Ch 6 notes filled out

6.1 The Greatest Common Factor   Date:

OBJECTIVES

1 Find the greatest common factor of a list of terms.

2 Factor out the greatest common factor.

3 Factor by grouping.

Factor *a number that divides another number evenly.*
ex: 2 is a factor of 6.

Common factor *a factor multiple numbers have in common*
ex: 2 is a common factor of 6, 18, and 36

Greatest common factor (GCF)
The largest factor multiple numbers share.
ex: The GCF of 6, 36, and 18 is 6

1) Find the greatest common factor for each list:

(a) $30, 45$

$30: \underline{3} \cdot 2 \cdot \underline{5}$

$45: \underline{3} \cdot 3 \cdot \underline{5}$

30 and 45 have a 3 and a 5 in common, so their GCF is 15.

(b) $72, 120, 432$

$72: \underline{2} \cdot \underline{2} \cdot 2 \cdot \underline{3} \cdot 3$

$120: \underline{2} \cdot \underline{2} \cdot 2 \cdot \underline{3} \cdot 5$

$432: \underline{2} \cdot \underline{2} \cdot 2 \cdot 2 \cdot \underline{3} \cdot 3 \cdot 3$

$GCF = 2 \cdot 2 \cdot 2 \cdot 3 = 24$

(c) $10, 11, 14$

$10: 2 \cdot 5$

$11: 1 \cdot 11$

$14: 2 \cdot 7$

No common factors
$\Rightarrow GCF = 1$

d) $21m^7, 18m^6, 54m^8, 24m^5$

$21m^7: \underline{3} \cdot 7 \cdot \underline{m^5} \cdot m^2$

$18m^6: \underline{3} \cdot 6 \cdot \underline{m^5} \cdot m$

$54m^8: \underline{3} \cdot 18 \cdot \underline{m^5} \cdot m^3$

$24m^5: \underline{3} \cdot 8 \cdot \underline{m^5}$

$GCF = 3m^5$

e) $x^4y, x^7y^5, x^3y^7, y^8$

$x^4y: x^4 \cdot \underline{y}$

$x^7y^5: x^7 \cdot \underline{y} \cdot y^4$

$x^3y^7: x^3 \cdot \underline{y} \cdot y^6$

$y^8: y^7 \cdot \underline{y}$

$GCF = y$

Factoring out the greatest common factor
Pull out GCF, in ( ) write left over factors
ex: $6 + 12 = 6(1 + 2)$

Factor using the GCF:

a) $5y^2 + 10y$

$\underline{5} \cdot y \cdot y + \underline{5} \cdot 2 \cdot y$

$5y(y + 2)$

b) $20m^5 + 10m^4 + 15m^3$

$\underline{5} \cdot 4 \cdot \underline{m^3} \cdot m^2 + \underline{5} \cdot 2 \cdot \underline{m^3} \cdot m + \underline{5} \cdot 3 \cdot \underline{m^3}$

$5m^3(4m^2 + 2m + 3)$

c) $x^5 + x^3$

d) $-4x^3 - 12x^2$ *whenever front term is negative, pull out a −. This will change the sign of every term

$-4 \cdot x^2 \cdot x - 4 \cdot 3 \cdot x^2$

$-4x^2(x+3)$

e) $-15x+3$

$-3 \cdot 5 \cdot x - 3 \cdot -1$

$-3(5x-1)$

f) $16x^2 - 12x$

$4 \cdot 4 \cdot x \cdot x - 3 \cdot 4 \cdot x$

$4x(4x-3)$

g) $36x^4 - 42x^2$

$6 \cdot 6 \cdot x^2 \cdot x^2 - 7 \cdot 6 \cdot x^2$

$6x^2(6x^2-7)$

h) $20m^7p^2 - 36m^3p^4$

$4 \cdot 5 \cdot m^3 \cdot m^4 \cdot p^2 - 9 \cdot 4 \cdot m^3 \cdot p^2 \cdot p^2$

$4m^3p^2(5m^4-9p^2)$

i) $15x^3y^2 - 20x^2y^3 + 12x^4y$

$15 \cdot x^2 \cdot x \cdot y \cdot y - 20 \cdot x^2 \cdot y \cdot y^2 + 12 \cdot x^2 \cdot x^2 \cdot y$

$x^2y(15xy-20y^2+12x^2)$

j) $4x^5 - 8x^4 - 4x^3$

$4 \cdot x^3 \cdot x^2 - 2 \cdot 4 \cdot x^3 \cdot x - 1 \cdot 4 \cdot x^3$

$4x^3(x^2-2x-1)$

6.2 Factoring Trinomials

Multiply the following binomials:
$(x + 3)(x − 1)$

$1x^2 - 1x + 3x - 3$

$x^2 + 2x - 3$

*middle term: add two constants

*end term: multiply two constants

Factor the following trinomials:

1) $x^2 + 9x + 14$

14
1   14
2   7

$(x+2)(x+7)$
+2x   +7x ✓

2) $x^2 - 9x + 20$

20
20 1
10 2
5   4

$(x-5)(x-4)$
−5x
−4x ✓

3) $x^2 + x - 6 = x^2 + 1x - 6$

$\begin{array}{c} 6 \\ \wedge \\ 6 \quad 1 \\ \boxed{2 \quad 3} \end{array}$

$(x + 3)(x - 2)$
$\underbrace{\quad}_{+3x \quad -2x} \checkmark$

4) $p^2 - 2p - 15$

$\begin{array}{c} 15 \\ \wedge \\ 15 \quad 1 \\ \boxed{5 \quad 3} \end{array}$

$(x + 3)(x - 5)$
$\underbrace{\quad}_{+3x \quad -5x} \checkmark$

5) $x^2 - 5x + 12$

$\begin{array}{cc} & 12 \\ & \wedge \\ 12 & 1 \times \\ 6 & 2 \times \\ 4 & 3 \times \end{array}$
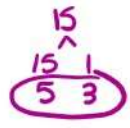
Not factorable

6) $k^2 - 8k + 11$

$\begin{array}{c} 11 \\ \wedge \\ 1 \quad 11 \times \end{array}$
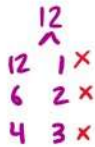
Not factorable

7) $z^2 - 2z - 3$

$\begin{array}{c} 3 \\ \wedge \\ 1 \quad 3 \end{array}$

$(z + 1)(z - 3)$
$\underbrace{\quad}_{+z \quad -3z} \checkmark$

8) $-x^2 + 3x + 28$ *Need to factor out the $-$ *

$-(x^2 - 3x - 28)$
$\downarrow$
$-(x - 7)(x + 4)$
$\underbrace{\quad}_{-7x \quad +4x} \checkmark$

$\begin{array}{c} 28 \\ \wedge \\ 1 \quad 28 \\ 2 \quad 14 \\ \boxed{4 \quad 7} \end{array}$

9) $w^3 - 10w^2 + 25w$

$w(w^2 - 10w + 25)$
$\downarrow$
$w(w - 5)(w - 5)$
$\underbrace{\quad}_{-5w \quad -5w} \checkmark$

$\begin{array}{c} 25 \\ \wedge \\ 25 \quad 1 \\ \boxed{5 \quad 5} \end{array}$

10) $-2x^2 + 14x - 24$

$-2(x^2 - 7x + 12)$
$\downarrow$
$-2(x - 3)(x - 4)$
$\underbrace{\quad}_{-3x \quad -4x}$

$\begin{array}{c} 12 \\ \wedge \\ 12 \quad 1 \\ 6 \quad 2 \\ \boxed{3 \quad 4} \end{array}$

11) $2a^2b - 10ab + 8b$

$2b(a^2 - 5a + 4)$
$\downarrow$
$2b(a - 1)(a - 4)$
$\underbrace{\quad}_{-1a \quad -4a}$

$\begin{array}{c} 4 \\ \wedge \\ 1 \quad 4 \\ 2 \quad 2 \end{array}$

12) $3x^2 - 24x + 12$

$\boxed{3(x^2 - 8x + 4)}$

can't be factored further

$\begin{array}{c} 4 \\ \wedge \\ 4 \quad 1 \\ 2 \quad 2 \end{array}$

13) $4x^5 - 28x^4 + 40x^3$

$4x^3(x^2 - 7x + 10)$
$\downarrow$
$4x^3(x - 5)(x - 2)$
$\underbrace{\quad}_{-5x \quad -2x} \checkmark$

$\begin{array}{c} 10 \\ \wedge \\ 10 \quad 1 \\ \boxed{5 \quad 2} \end{array}$

3
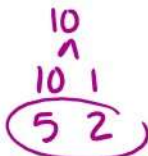
6.2 Day 2
-Dice Game
-Practice problems

Extra examples if needed:

For #1 – 9, factor out the greatest common factor.

1) $10x - 10y$

$10(x-y)$

2) $8x^2 + 20y$

$4(2x^2+5y)$

3) $4x^2 - 4x$

$4x(x-1)$

4) $2m^2 + 6mn$

$2m(m+3n)$

5) $9a^5 + a^3$

$a^3(9a^2+1)$

6) $6w^3 - 14w^2$

$2w^2(3w-7)$

7) $+11x^3y + 33xy + 44xy^2$

$-11xy(x^2-3+4y)$

8) $5a^2b^2 - 9cd + 15a$

Not factorable

9) $18xy - 24xz + 21z$

$3(6xy-8xz+7z)$

For #10 – 21, factor each expression completely.

10) $x^2 + 8x + 7$

$(x+1)(x+7)$

11) $k^2 - 7k + 10$

$(k-5)(k-2)$

12) $w^2 - 12w - 13$

$(w-13)(w+1)$

13) $y^2 - 4y - 45$

$(y-9)(y+5)$

14) $x^2 - 6x + 9$

$(x-3)(x-3)$

or $(x-3)^2$

15) $x^3 + 2x^2 - 15x$

$x(x^2+2x-15)$

$x(x+5)(x-3)$

16) $5y^2 - 25y - 120$

$5(y^2-5y-24)$

$5(y-8)(y+3)$

17) $-x^2 - 13x - 30$

$-(x^2+13x+30)$

$-(x+10)(x+3)$

18) $3x^2 + 30x + 75$

$3(x^2+10x+25)$

$3(x+5)(x+5)$

or $3(x+5)^2$

19) $-8x^2 + 16x - 8$

$-8(x^2-2x+1)$

$-8(x-1)(x-1)$

or $-8(x-1)^2$

20) $x^2 + 12x + 36$

$(x+6)(x+6)$

or $(x+6)^2$

21) $-2x^2 - 2x + 84$

$-2(x^2+1x-42)$

$-2(x+7)(x-6)$

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6.3 More on Factoring Trinomials   Date:

Factor the following trinomials completely:

1) $2x^2 - 11x + 5$

$(2x - 1)(1x - 5)$
$-1x \quad -10x$ ✓

2) $2y^2 - 13y - 7$

$(2y + 1)(1y - 7)$
$+1y \quad -14y$ ✓

3) $6r^2 + r - 1$

$(2r + 1)(3r - 1)$
$+3r \quad -2r$ ✓

4) $9y^2 + 6y + 1$

$(3y + 1)(3y + 1)$
$+3y \quad +3y$ ✓

5) $3x^2 - 5x + 2$

$(3x - 2)(1x - 1)$
$-2x \quad -3x$ ✓

6) $15x^2 - x - 6$

$(3x - 2)(5x + 3)$
$-10x \quad +9x$ ✓

7) $6x^2 + 5xy - 6y^2$

$(2x + 3y)(3x - 2y)$
$+9xy \quad -4xy$ ✓

8) $8x^2 + 14x - 15$

$(4x - 3)(2x + 5)$
$-6x \quad +20x$ ✓

9) $2m^2 + mn - 21n^2$

$(2m + 7n)(1m - 3n)$
$+7mn \quad -6mn$ ✓

Factor each trinomial completely (take out GCF first if necessary):

10) $6x^2 - 2x - 4$

$2(3x^2 - x - 2)$
$2(1x - 1)(3x + 2)$
$-3x \quad +2x$ ✓

11) $3r^3 - 21r^2 + 30r$

$3r(r^2 - 7r + 10)$
$3r(r - 5)(r - 2)$
$-5r \quad -2r$ ✓

12) $-4x^3 + 4x^2y + 3xy^2$

$-x(4x^2 - 4xy - 3y^2)$
$-x(2x - 3y)(2x + 1y)$
$-6x \quad +2x$ ✓

13) $28x^2 + 38xw - 6w^2$

$2(14x^2 + 19xw - 3w^2)$
$2(2x + 3w)(7x - 1w)$
$+21xw \quad -2xw$ ✓

14) $-6x^2 + 12x + 90$

$-6(x^2 - 2x - 15)$
$-6(x - 5)(x + 3)$
$-5x \quad +3x$ ✓

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6.4 Special Factoring Techniques Part 1

Factor the following binomials:

* Difference of Squares $x^2 - a^2 = (x-a)(x+a)$

a) $a^2 - 49$

$a^2 + 0a - 49$

$(a+7)(a-7)$

$+7a \quad -7a$ ✓

b) $y^2 - 36$

$y^2 - 6^2$

$(y+6)(y-6)$

c) $x^2 - 121$

$x^2 - 11^2$

$(x+11)(x-11)$

d) $25m^2 - 16$

$(5m)^2 - 4^2$

$(5m+4)(5m-4)$

e) $z^2 - r^2$

$(z+r)(z-r)$

f) $81y^2 - 36$

$(9y)^2 - 6^2$

$(9y+6)(9y-6)$

g) $9x^2 - 64$

$(3x)^2 - 8^2$

$(3x+8)(3x-8)$

h) $25p^2 - 49q^2$

$(5p)^2 - (7q)^2$

$(5p+7q)(5p-7q)$

i) $x^3 - 4x$

$x(x^2-4)$
↓
$x(x+2)(x-2)$

j) $36a^5 - 9a^3$

$9a^3(4a^2-1)$
↓
$9a^3(2a+1)(2a-1)$

k) $25x^4 - 100x^2$

$25x^2(x^2-4)$
↓
$25x^2(x+2)(x-2)$

l) $5a^2 - 405$

$5(a^2-81)$

$5(a+9)(a-9)$

6.4 Day 2 Mixed Factor practice

Step 1: Look for GCF
Step 2: a. If 3 terms - guess and ✓ b. If 2 terms: diff. of squares.

1) $x^2 - 5x + 6$

$(x-2)(x-3)$

2) $2x^2 - 8x - 10$
$2(x^2 - 4x - 5)$

$2(x-5)(x+1)$

3) $y^2 - y - 20$

$(y-5)(y+4)$

4) $16x^2 - 1$

$(4x+1)(4x-1)$

5) $x^2 - 7x - 8$

$(x-8)(x+1)$

6) $8x^2 - 2x - 45$

$(4x+9)(2x-5)$

7) $3a^2 + 24a + 45$
$3(a^2 + 8a + 15)$

$3(a+3)(a+5)$

8) $25w^2 - 20w + 4$

$(5w-2)(5w-2)$
or
$(5w-2)^2$

9) $x^2 + 6x - 27$

$(x+9)(x-3)$

10) $8x^2 - 63x - 81$

$(8x+9)(x-9)$

11) $49x^2 - 25$

$(7x-5)(7x+5)$

12) $2x^2 + 11x + 12$

$(2x+3)(x+4)$

13) $x^2 - 4$

$(x+2)(x-2)$

14) $7x^2 + 32x - 60$

$(7x-10)(x+6)$

15) $+x^3 + x^2 + 12x$
$-x(x^2 + x - 12)$

$-x(x+4)(x-3)$

16) $3x^3 - 3x$
$3x(x^2 - 1)$

$3x(x+1)(x-1)$

17) $n^2 - 8n - 20$

$(n-10)(n+2)$

18) $z^2 - 7z + 6$

$(z-6)(z-1)$

19) $a^2 + 10ab + 24b^2$

$(a+6b)(a+4b)$

20) $2x^2 + 5xy + 2y^2$

$(2x+y)(x+2y)$