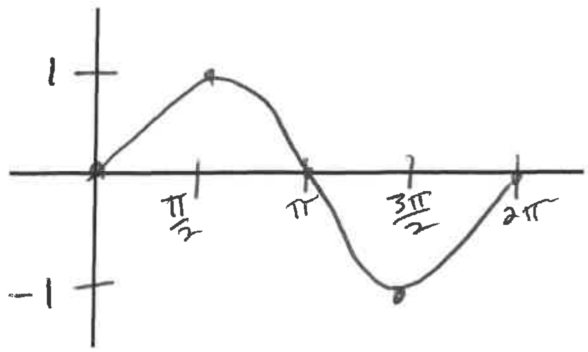
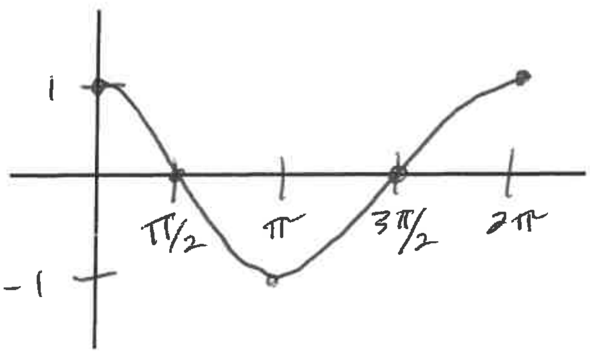
4.5 Graphs of Sin and Cos Functions

$y = \sin x$



x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
y	0	1	0	-1	0

$y = \cos x$



x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
y	1	0	-1	0	1

$y = A \sin(Bx - C) + D$ $\qquad$ $y = A \cos(Bx - C) + D$

- A = amplitude: distance from the midline to the maximum or minimum (vertical stretch/comp)
- B = horizontal stretch or compression: change in the period of the function
  - Period $= 2\pi/B$

$Bx - C = 0$

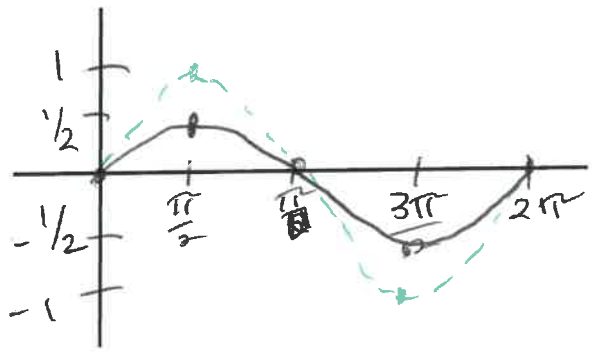
- C = horizontal shift (translation): phase shift $= C/B$
- D = vertical shift (translation): move the midline up or down

Always list
A –
P –
PS – VS –

→ Don't have to list reflection.

1) *Determine the amplitude of y=1/2 sin x. Then graph both y=sin x and y=1/2 sin x for 0 ≤ x ≤ 2π.*

**draw original graph dashed*



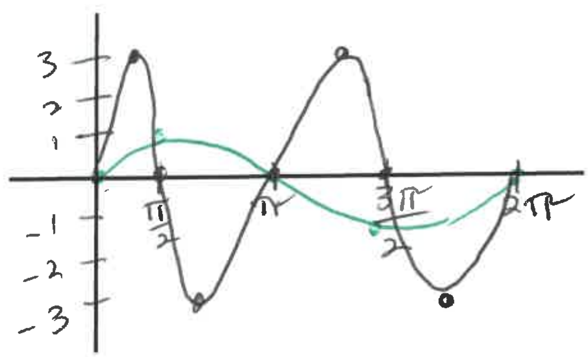
$A = \frac{1}{2}$

$P = 2\pi$

$PS = 0$

$VS = 0$

2) *Determine the amplitude and period of y=3sin 2x. Then graph the functions for 0 ≤ x ≤ 2π.*



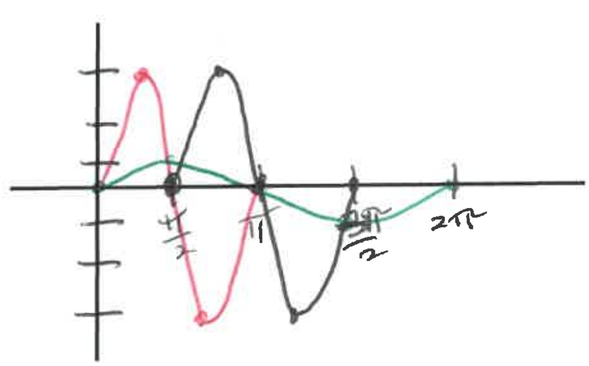
$A = 3$

$P = 2\pi/2 = \pi$

$PS = \begin{matrix} 2x = 0 \\ x = 0 \end{matrix}$

$VS = 0$

3) *Determine the amplitude, period and phase shift of y = 3 sin(2x - π). Then graph one period of the function.*



$A = 3$

$P = \frac{2\pi}{2} = \pi$

$PS = 2x - \pi = 0$

$\quad +\pi \quad +\pi$

$2x = \pi$

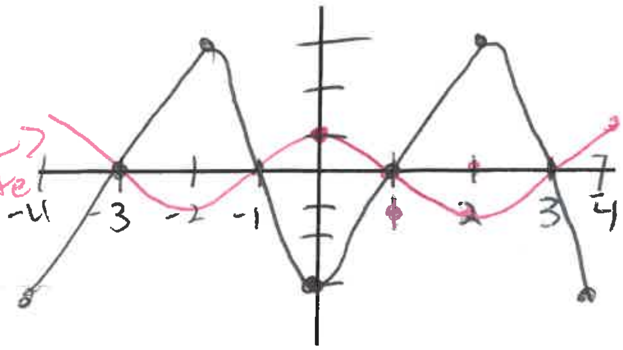
$x = \pi$ rig

$VS = 0$

→ Graph Amp. & period 1st

4) *Determine the amplitude and period of* $y = -3\cos\frac{\pi}{2}x$ *Then graph the function for* $-4 \le x \le 4$.

not accurate



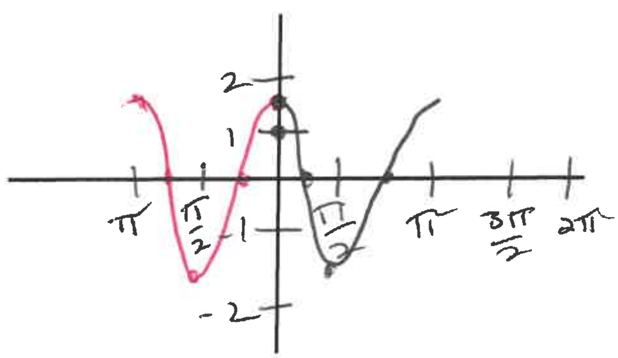
$A = |-3| = 3$

$P = 2\pi / \frac{\pi}{2} = 4$

$PS = 0$

$VS = 0$

5) *Determine the amplitude, period and phase shift of* $y = \frac{3}{2}\cos(2x + \pi)$. *Then graph one period of the function.*



$A = \frac{3}{2}$

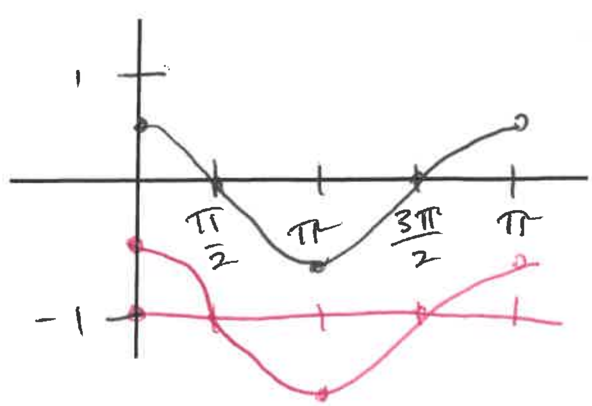
$P = \frac{2\pi}{2} = \pi$

$PS = \leftarrow \pi$

$VS = 0$

6) *Graph one period of the function* $y = \frac{1}{2}\cos x - 1$.

$A = \frac{1}{2}$

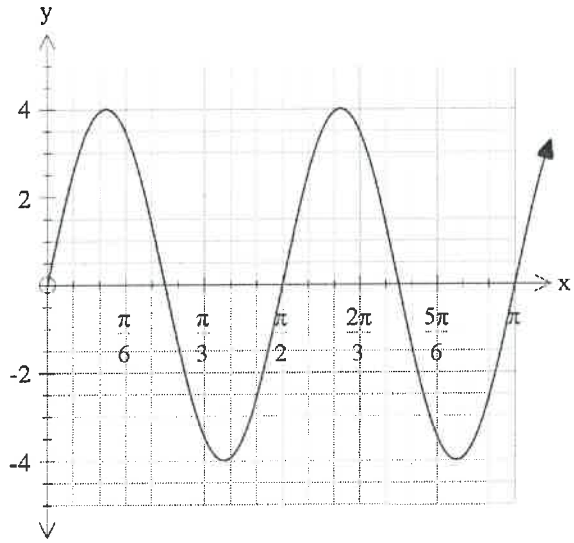


$P = 2\pi$

$PS = 0$

$VS = -1$

7) *Write an equation for the curve:*



$$y = 4\sin(4x)$$

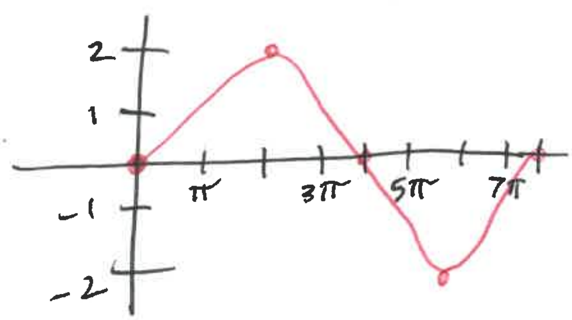
$y = 2\sin\frac{1}{4}x$

$A = 2$

$P = \frac{2\pi}{\frac{1}{4}} = 8\pi$

$PS = 0$

$VS = 0$



x	y	New x
0	0	0
$\frac{\pi}{2}$	1	$\frac{\pi}{2}\cdot 4 = 2\pi$
π	0	$\pi\cdot 4 = 4\pi$
$\frac{3\pi}{2}$	-1	$\cdot 4 = 6\pi$
2π	0	$\cdot 4 = 8\pi$

$y = -2\sin(2\pi x + 4\pi)$

$A = 2$

$P = \frac{2\pi}{2\pi} = 1$

$PS \Rightarrow 2\pi x + 4\pi = 0$

$x = -2$

$VS = 0$

x	y (A)	New x (Period)	PS
0	0	$\div 2\pi = 0$	$-2 = -2$
$\pi/2$	2	$\div 2\pi = 1/4$	$-2 = -7/4$
π	0	$\div 2\pi = 1/2$	$-2 = -3/2$
$3\pi/2$	-2	$\div 2\pi = 3/4$	$-2 = -5/4$
2π	0	$\div 2\pi = 1$	$-2 = -1$

4.6 Graphs of Other Trig Functions

Tangent:

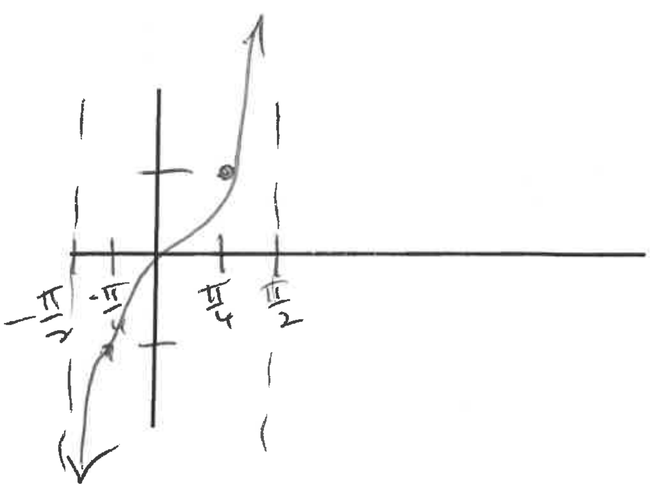
* *slope, sin/cos, reciprocal of cotangent is 0 at 0π, π, 2π, 3π etc (Integer multiples of π)*
* *undefined at $\frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$ etc (at odd integer multiples of $\frac{\pi}{2}$). These are vertical asymptotes.*

$y = \tan x$

$-\frac{\pi}{2}$ $-\frac{\pi}{4}$ 0 $\frac{\pi}{4}$ $\frac{\pi}{2}$

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\pi/3$	$\pi/2$
y	0	$\sqrt{3}/3$ 0.6	1	$\sqrt{3}$ 1.7	und

und -1 0 1 und

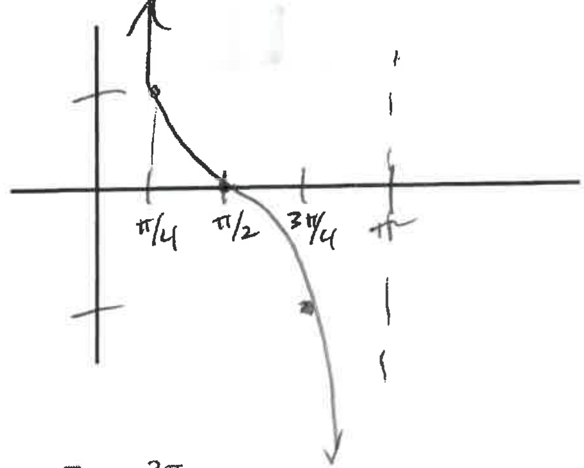


$y = A \tan(Bx - C) + D$

* A = Changes the height of $\frac{\pi}{4}$ & $-\frac{\pi}{4}$
* B = Change in the period: $P = \frac{\pi}{B}$
* C = Phase shift: $P.S. = \frac{C}{B}$ (left or right)
* D = moves midline up or down

$y = \cot x$ (has asymptotes where tan has zeros)

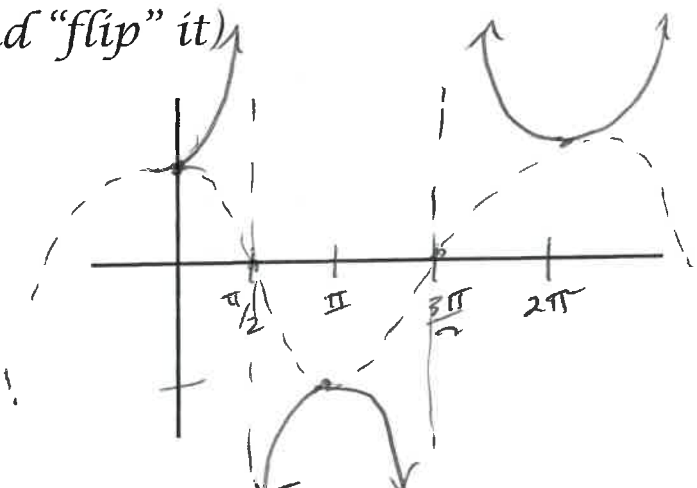
x	y
0	und
$\pi/4$	1
$\pi/2$	0
$3\pi/4$	-1
π	und



$y = A \cot(Bx - C) + D$

* A = Changes the height of $\frac{\pi}{4}$ & $\frac{3\pi}{4}$
* B = Change in the period: $P = \frac{\pi}{B}$
* C = Phase shift: $P.S. = \frac{C}{B}$ (left or right)
* D = moves midline up or down

$y = \sec x$ (draw cos and "flip" it)

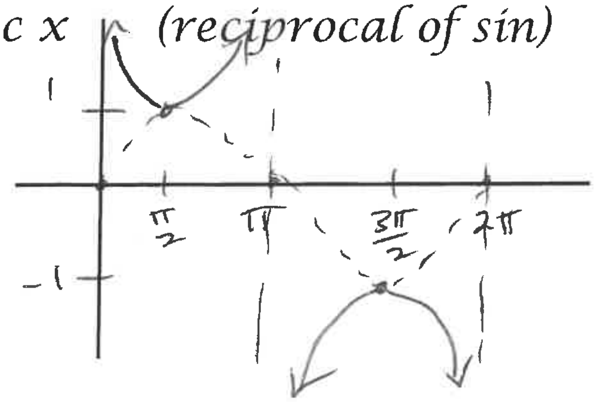


so draw cos and flip!

$y = A \sec(Bx - C) + D$

* A = Location of the vertices of the parabola
* B = Change in the period: $P = \frac{2\pi}{B}$
* C = Phase shift: $P.S. = \frac{C}{B}$ (left or right)
* D = moves midline up or down

$y = \csc x$ (reciprocal of sin)



draw sin first.

$y = A \csc (Bx - C) + D$

* A = Location of the vertices of the parabola
* B = Change in the period: $P = \frac{2\pi}{B}$
* C = Phase shift: $P.S. = \frac{C}{B}$ (left or right)
* D = moves midline up or down

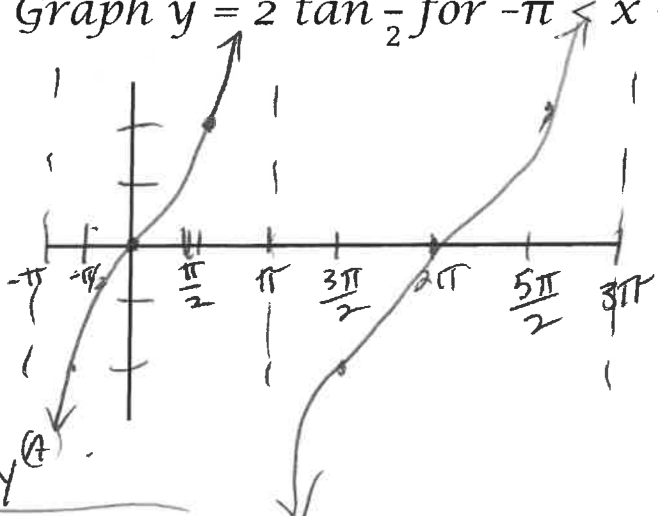
1) Graph $y = 2 \tan \frac{x}{2}$ for $-\pi \leq x < 3\pi$

$A = 2$

$P = \frac{\pi}{\frac{1}{2}} = 2\pi$

$PS = 0$

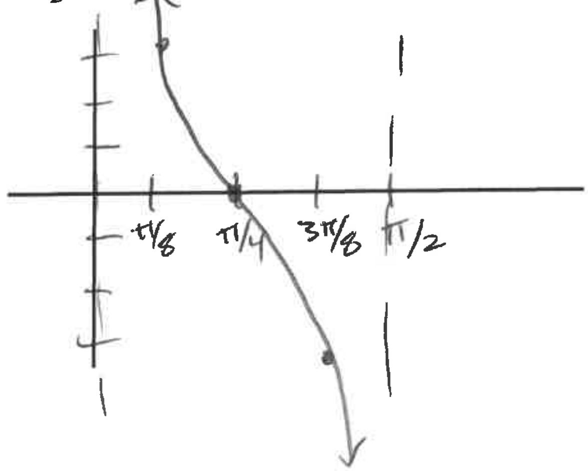
$VS = 0$



	x	y (A)
$-\pi/2 \cdot 2$	$-\pi$	und
$-\pi/4 \cdot 2$	$-\frac{\pi}{2}$	-2
$0 \cdot 2$	0	0
$\pi/4 \cdot 2$	$\frac{\pi}{2}$	2
$\pi/2 \cdot 2$	π	und

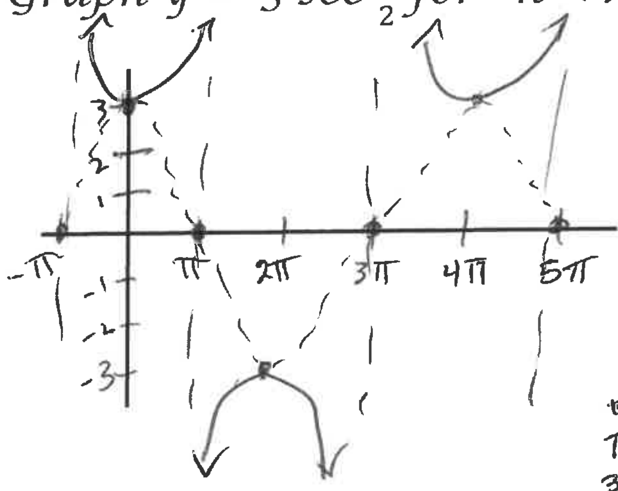
$A = 3$
$P = \frac{2\pi}{2} = \pi$

2) *Graph* $y = 3 \cot 2x$



x	y
$0 \div 2 = 0$	und
$\pi/4 \div 2 = \pi/8$	3
$\pi/2 \div 2 = \pi/4$	0
$3\pi/4 \div 2 = 3\pi/8$	-3
$\pi \div 2 = \pi/2$	und

3) *Graph* $y = -3 \sec \frac{x}{2}$ *for* $-\pi < x < 5\pi$



Graph as cos 1st

$A = -3$

$P = \frac{2\pi}{\frac{1}{2}} = 4\pi$

	x	y
$0 \cdot 2$	0	3
$\pi/2 \cdot 2$	π	0
$\pi \cdot 2$	2π	-3
$3\pi/2 \cdot 2$	3π	0
$2\pi \cdot 2$	4π	3

4.7 Inverse Trig Functions

Sin⁻¹: inverse of the restricted sine function y=sin x, $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$.

$y = \sin^{-1} x$ *means* $\sin y = x$

* *This does not mean* $\frac{1}{\sin x}$

The horizontal line test tells us whether the function has an inverse or not.

- One way to graph $y = \sin^{-1} x$ is to take points on the graph of the restricted sine function and reverse the order of the coordinates.
- Another way to graph $y = \sin^{-1} x$ is to reflect the graph of the restricted sine function about the line $y = x$.

For inverse trig functions, there is a restriction on the range

$\sin^{-1}(\sin x)$ $(-\frac{\pi}{2} \le \sin \le \frac{\pi}{2})$ $\quad$ $(-\frac{\pi}{2} \le \csc \le \frac{\pi}{2})$

$\sin(\sin^{-1}x)$ $[-1, 1]$

$\cos^{-1}(\cos x)$ $(0 \le \cos \le \pi)$ $\quad$ $(0 \le \sec \le \pi)$

$\cos(\cos^{-1}x)$ $[-1, 1]$

$\tan^{-1}(\tan x)$ $(-\frac{\pi}{2} \le \tan \le \frac{\pi}{2})$ $\quad$ $(0 \le \cot \le \pi)$

$\tan(\tan^{-1}x) = \mathbb{R}$

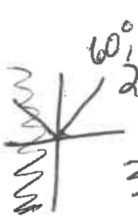
* must pass the horizontal line test

* choose phase surrounding the origin $-\frac{\pi}{2} \to \frac{\pi}{2}$ for sin

* $0 \to \pi$ for cos

1) Find the exact value of $\sin^{-1}(\frac{\sqrt{3}}{2})$

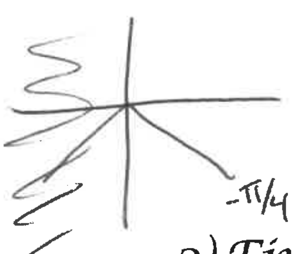
1.) Let $\theta = \sin^{-1} x$ $\quad$ $\theta = \sin^{-1}\frac{\sqrt{3}}{2}$



2.) Rewrite $\theta = \sin^{-1} x$ as $\sin\theta = x$, where $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$. $\sin\theta = \frac{\sqrt{3}}{2}$

3.) $\sin\theta = \frac{\sqrt{3}}{2}$ is $\frac{\pi}{3}$. Thus, $\theta = \frac{\pi}{3}$. Because θ, in step 1, represents $\sin^{-1}\frac{\sqrt{3}}{2}$, we conclude that $\sin^{-1}\frac{\sqrt{3}}{2} = \boxed{\frac{\pi}{3}}$

2) Find the exact value of $\sin^{-1}(-\frac{\sqrt{2}}{2})$



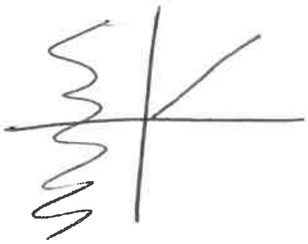
$-\pi/4$

3) Find the exact value of $\cos^{-1}\left(-\frac{1}{2}\right)$

$\frac{2\pi}{3}$

$* \ -\frac{\pi}{2} \to \frac{\pi}{2}$

4) *Find the exact value of* $\tan^{-1}(1)$



$\pi/4$

Find the exact values if possible

5) $\cos(\cos^{-1} 0.7)$

cancel out

$\boxed{0.7}$

$0 \le \cos \le \pi$

0.7 is between

0 & 3.14

6) $\sin(\sin^{-1} \pi)$

cancel out

$-\pi/2 \le \sin \le \pi/2$

π is not between

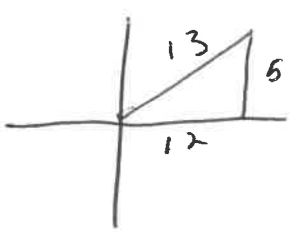
$-\pi/2$ & $\pi/2$

$\therefore \sin^{-1}(0) = 0\pi \Rightarrow$ The $\angle$ in $[-\pi/2, \pi/2)$ whose

$\sin = 0$ is 0π.

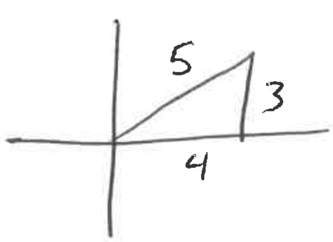
7) $\cos(\tan^{-1} \frac{5}{12})$

$-\pi/2 \le \tan \le \frac{\pi}{2}$



$\therefore \cos(\tan^{-1} \frac{5}{12}) = \frac{12}{13}$

8) $\sin(\tan^{-1} \frac{3}{4})$



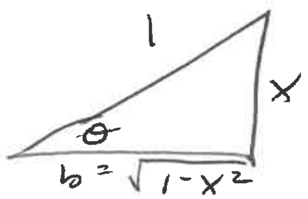
$\sin(\tan^{-1} \frac{3}{4}) = \frac{3}{5}$

4.7-3

9) *If $0 < x \le 1$, write $\cos(\sin^{-1} x)$ as an algebraic expression.*

$-\pi/2 \le x \le \pi/2$

$\sin\theta = \frac{x}{1}$



$\cos\theta = \frac{\sqrt{1-x^2}}{1} = \boxed{\sqrt{1-x^2}}$

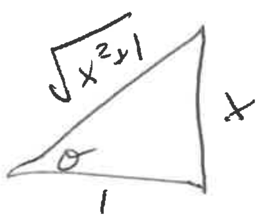
$x^2 + b^2 = 1^2$

$b^2 = 1 - x^2$

10) *If $x > 0$, write $\sec(\tan^{-1} x)$ as an algebraic expression.*

$-\pi/2$ to $\pi/2$

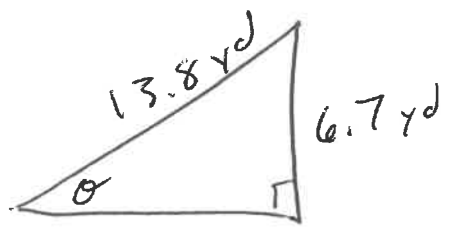
$\tan\theta = \frac{x}{1}$



$\sec = \frac{\sqrt{x^2+1}}{1} = \boxed{\sqrt{x^2+1}}$

4.8 Applications of Trig Functions

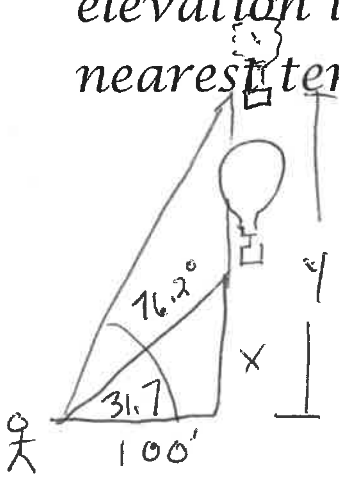
1) A guy wire is 13.8 yards long and is attached from the ground to a pole 6.7 yards above the ground. Find the angle, to the nearest tenth of a degree that the wire makes with the ground.



$\sin^{-1}\frac{6.7}{13.8}$

$\boxed{29.05°}$

2) You are taking your first hot-air balloon ride. Your friend is standing on level ground, 100 ft away from your point of launch, making a video of the terrified look on your rapidly ascending face. How rapidly? At one instant, the angle of elevation from the video camera to your face is 31.7°. One minute later, the angle of elevation is 76.2°. How far did you travel to the nearest tenth of a foot, during that minute?

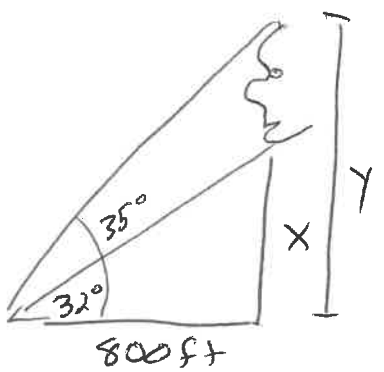


$\tan 31.7 = \frac{x}{100}$ $\quad$ $\tan 76.2 = \frac{y}{100}$

$x = 61.8$ ft $\quad$ $y = 407.1$ ft

$$\begin{array}{r} 407.1 \\ -\ 61.8 \\ \hline 345.3 \text{ ft/min} \end{array}$$

3) *You are standing on level ground 800 ft from mt. Rushmore looking at the sculpture of Abraham Lincoln's face. The angle of elevation to the bottom of the sculpture is 32° and the angle of elevation to the top is 35°. Find the height of the sculpture of Lincoln's face to the nearest tenth of a foot.*



$\tan 32 = \frac{x}{800}$ $\tan 35 = \frac{y}{800}$

$x = 499.9$ $y = 560.2$

$$\begin{array}{r} 560.2 \\ -\ 499.9 \\ \hline \boxed{60.3\text{ ft}} \end{array}$$

* Bearing is used to specify the location of one part relative to another.
- acute ∠ measure (degrees) between Ray & N-S line.

4) *Find the bearing from O to B*

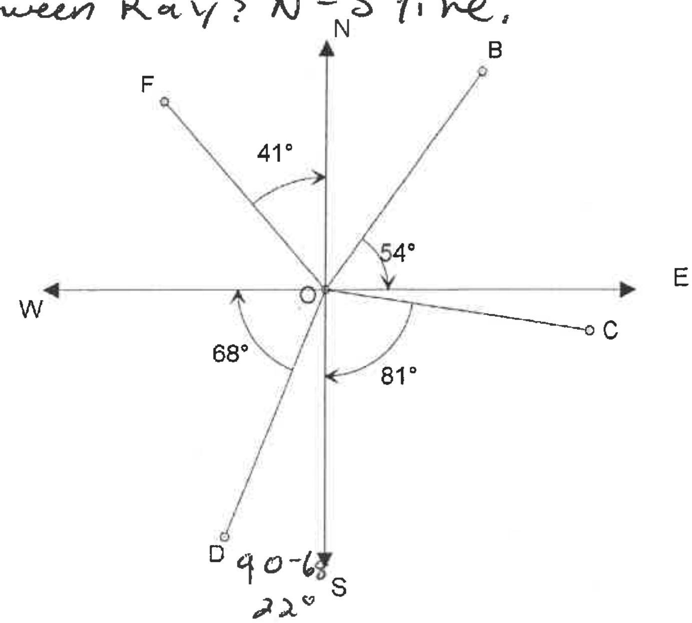
N 36° E

5) *Find the bearing from O to F*

N 41° W

6) *Find the bearing from O to D*

S 22° W



7) You leave the entrance to a system of hiking trails and hike 2.3 miles on a bearing of S 31° W. Then the trail turns 90° clockwise and you hike 3.5 miles on a bearing of N 59° W.

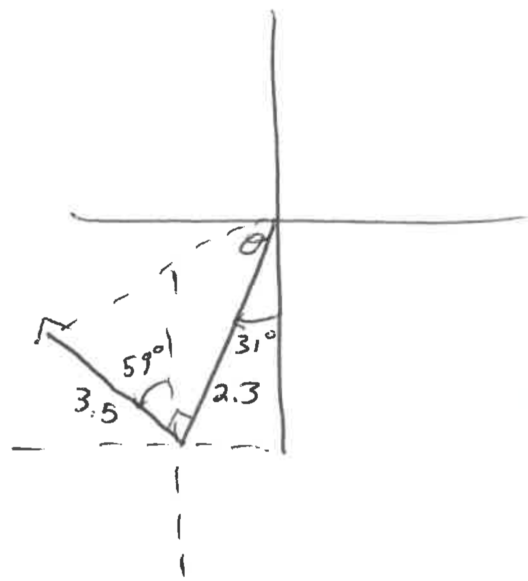
a) How far are you from the entrance?

b) What is your bearing from the entrance?

a.) $a^2+b^2=c^2$

$2.3^2+3.5^2=c^2$

$c=4.19$ mi



b.) $\tan\theta=\frac{3.5}{2.3}$

$\theta=\tan^{-1}\frac{3.5}{2.3}$

$\theta=56.69°$

+31

$\boxed{S\ 87.69°\ W}$

D axis (used for distance)

Simple Harmonic Motion -> up & down oscillations

Trig functions

* Model phenomena that occur again & again

- vibrating guitar string
- swinging pendulem
- radio waves/TV waves.

$d=a\cos\omega t$ or $d=a\sin\omega t$

amplitude $|a|$

period of the motion is $\frac{2\pi}{\omega}$ where $\omega>0$

frequency $f=\frac{\omega}{2\pi}$ where $\omega>0$

also, $f=\frac{1}{\text{period}}$

8) *A ball on a spring is pulled 4 inches below its rest position and then released. The period of the motion is 6 seconds. Write the equation for the balls simple harmonic motion.*

period: $6 = \frac{2\pi}{\omega}$ $\quad a = -4$

$2\pi = 6\omega$

$\omega = \pi/3$

$$\boxed{d = -4\cos \tfrac{\pi}{3} t}$$

* cos because it starts at (0, −4) like a cos graph.

9) *A weight is attached to a spring is pulled down 6 inches below the equilibrium position. Assuming that the frequency of the system is $\frac{5}{\pi}$ cycles per second, determine a trig model that gives the position of the weight at time t seconds.*

frequency $= \frac{\omega}{2\pi} = \frac{5}{\pi}$

$\omega \cancel{\pi} = 10 \cancel{\pi}$

$\omega = 10$

$$\boxed{d = -6\cos 10t}$$