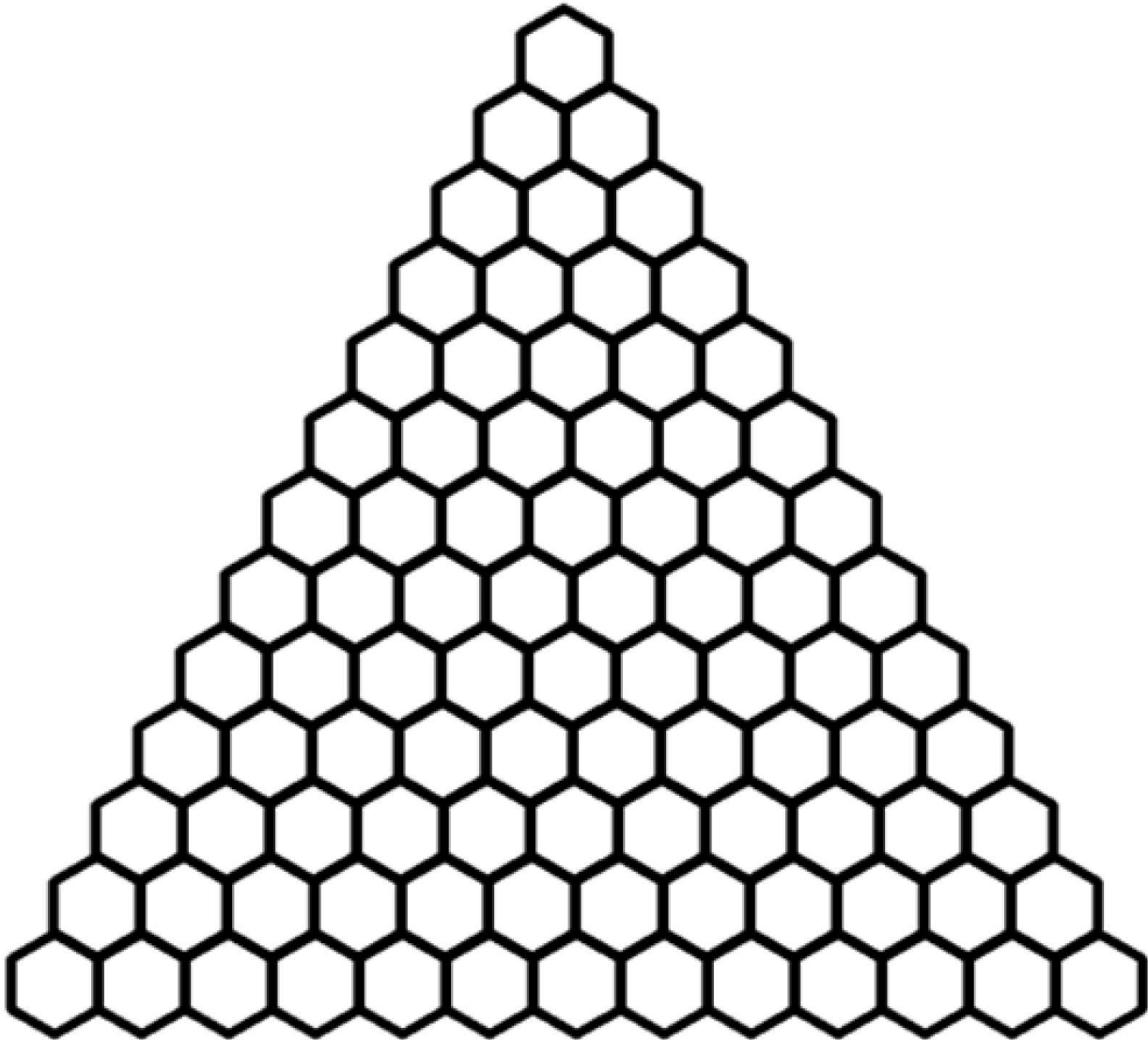


Pascal's Triangle

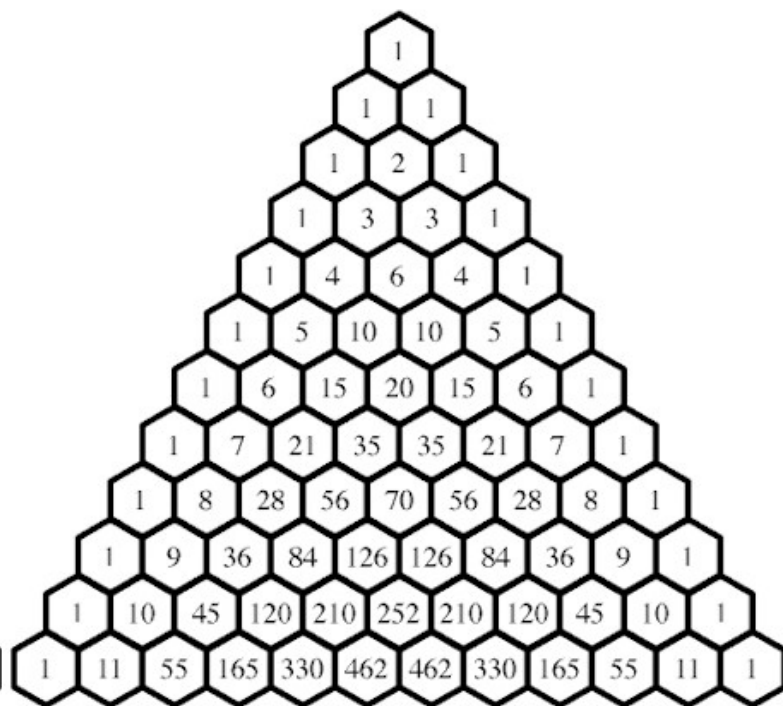
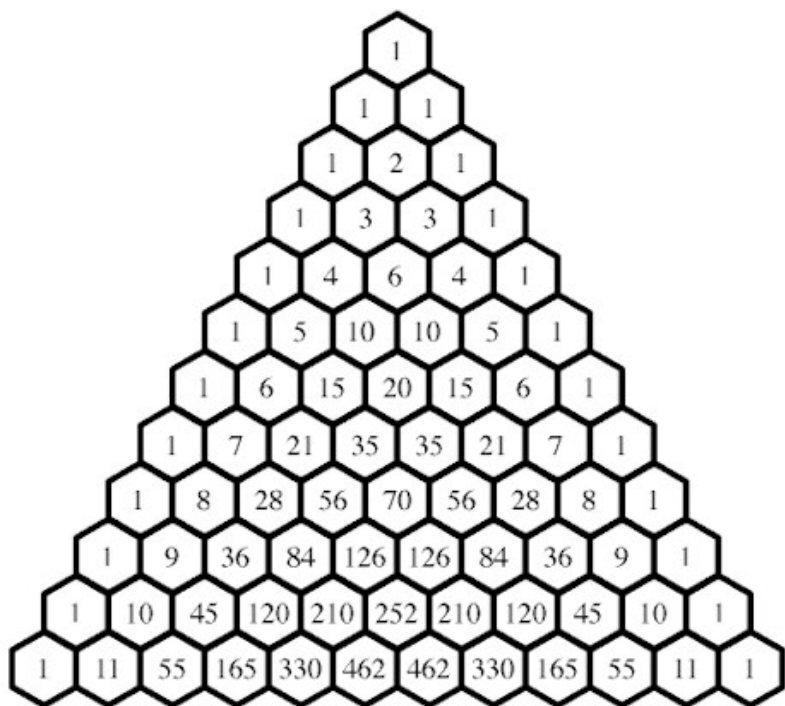
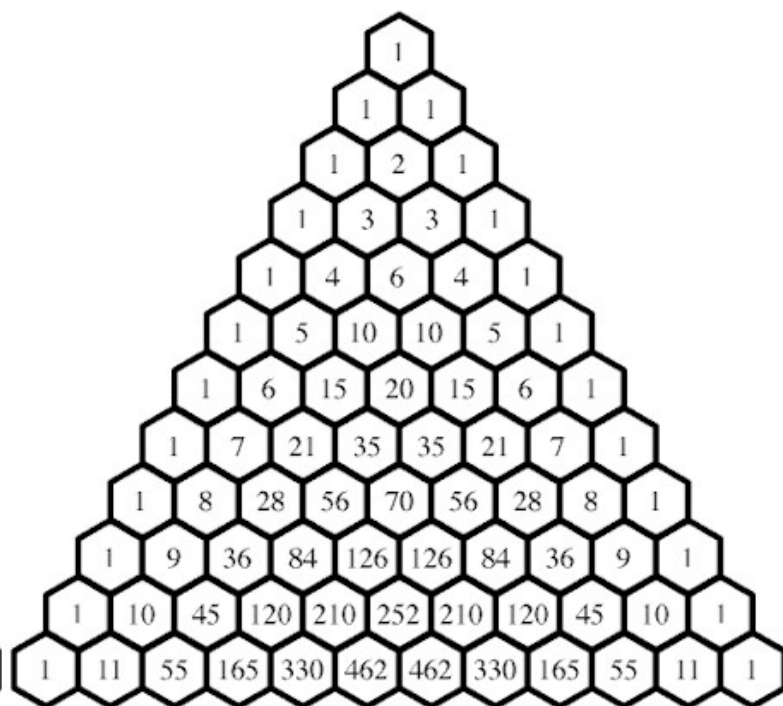
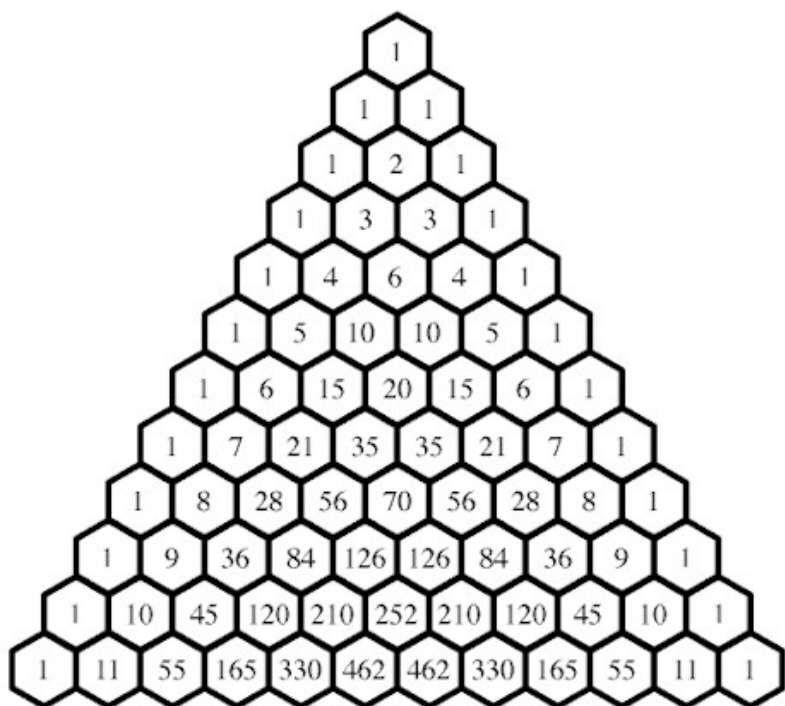


## Pascal's Patterns

1. There are over 100 patterns and mathematical ideas that come from Pascal's Triangle. Take a good look at Pascal's Triangle and see if you can find any patterns in it. List at least two below.
  
2. If you look at the diagonal just inside the ones, what do you see?
  
3. Find the sum of each row of Pascal's and list them on one side of your triangle. What patterns are emerging from these sums. Describe the patterns below.
  
4. The Fibonacci sequence is present in this triangle. Describe where the Fibonacci sequence is found.
  
5. Take a look at row 7, what do you notice about all of the numbers in that row. Describe below.
  
6. Make each row into a single digit number by using each element as a digit of the number (carrying over when an element itself has more than one digit). What do you notice about these new numbers?

Row #	Actual Row	Single Digit	Pattern
Row 0	1	1	
Row 1	1 1	11	
Row 2	1 2 1	121	
Row 3			
Row 4			
Row 5	1 5 10 10 5 1	161051	
Row 6	1 6 15 20 15 6 1	1771561	
Row 7	1 7 21 35 35 21 7 1	19487171	
Row 8	1 8 28 56 70 56 28 8 1	214358881	

7. Color the odd numbers in the triangle on page 4, what patterns or formations are emerging?
8. Color the even numbers in a different triangle on page 4, what patterns or formations are emerging?
9. Color the numbers in Pascal's Triangle that are divisible by 3. What patterns are forming?
10. What pattern do you think will form if you color in all numbers divisible by 5? Divisible by 7? Now choose one pattern to color in the last triangle on page 4. Was your guess correct?
11. Describe the symmetry in Pascal's Triangle.
12. There is a pattern known as a hockey stick in Pascal's Triangle, can you find it and describe the pattern?
13. Find the triangular numbers. The first three numbers are 1, 3, 6. Describe how the triangular numbers are found.
14. Last but not least, try to find the square numbers in this triangle. These are numbers like 4, 9, 16, 25. Hint they are found in the same place the triangular numbers are found. Describe how you found these.



## Exploring the Binomial Expansion Theorem and Pascal's Triangle Relationships

1. Multiply the following binomials.

a.  $(x + 2)^2$

b.  $(x - 5)^2$

c.  $(4m + 3n)^2$

d.  $(r + 2)^3$

2. Exploration. Fill in the following table.

\*Hint: to expand  $(x + y)^3$ , you can multiply  $(x + y)^2$  by  $(x + y)^1$

Product	Expansion
$(x + y)^0$	
$(x + y)^1$	
$(x + y)^2$	
$(x + y)^3$	
$(x + y)^4$	
$(x + y)^5$	

3. Write conjectures about the number of terms and about symmetry in the terms of the expansion in any row of the table. Verify your conjectures by filling in the row that would follow  $(x + y)^5$ .

4. Do you notice any relationships between the expansion and Pascal's Triangle? Write these below.

5. Use the pattern you saw to try to expand  $(x + y)^{11}$ .

6. Expand the following binomials.

a.  $(a + b)^6$

b.  $(p + q)^4$

Expand the binomials below.

c.  $(x - y)^3$  \*hint: think of this one as  $(x + (-y))^3$

d.  $(y + 3)^4$  Now complete the multiplication below.

7. Fill in the blanks below to expand  $(2x - 3y)^4$ . Remember think of this one as  $(2x + (-3y))^4$

8. Try to expand the binomials below using the pattern above.

a.  $(a + 2b)^4$

b.  $(2x + 3)^3$

c.  $(3x + 2y)^5$

d.  $(2x - 2y)^4$  (be careful on this one! Remember think of the binomial as  $(2x + (-2y))^4$ )

## Binomial Expansion Practice

Expand the following binomials.

a.  $(a + b)^4$

---

b.  $(x + y)^6$

---

c.  $(x + 2)^3$

---

d.  $(2x + b)^5$

---

e.  $(x - y)^3$

---

f.  $(h - 2t)^7$

---

g.  $(3x + 2y)^4$

---

## Binomial Expansion Practice #2

Expand the following binomials:

1.  $(a + b)^3$

2.  $(2 + x)^5$

3.  $(y + 4)^7$

4.  $(6 + x)^6$

5.  $(x - y)^5$

6.  $(x - 2y)^9$

7.  $(-x + 4y)^4$

8.  $(2m - 3q)^6$

9. Find the sum of the 12<sup>th</sup> row of Pascal's Triangle.

10. Name some patterns that are found in Pascal's Triangle.



## Pascal's and Binomial Expansion Review

1. Describe 3 patterns found within pascal's triangle.
2. What is the relationship between pascal's triangle and binomial expansion?
3. How many terms are in the expansion of  $(x + y)^{13}$ ?

### For #4-8 expand the binomials:

4.  $(a + b)^9$

5.  $(2 + x)^5$

6.  $(x - 2y)^6$

7.  $(y + 4)^7$

8.  $(-x + 4y)^4$

For #9-12 use the binomial  $(r + s)^{12}$  to answer the questions.

9. What is the exponent of  $r$  in the term that contains  $s^8$ ?

10. What is the exponent of  $s$  in the terms that contains  $r^5$ ?

11. What are the exponents for  $r$  and  $s$  in the term that has a coefficient of 924?

12. What is the coefficient of the term with  $s^3$ ?

13. What is the sum of the 7<sup>th</sup> row in pascals triangle? The 12<sup>th</sup>? The 20<sup>th</sup>?