Ch 4 Notes

Thursday, December 9, 2021 9:59 AM

4.1 Solving Systems of Linear Equations by Graphing

EXAMPLE 1 Determining Whether an Ordered Pair Is a Solution

Decide whether the ordered pair $(4, -3)$ is a solution of each equation.

x, y

(a) $x + 4y = -8$

$4 + 4(-3) = -8$
$4 - 12 = -8$
$-8 = -8$ ✓

yes $(4, -3)$ is a solution

(b) $2x + 5y = -7$

$2(4) + 5(-3) = -7$
$8 - 15 = -7$
$-7 = -7$ ✓

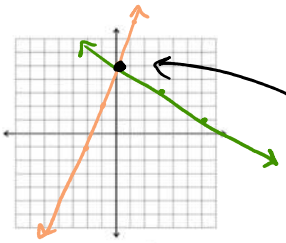
yes solution

EXAMPLE 2 Solving a System by Graphing

Solve the system of equations by graphing both equations on the same axes.

$y = \frac{-2}{3}x + 5$ $m = -\frac{2}{3}$ $b = 5$

$y = 3x + 5$ $m = 3$ $b = 5$



solution is the point of intersection

(0, 5)

Solve the system of equations by graphing.

2b)

$y = \frac{1}{2}x - 3$ $m = \frac{1}{2}$ $b = -3$

$y = \frac{1}{2}x + 1$ $m = \frac{1}{2}$ $b = 1$

parallel lines

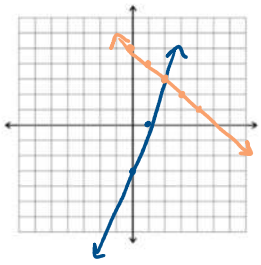
No Solution

2c)

$y = 3x - 3$

$y = -x + 5$

(2, 3)

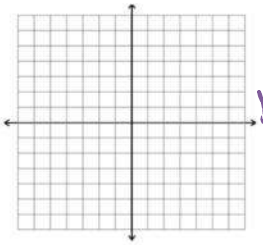


EXAMPLE 3 Solving Special Systems by Graphing

Solve each system by graphing.

(a) $y = -2x + 2$
$y = -2x + 8$

same slope = parallel



No Solution

(b) $y = \frac{-2}{5}x + 2$
$6x + 15y = 30$

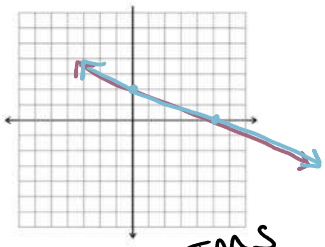
$m = -\frac{2}{5}$ $b = 2$

$6x + 15y = 30$
$-6x \quad -6x$

$\frac{15y}{15} = \frac{-6x}{15} + \frac{30}{15}$

$y = -\frac{6x}{15} + 2$

$y = -\frac{2x}{5} + 2$



Infinitely many Solutions

IMS

Three cases for solutions of systems

one solution (x, y) — No Solution — parallel lines

IMS — same line

Classification:

EXAMPLE 4 Identifying the Three Cases by Using Slopes

Describe each system without graphing. State the number of solutions.

(a) $3x + 2y = 6$
$-2y = 3x - 5$

(b) $2x - y = 4$
$x = \frac{y}{2} + 2$

(c) $x - 3y = 5$
$2x + y = 8$

4.2 Solving Systems of Linear Equations by Substitution

EXAMPLE 1 Using the Substitution Method

Solve the system by the substitution method.

$3x + 5y = 26$

$y = 2x$

$3x+5(2x)=26$

$3x+10x=26$

$\frac{13x}{13} = \frac{26}{13}$

$x=2$

$y=2(2)$

$y=4$

$(2,4)$

$3(2)+5y=26$

$6+5y=26$

$-6 \quad -6$

$\frac{5y}{5} = \frac{20}{5}$

$y=4$

EXAMPLE 2 Using the Substitution Method

Solve the system by the substitution method.

$2x + 5y = 7$

$x = -1 - y$

$2(-1-y)+5y=7$

$-2-2y+5y=7$

$-2+3y=7$

$+2 \quad +2$

$\frac{3y}{3} = \frac{9}{3}$

$y=3$

$(-4, 3)$

$x \quad y$

$x=-1-(3)$

$=-1+3$

$=-4$

EXAMPLE 3 Using the Substitution Method

Use substitution to solve the system.

$y = 4 - 2x$

$5x + 3y = 10$

EXAMPLE 4 Solving an Inconsistent System by Substitution

Use substitution to solve the system.

$x = 5 - 2y$

$2x + 4y = 6$

$x = 5 - 2y$

$2x + 4y = 6$

$2(5-2y) + 4y = 6$

$10 - 4y + 4y = 6$

$-10 \quad -10$

$-4y + 4y = -4$

variables cancelled

False

$0 = -4$

No Solution

$x + 2y = 5$

$-x \quad -x$

$\frac{2y}{2} = \frac{-x}{2} + \frac{5}{2}$

$y = -\frac{1}{2}x + \frac{5}{2}$

$2x + 4y = 6$

$-2x \quad -2x$

$\frac{4y}{4} = \frac{-2x}{4} + \frac{6}{4}$

$y = -\frac{1}{2}x + \frac{3}{2}$

same slope, diff y-int

EXAMPLE 5 Solving a System with Dependent Equations by Substitution

Solve the system by the substitution method.

$-y = 4 - 3x$

$-9x + 3y = -12$

$y = -4 + 3x$

$-9x + 3(-4 + 3x) = -12$

$-9x - 12 + 9x = -12$

variables cancelled

$-12 = -12$ True

IMS

(,)

EXAMPLE 6 Using the Substitution Method with Fractions as Coefficients

Solve the system by the substitution method.

$12x + y = 8$

$2x + 3y = -10$

$12x + y = 8$

$-12x \quad -12x$

$y = -12x + 8$

4.3 Solving Systems of Linear Equations by Elimination

Panera Bread is known for their delicious bagels, a popular choice for breakfast or a mid-afternoon snack. It's also a beloved item to bring to work. Peyton, Kelly, Carter, and Cecily take turns bringing food for their department each Friday.

1. Peyton's Panera Bread order is shown below. What is a *possible* cost for a single bagel and what is a *possible* cost for one tub of cream cheese? Explain your thinking.

Cinnamon Crunch Bagel
Individual · 420 Cal
Customize · Special Instructions

× Qty 8

Plain Cream Cheese Spread Tub
8 oz
110 Cal per 1 oz

Subtotal	$19.50
Taxes	$0.45
Add Promo Code	
Total	$19.95

$2.20(8) = 17.60
1.90(1) = 1.90
19.50
cream cheese

2. Kelly bought 12 Cinnamon Crunch bagels and 3 tubs of plain cream cheese. If Panera charges fairly, how much should Kelly's order cost? How do you know?

Tax 10.75
$31.05
$29.70
$34.50
$31.50

What do you think it means to charge fairly?

3. Assuming that Panera charges fairly, find the cost of a single bagel and a single tub of plain cream cheese or explain why this is not possible.

4. Carter loves Panera, so when it was his turn to bring food he ordered more Cinnamon Crunch bagels than Peyton and double the amount of cream cheese. His order is shown.
   a) How does Carter's subtotal compare to Peyton's?
   b) Can you figure out the cost of a single bagel and the cost of a single tub of cream cheese?

Cinnamon Crunch Bagel
Individual · 420 Cal
Customize · Special Instructions

× Qty 13

Plain Cream Cheese Spread Tub
8 oz
110 Cal per 1 oz
Special Instructions

× Qty 4

2.20(13) = 28.60
1.90(4) = 7.60
35.25
36.20

5. Cecily went to Panera and bought 7 bagels, 1 tub of cream cheese, and a salad. Her subtotal (before tax) was $23.71. Find the cost of her salad.

Subtotal	$34.53
Taxes	$0.91
Add Promo Code	
Total	$35.44

EXAMPLE 1 Using the Elimination Method

Use the elimination ~~method to~~ solve the system.

$1x + y = 5$
$+\ 1x - y = 3$
$2x = 8$
$\frac{2x}{2} = \frac{8}{2}$
$x = 4$

(4, 1)

$4 + y = 5$
$y = 1$

EXAMPLE 2 Using the Elimination Method

Solve the system.

$x + 4y = 5$
$4(2x - y = 1)$

(1, 1)

$1x + 4y = 5$
$+\ 8x - 4y = 4$
$9x = 9$
$\frac{9x}{9} = \frac{9}{9}$
$x = 1$

$(1) + 4y = 5$
$1 + 4y = 5$
$-1 \quad -1$
$4y = 4$
$y = 1$

EXAMPLE 3 Using the Elimination Method

Solve the system.

$-5(2x + 3y = -15)$
$2(5x + 2y = 1)$

$-10x - 15y = 75$
$+\ 10x + 4y = 2$
$-11y = 77$
$y = -7$

(3, -7)

$5x + 2(-7) = 1$
$5x - 14 = 1$
$5x = 15$

3b) $2x + y = 8$
$5x - 2y = -16$

(0, 8)

$2(0) + y = 8$
$0 + y = 8$
$y = 8$

$4x + 2y = 16$
$5x - 2y = -16$
$9x = 0$
$x = 0$

3c) $6x - 2y = -22$
$-3x + 4y = 17$

(-3, 2)

$9x = -27$
$x = -3$

$6(-3) - 2y = -22$
$-18 - 2y = -22$
$+18 \quad +18$
$-2y = -4$
$y = 2$

EXAMPLE 5 Solving Special Systems Using the Elimination Method

Solve each system by the elimination method.

a) $2(2x + 4y = 5)$
$4x + 8y = -9$

$-4x - 8y = -10$
$0 = -19$

variables cancel
False

No Solution

b. $3x - y = 4$
$-9x + 3y = -12$

4.4 Applications of Linear Systems

Example 1: A coffee shop sells teas for $4 each and coffees for $5 each. If the coffee shop sold 9 drinks for a total of $40, how many of each type of drink were sold?

x = # tea y = # coffee

a) Write two equations to model this situation.

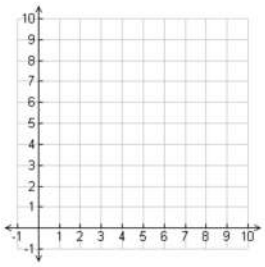
$-4(x + y = 9)$
$4x + 5y = 40$

$-4x - 4y = -36$
$+ 4x + 5y = 40$
$y = 4$

b) Solve the system by graphing.

$x + (4) = 9$
$x + 4 = 9$
$-4 \quad -4$
$x = 5$

Teas = 5
Coffees = 4



EXAMPLE 2 Solving a Problem about Quantities and Costs

For a production of the musical *Wicked* at the Ford Center in Chicago, main floor tickets cost $148, while the best balcony tickets cost $65. Suppose that the members of a club spent a total of $2614 for 30 tickets to *Wicked*. How many tickets of each kind did they buy? (*Source:* www.ticketmaster.com)

x = # main floor
y = # balcony

$-65(x + y = 30)$
$148x + 65y = 2614$

$-65x - 65y = -1950$
$148x + 65y = 2614$

$\frac{83x}{83} = \frac{6664}{83}$

$x = 8$

$8 + y = 30$
$y = 22$

8 main floor, 22 balcony

Example 3:
Jonathan, a second grader, counted the money in his piggy bank. He had only quarters and dimes. When he added up his money, he had 39 coins worth a total of $7.50. How many coins of each kind did he have?

q = # quarters
d = # dimes

$-.10(q + d = 39)$
$.25q + .10d = 7.50$

$-.1q - .1d = -3.9$
$.25q + .1d = 7.50$

$\frac{.15q}{.15} = \frac{3.6}{.15}$

$q = 24$

$24 + d = 39$
$d = 15$

24 quarters, 15 dimes

Example 4:
Lindsey and Gretchen work at two different hair salons and pay different amounts for their station. Lindsey pays $140 for rent, and $10 per customer that she works on that month. Gretchen only pays $100 for rent, but has to pay $18 per customer. How many customers would it take for them to pay the same amount?

Lindsey $140 rent $10/customer
Gretchen $100 rent $18/customer

x = # customers
y = Total cost

$y = 140 + 10x$
$y = 100 + 18x$

$100 + 18x = 140 + 10x$
$-10x \quad -10x$
$100 + 8x = 140$
$-100 \quad -100$
$\frac{8x}{8} = \frac{40}{8}$
$x = 5$

$y = 140 + 10(5)$
$= 140 + 50$
$= 190$

5 customers they pay $190

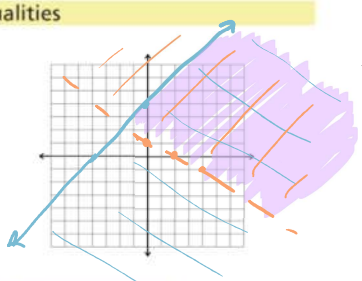
4.5 Solving Systems of Linear Inequalities Date:

Inequality symbols		Shading rules	
$>$ $<$	dashed line	$>$, $\geq$	above
$\geq$ $\leq$	solid line	$<$, $\leq$	below

EXAMPLE 1 Solving a System of Linear Inequalities

Graph the solution set of the system.

$$\begin{cases} y > -\frac{1}{2}x + 1 \\ y \leq x + 4 \end{cases}$$



overlap is solution

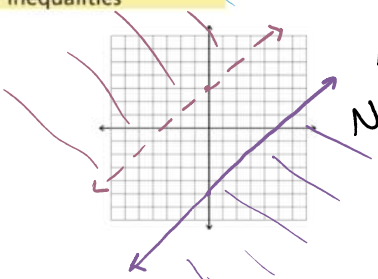
EXAMPLE 2 Solving a System of Linear Inequalities

EXAMPLE 2 Solving a System of Linear Inequalities

Graph the solution set of the system.

$$\begin{cases} y > x + 3 \\ -x + y \leq -5 \end{cases}$$

↳ $y \leq x - 5$

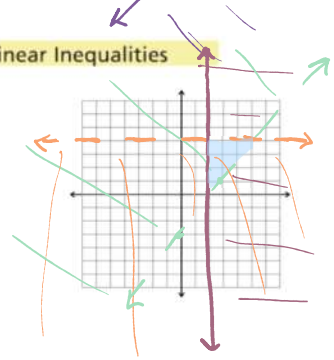


No overlap, No Solution

EXAMPLE 3 Solving a System of Three Linear Inequalities

Graph the solution set of the system.

$$\begin{cases} y > \frac{4}{3}x - 3 \\ x \geq 2 \\ y < 4 \end{cases}$$



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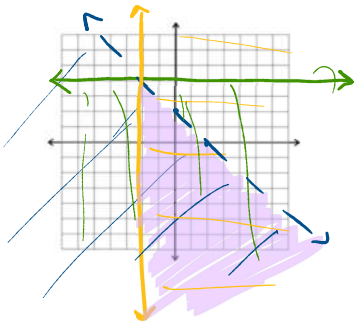
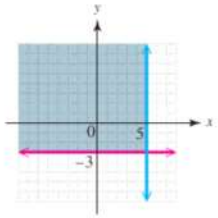
Example 4:

$$\begin{cases} x + y < 2 \\ x \geq -2 \\ y \leq 4 \end{cases}$$

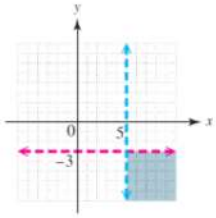


Example 5

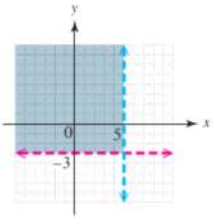
Write the equations for the system graphed.



$$\begin{cases} x \leq 5 \\ y \geq -3 \end{cases}$$



$$\begin{cases} x > 5 \\ y < -3 \end{cases}$$



$$\begin{cases} x < 5 \\ y > -3 \end{cases}$$