $\tan(112.5^{\circ})$

5.3 Day 2 Notes: More Practice with Double and Half-Angles

Examples

1. If $\cos\theta = \frac{24}{25}, \theta$ lies in quadrant IV, find the following:

   a) $\sin 2\theta$

   b) $\cos 2\theta$

   c) $\tan 2\theta$

2. Find the exact value without a calculator (angles are given in degrees): $\cos^2 75 - \sin^2 75$

3. Find the exact value without a calculator: $\frac{2\tan\left(\frac{\pi}{8}\right)}{1-\tan^2\left(\frac{\pi}{8}\right)}$

4. Verify the identity: $(\sin\theta + \cos\theta)^2 = 1 + \sin 2\theta$

5. Verify the identity: $\cot x = \frac{\sin 2x}{1-\cos 2x}$

6. Find the exact value of $\tan 75^{\circ}$

7. If $\tan\alpha = \frac{7}{24}$, find $2\sin\left(\frac{\alpha}{2}\right)\cos\left(\frac{\alpha}{2}\right)$

8. If $\sec \alpha = -\frac{13}{5}$ and $\frac{\pi}{2} < \alpha < \pi$, find the following:

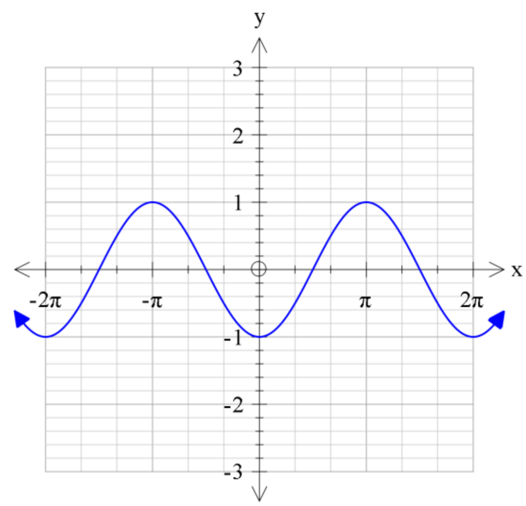
a) $\sin \frac{\alpha}{2}$

b) $\cos \frac{\alpha}{2}$

c) $\tan \frac{\alpha}{2}$

9. Verify the identity: $2 \tan\left(\frac{\alpha}{2}\right) = \frac{\sin^2\alpha + 1 - \cos^2\alpha}{\sin\alpha(1+\cos\alpha)}$

10. Use the graph shown to complete the identity. Then verify.
$\sin^2\left(\frac{x}{2}\right) - \cos^2\left(\frac{x}{2}\right) = ?$



Notes on Power Reducing Formulas

*Double angles are used to derive the power reducing formulas

In calculus, by reducing the power, we can better explore the relationship between a function and how it changes at every instant in time. (Used by athletes to increase throwing distance)

Power-Reducing Formulas

$\sin^2\theta = \frac{1-\cos 2\theta}{2}$	$\cos^2\theta = \frac{1+\cos 2\theta}{2}$	$\tan^2\theta = \frac{1-\cos 2\theta}{1+\cos 2\theta}$

Examples:

1) Write an equivalent expression for $\cos^4 x$ that does not contain powers of trigonometric functions greater than 1.

2) Write an equivalent expression for $8\sin^4 x$ that does not contain powers of trigonometric functions greater than 1.

3) Write an equivalent expression for $2\sin^2 x\cos^2 x$ that does not contain powers of trigonometric functions greater than 1.

4) Verify the identity: $\sin 4t = 4\sin t\cos^3 t - 4\sin^3 t\cos t$

5) Verify the identity: $2\tan\left(\frac{x}{2}\right) = \frac{2\sin x - \sin 2x}{\sin^2 x}$

5.5 Day 1 Notes: Solving Trig Equations

- Objective: Students will be able to solve trig equations for a variable.

Strategies for solving trig equations:

- Isolate the trig expression.
  - Collect all terms on one side.
  - Factor (GCF or trinomial)
  - Use a trig identity to write all terms with the same trig function.
- Use the unit circle to solve for the argument.
  - If the argument has a variable multiplied by *n*, set the argument equal to the values in the unit circle (go around the circle *n* times)
  - Solve each equation for the variable.

Examples: Solve each equation for the variable, in radians.

1. $5\sin x = 3\sin x + \sqrt{3}$

2. $\tan 2x = \sqrt{3}$ if $0 \le x < 2\pi$

3. $\sin \frac{x}{3} = \frac{1}{2}$ if $0 \le x < 2\pi$

4. $2\sin^2 x - 3\sin x + 1 = 0, \quad 0 \le x < 2\pi$

5. $4\cos^2 x - 3 = 0, \quad 0 \le x < 2\pi$

6. $\sin x \tan x = \sin x, \quad 0 \le x < 2\pi$

7. $2\sin^2 x - 3\cos x = 0,\ 0 \le x < 2\pi$

8. $\cos 2x + \sin x = 0,\ 0 \le x < 2\pi$

9. $\sin x \cos x = -\frac{1}{2},\ 0 \le x < 2\pi$

10. $\cos x - \sin x = -1,\ 0 \le x < 2\pi$

11. **Calculator allowed:** $\cos^2 x + 5\cos x + 3 = 0,\ 0 \le x < 2\pi$; round to four decimals

5.5 Day 2 Notes: Solving Trig Equations, continued.

Examples:

1. Use substitution to determine whether the given *x*-value is a solution: $\cos x = -\frac{1}{2}$, $x = \frac{2\pi}{3}$

For #2 – 7: Solve each equation for the variable (in radians).

2. $2 \cos x + \sqrt{3} = 0$

3. $\sin 4x = -\frac{\sqrt{2}}{2}$, $[0, 2\pi)$

4. $\cos^2 x + 2 \cos x - 3 = 0$, $[0, 2\pi)$

5. $9 \tan^2 x - 3 = 0$, $[0, 2\pi)$

6. $\cot x (\tan x + 1) = 0$, $[0, 2\pi)$

7. $4 \sin^2 x + 4 \cos x - 5 = 0$, $[0, 2\pi)$

8. **Solve for *x* in radians:** $\cos 2x = \cos x$; $[0, 2\pi)$

9. **Use a calculator to solve for *x* in radians:** $4\tan^2 x - 8\tan x + 3 = 0$; $[0, 2\pi)$; round to four decimals

10. **Solve for *x* in radians:** $\cos x - 5 = 3\cos x + 6$