SENSE-MAKING Determine whether $\triangle MNO \cong \triangle QRS$. Explain using rigid motions.

9. $M(0, -1)$, $N(-1, -4)$, $O(-4, -3)$, $Q(3, -3)$, $R(4, -4)$, $S(3, 3)$

SOLUTION:

Use the Distance Formula to find the lengths of $\overline{MN}, \overline{NO}$ and $\overline{OM}$.

$\overline{MN}$ has end points $M(0, -1)$ and $N(-1, -4)$.

$$MN = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Substitute.

$$\begin{aligned} MN &= \sqrt{(-1-0)^2 + (-4-(-1))^2} && \text{Substitute.} \\ &= \sqrt{(-1)^2 + (-3)^2} && \text{Subtraction.} \\ &= \sqrt{1+9} && \text{Square terms.} \\ &= \sqrt{10} && \text{Addition.} \end{aligned}$$

$\overline{NO}$ has end points $N(-1, -4)$ and $O(-4, -3)$.

$$NO = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Substitute.

$$\begin{aligned} NO &= \sqrt{(-4-(-1))^2 + (-3-(-4))^2} && \text{Substitute.} \\ &= \sqrt{(-3)^2 + (1)^2} && \text{Subtraction.} \\ &= \sqrt{9+1} && \text{Square terms.} \\ &= \sqrt{10} && \text{Addition.} \end{aligned}$$

$\overline{OM}$ has end points $O(-4, -3)$ and $M(0, -1)$.

$$OM = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Substitute.

$$\begin{aligned} OM &= \sqrt{(0-(-4))^2 + (-1-(-3))^2} && \text{Substitute.} \\ &= \sqrt{(4)^2 + (2)^2} && \text{Subtraction.} \\ &= \sqrt{16+4} && \text{Square terms.} \\ &= \sqrt{20} && \text{Addition.} \end{aligned}$$

Similarly, find the lengths of $\overline{QR}, \overline{RS}$ and $\overline{SQ}$.

$\overline{QR}$ has end points $Q(3,-3)$ and $R(4, -4)$.

$$QR = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Substitute.

$$\begin{aligned} QR &= \sqrt{(4-3)^2 + (-4-(-3))^2} && \text{Substitute.} \\ &= \sqrt{(-1)^2 + (-1)^2} && \text{Subtraction.} \\ &= \sqrt{1+1} && \text{Square terms.} \\ &= \sqrt{2} && \text{Addition.} \end{aligned}$$

$\overline{RS}$ has end points $R(4, -4)$ and $S(3, 3)$.

$$RS = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Substitute.

$$\begin{aligned} RS &= \sqrt{(3-4)^2 + (3-(-4))^2} && \text{Substitute.} \\ &= \sqrt{(-1)^2 + (7)^2} && \text{Subtraction.} \\ &= \sqrt{1+49} && \text{Square terms.} \\ &= \sqrt{50} && \text{Addition.} \end{aligned}$$

$\overline{SQ}$ has end points $S(3, 3)$ and $Q(3, -3)$.

$$SQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Substitute.

$$\begin{aligned} SQ &= \sqrt{(3-3)^2 + (-3-3)^2} && \text{Substitute.} \\ &= \sqrt{(0)^2 + (-6)^2} && \text{Subtraction.} \\ &= \sqrt{0+36} && \text{Square terms.} \\ &= 6 && \text{Addition.} \end{aligned}$$

There are not rigid motions that map one triangle onto the other. The corresponding sides are not congruent, so the triangles are not congruent.

10. $M(0, -3)$, $N(1, 4)$, $O(3, 1)$, $Q(4, -1)$, $R(6, 1)$, $S(9, -1)$

SOLUTION:

Use the Distance Formula to find the lengths of $\overline{MN}, \overline{NO}$ and $\overline{OM}$.

$\overline{MN}$ has end points $M(0, -3)$ and $N(1, 4)$.

$$MN = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Substitute.

$$MN = \sqrt{(1-0)^2 + (4-(-3))^2} \quad \text{Substitute.}$$
$$= \sqrt{(1)^2 + (7)^2} \quad \text{Subtraction.}$$
$$= \sqrt{1+49} \quad \text{Square terms.}$$
$$= \sqrt{50} \quad \text{Addition.}$$

$\overline{NO}$ has end points $N(1, 4)$ and $O(3, 1)$.

$$NO = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Substitute.

$$NO = \sqrt{(3-1)^2 + (1-4)^2} \quad \text{Substitute.}$$
$$= \sqrt{(2)^2 + (-3)^2} \quad \text{Subtraction.}$$
$$= \sqrt{4+9} \quad \text{Square terms.}$$
$$= \sqrt{13} \quad \text{Addition.}$$

$\overline{OM}$ has end points $O(3, 1)$ and $M(0, –3)$.

$$OM = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Substitute.

$$OM = \sqrt{(0-3)^2 + (-3-1)^2} \quad \text{Substitute.}$$
$$= \sqrt{(-3)^2 + (-4)^2} \quad \text{Subtraction.}$$
$$= \sqrt{9+16} \quad \text{Square terms.}$$
$$= \sqrt{25} \quad \text{Addition.}$$
$$= 5 \quad \text{Simplify.}$$

Similarly, find the lengths of $\overline{QR}, \overline{RS}$ and $\overline{SQ}$.

$\overline{QR}$ has end points $Q(4,–1)$ and $R(6, 1)$.

$$QR = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Substitute.

$$QR = \sqrt{(6-4)^2 + (1-(-1))^2} \quad \text{Substitute.}$$
$$= \sqrt{(2)^2 + (2)^2} \quad \text{Subtraction.}$$
$$= \sqrt{4+4} \quad \text{Square terms.}$$
$$= \sqrt{8} \quad \text{Addition.}$$

$\overline{RS}$ has end points $R(6, 1)$ and $S(9, –1)$.

$$RS = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Substitute.

$$RS = \sqrt{(9-6)^2 + (-1-1)^2} \quad \text{Substitute.}$$
$$= \sqrt{(3)^2 + (-2)^2} \quad \text{Subtraction.}$$
$$= \sqrt{9+4} \quad \text{Square terms.}$$
$$= \sqrt{13} \quad \text{Addition.}$$

$\overline{SQ}$ has end points $S(9, –1)$ and $Q(4, –1)$.

$$SQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Substitute.

$$SQ = \sqrt{(4-9)^2 + (-1-(-1))^2} \quad \text{Substitute.}$$
$$= \sqrt{(-5)^2 + (0)^2} \quad \text{Subtraction.}$$
$$= \sqrt{25} \quad \text{Square terms.}$$
$$= 5 \quad \text{Simplify.}$$

There are not rigid motions that map one triangle onto the other. The corresponding sides are not congruent, so the triangles are not congruent.

11. $M(4, 7)$, $N(5, 4)$, $O(2, 3)$, $Q(2, 5)$, $R(3, 2)$, $S(0, 1)$

SOLUTION:

Use the Distance Formula to find the lengths of $\overline{MN}, \overline{NO}$ and $\overline{OM}$.

$\overline{MN}$ has end points $M(4, 7)$ and $N(5, 4)$.

$$MN = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Substitute.

$$MN = \sqrt{(5-4)^2 + (4-7)^2} \quad \text{Substitute.}$$
$$= \sqrt{(1)^2 + (-3)^2} \quad \text{Subtraction.}$$
$$= \sqrt{1+9} \quad \text{Square terms.}$$
$$= \sqrt{10} \quad \text{Addition.}$$

$\overline{NO}$ has end points $N(5, 4)$ and $O(2, 3)$.

$$NO = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Substitute.

$$\begin{aligned} NO &= \sqrt{(2-5)^2+(3-4)^2} && \text{Substitute.} \\ &= \sqrt{(-3)^2+(-1)^2} && \text{Subtraction.} \\ &= \sqrt{9+1} && \text{Square terms.} \\ &= \sqrt{10} && \text{Addition.} \end{aligned}$$

$\overline{OM}$ has end points $O(2, 3)$ and $M(4,7)$.

$$OM = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

Substitute.

$$\begin{aligned} OM &= \sqrt{(4-2)^2+(7-3)^2} && \text{Substitute.} \\ &= \sqrt{(2)^2+(4)^2} && \text{Subtraction.} \\ &= \sqrt{4+16} && \text{Square terms.} \\ &= \sqrt{20} && \text{Addition.} \end{aligned}$$

Similarly, find the lengths of $\overline{QR}, \overline{RS}$ and $\overline{SQ}$.

$\overline{QR}$ has end points $Q(2, 5)$ and $R(3, 2)$.

$$QR = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

Substitute.

$$\begin{aligned} QR &= \sqrt{(3-2)^2+(2-5)^2} && \text{Substitute.} \\ &= \sqrt{(1)^2+(-3)^2} && \text{Subtraction.} \\ &= \sqrt{1+9} && \text{Square terms.} \\ &= \sqrt{10} && \text{Addition.} \end{aligned}$$

$\overline{RS}$ has end points $R(3, 2)$ and $S(0, 1)$.

$$RS = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

Substitute.

$$\begin{aligned} RS &= \sqrt{(0-3)^2+(1-2)^2} && \text{Substitute.} \\ &= \sqrt{(-3)^2+(-1)^2} && \text{Subtraction.} \\ &= \sqrt{9+1} && \text{Square terms.} \\ &= \sqrt{10} && \text{Addition.} \end{aligned}$$

$\overline{SQ}$ has end points $S(0, 1)$ and $Q(2, 5)$.

$$SQ = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

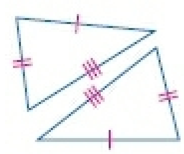
Substitute.

$$\begin{aligned} SQ &= \sqrt{(2-0)^2+(5-1)^2} && \text{Substitute.} \\ &= \sqrt{(2)^2+(4)^2} && \text{Subtraction.} \\ &= \sqrt{4+16} && \text{Square terms.} \\ &= \sqrt{20} && \text{Addition.} \end{aligned}$$

So, $\overline{MN} \cong \overline{QR}, \overline{NO} \cong \overline{RS}$ and $\overline{OM} \cong \overline{SQ}$.

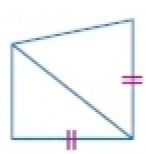
A translation maps one triangle onto the other. Each pair of corresponding sides has the same measure, so they are congruent. $\triangle MNO \cong \triangle QRS$ by SSS.

CONSTRUCT ARGUMENTS Determine which postulate can be used to prove that the triangles are congruent. If it is not possible to prove congruence, write *not possible*. If it is possible, describe the rigid motions that map one triangle onto the other.

16. 

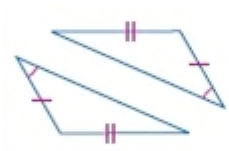
SOLUTION:

The corresponding sides are congruent; SSS. And, a combination of a rotation and a reflection map one triangle onto the other.

17. 

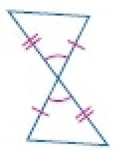
SOLUTION:

The triangles have two corresponding sides congruent but no information is given about the included angle or the third pair of sides; not possible.

18. 

SOLUTION:

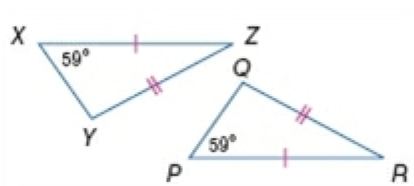
Information is given about two pairs of corresponding sides and one pair of corresponding angles but the congruent angles are not in the interior of the congruent sides; not possible.

19. 

SOLUTION:

The triangles have two pairs of corresponding sides congruent and the interior angles are congruent; SAS. And, a rotation maps one triangle onto the other.

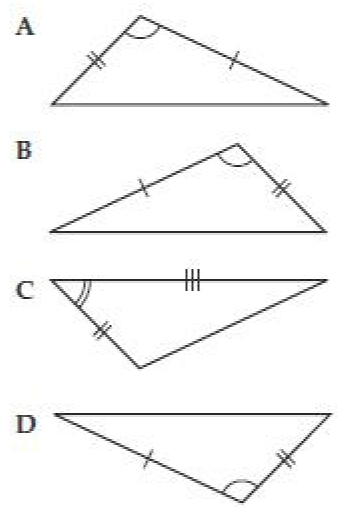
31. **ERROR ANALYSIS** Bonnie says that $\Delta PQR \cong \Delta XYZ$ by SAS. Shada disagrees. She says that there is not enough information to prove that the two triangles are congruent. Is either of them correct? Explain.



SOLUTION:

Shada is correct. For SAS, the angle must be the included angle and here it is not included.

38. Select the triangles that can be proven congruent by SSS or SAS.



SOLUTION:

Considering the given triangles, choices A, B, and D can be proven congruent by SSS (Side-Side-Side) or SAS (Side-Angle-Side).