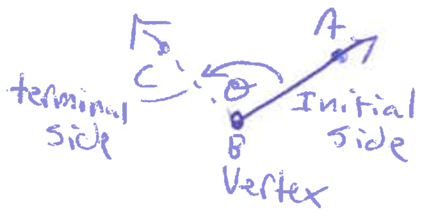
Ch 4.1 Angles & Radian Measure

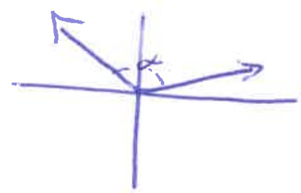
① Recognize and use the vocabulary of angles



Standard position

→ Vertex on origin of rect. coord. system

→ Initial side along positive x-axis.



Terminal side is in quadrant II ∴ angle α lies in quadrant II

② Use degree measure

a.) Acute, Rt, obtuse, straight ∠.

b.) $1° = \frac{1}{360}$ of a rotation

$1 \text{ min} = \frac{1}{60}$ degree

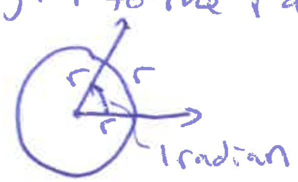
$1 \sec = \frac{1}{60 \cdot 60} = \frac{1}{3600}$ degree

Convert $31° 47' 12'' = 31 + \frac{47}{60} + \frac{12}{3600} = 31.787°$

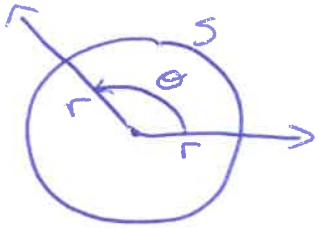
→ on Calc use "° ' ''" button.

③ Use radian measure

Definition of a Radian - One Radian is the measure of the central ∠ of a circle that intercepts an arc equal in length to the radius of the circle.



Radian Measure -



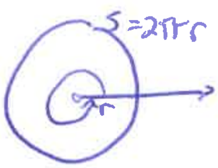
$\theta = \frac{s}{r}$ radians

Ex 1 A central ∠, θ, in a circle of radius 12 feet intercepts an arc of length 42 feet, What is the radian measure of θ?

$$\theta = \frac{s}{r} = \frac{42 \cancel{\text{feet}}}{12 \cancel{\text{feet}}} = 3.5$$

Thus, the radian measure of θ is 3.5,

(4) Convert between degrees and radians,



$$\theta = \frac{s}{r} = \frac{2\pi \cancel{r}}{\cancel{r}} = 2\pi$$

∴ we know a rotation of 360° = 2π radians

$$\frac{360°}{2} = \frac{2\pi \text{ radians}}{2}$$

$$180° = \pi \text{ radians}$$

Using this relationship:

1.) To convert degrees to radians, mult. degrees by $\frac{\pi \text{ radians}}{180°}$

2.) To convert radians to degrees, mult. radians by $\frac{180}{\pi \text{ radians}}$.

Ex 2 Convert each angle in degrees to radians:

a.) $60°$

$60° \cdot \frac{\pi \text{ radians}}{180°}$

$= \frac{60\pi \text{ radians}}{180}$

$= \frac{\pi}{3}$ radians

b.) $270°$

$270 \cdot \frac{\pi \text{ radians}}{180}$

$= \frac{3\pi}{2}$ radians

c.) $-300°$

$-300 \cdot \frac{\pi \text{ radians}}{180}$

$= -\frac{5\pi}{3}$ radians

Ex 3 Convert each ∠ in radians to degrees.

a.) $\frac{\pi}{4}$ radians

$\frac{\pi \text{ radians}}{4} \cdot \frac{180°}{\pi \text{ radians}}$

$= \frac{180°}{4} = 45°$

b.) $-\frac{4\pi}{3}$ radians

$= \frac{-4\pi \text{ radians}}{3} \cdot \frac{180°}{\pi \text{ radians}}$

$= \frac{-4 \cdot 180}{3} = -240°$

c.) 6 radians

$6 \text{ radians} \cdot \frac{180°}{\pi \text{ radians}}$

$= \frac{6 \cdot 180°}{\pi} = 343.8°$

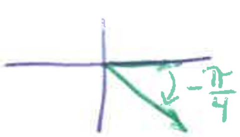
⑤ Draw angles in standard position

Ex

a.) $\theta = -\frac{\pi}{4}$

method 1: $\theta = -\frac{1}{8} \cdot 2\pi$

Method 2: $\theta = -\frac{\pi}{4}$

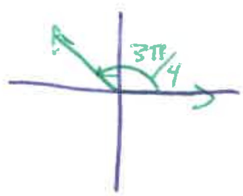


b.) $\alpha = \frac{3\pi}{4}$

M1: $\alpha = \frac{3}{8} \cdot 2\pi$

M2: $\alpha = \frac{2\pi}{4} + \frac{\pi}{4}$

$= \frac{\pi}{2} + \frac{\pi}{4}$

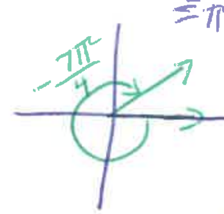


c.) $\beta = -\frac{7\pi}{4}$

M1: $\beta = -\frac{7}{8} \cdot 2\pi$

M2: $\beta = \frac{4\pi}{4} + \frac{2\pi}{4} + \frac{1\pi}{4}$

$= \pi + \frac{\pi}{2} + \frac{\pi}{4}$



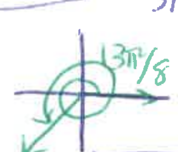
D.) $\gamma = \frac{13\pi}{4}$

M1: $\gamma = \frac{13}{8} \cdot 2\pi$

$= 1\frac{5}{8}$

M2: $\frac{12\pi}{4} + \frac{\pi}{4}$

$3\pi + \frac{\pi}{4}$



(6.) Find Coterminal ∠s.

Ex) Find a positive angle less than 360° that is coterminal with each of the following:

a.) a 400° angle

$400° - 360° = \boxed{40°}$

b.) a −135° angle

$-135 + 360 = \boxed{225°}$

Ex) Find " " 2π " " :

a.) a $\frac{13\pi}{5}$ angle

$\frac{13\pi}{5} - 2\pi = \frac{13\pi}{5} - \frac{10\pi}{5} = \boxed{\frac{3\pi}{5}}$

b.) a $-\frac{\pi}{15}$ angle

$-\frac{\pi}{15} + 2\pi = -\frac{1\pi}{15} + \frac{30\pi}{15} = \boxed{\frac{29\pi}{15}}$

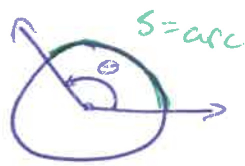
-> What about an angle like 855°?

↳ subtract 360° · 2

(7) Find the length of a circular arc.

$s = r\theta$

in radians



Ex) A circle has a radius of 6 inches. Find the length of the arc intercepted by a central ∠ of 45°. Express arc length in terms of π. Then round your answer to two decimal places.

$45° \cdot \frac{\pi \text{ radians}}{180°}$

$= \frac{\pi}{4}$ radians

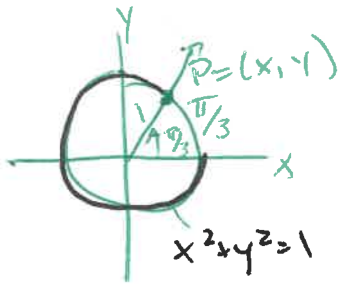
$s = r\theta$

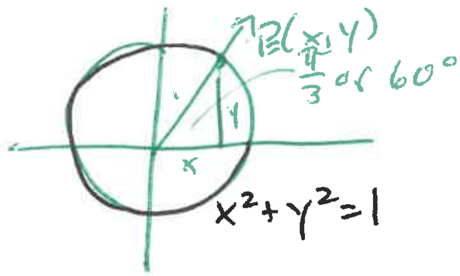
$s = (6 \text{ inches})\left(\frac{\pi}{4}\right) = \frac{6\pi}{4}$ inches

≈ 4.71 inches

Chapter 4.3 Right $\triangle$ Trigonometry

Use rt. $\triangle$s to evaluate Trig Functions



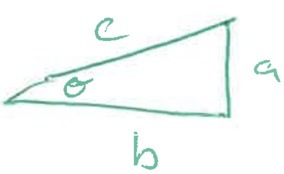


$\sin\frac{\pi}{3} = \sin 60° = y = \frac{y}{1}$

$\cos\frac{\pi}{3} = \cos 60° = x = \frac{x}{1}$

We know SOHCAHTOA

<u>tangent is slope</u>

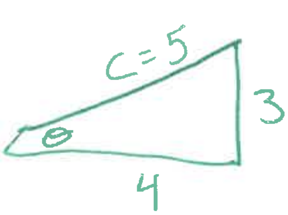


$\sin\theta = \frac{a}{c}$ $\qquad$ $\csc\theta = \frac{c}{a}$

$\cos\theta = \frac{b}{c}$ $\qquad$ $\sec\theta = \frac{c}{b}$

$\tan\theta = \frac{a}{b}$ $\qquad$ $\cot\theta = \frac{b}{a}$

Ex] Find all 6 values

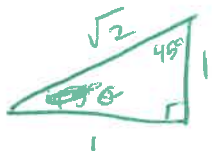


$\sin\theta = \frac{3}{5}$ $\qquad$ $\csc\theta = \frac{5}{3}$

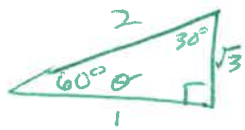
$\cos\theta = \frac{4}{5}$ $\qquad$ $\sec\theta = \frac{5}{4}$

$\tan\theta = \frac{3}{4}$ $\qquad$ $\cot\theta = \frac{4}{3}$

② Find Function values for $30°(\frac{\pi}{6})$, $45°(\frac{\pi}{4})$, and $60°(\frac{\pi}{3})$.



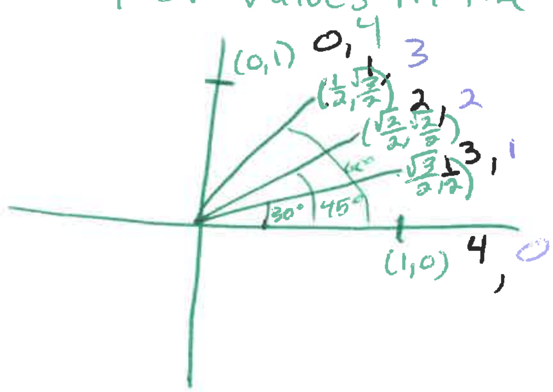
so $\sin\theta = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$



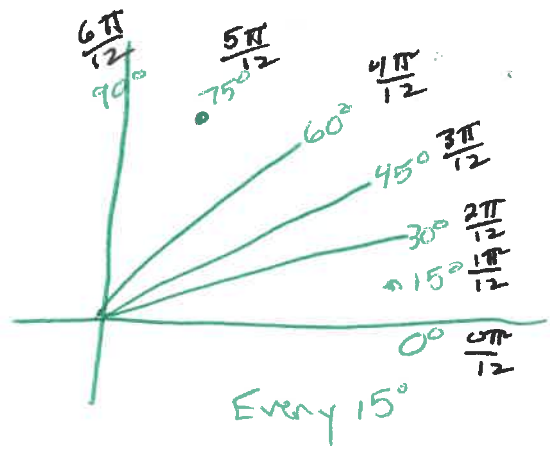
$\sin\theta = \frac{\sqrt{3}}{2}$

Easy way to remember

For values in the coordinate using degrees



For radians



in black $\frac{\sqrt{\ }}{2}$ in blue $\frac{\sqrt{\ }}{2}$

Ex) find csc 45°, sec 45°, cot 45°

these are all recipricals of sin, cos, tan

a.) $\sin 45° = \frac{\sqrt{2}}{2}$ b.) $\cos 45° = \frac{\sqrt{2}}{2}$ c.) $\tan 45° = 1$

$\csc = \frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\cancel{2}\sqrt{2}}{\cancel{2}} = \sqrt{2}$ $\sec = \sqrt{2}$ $\cot 45° = 1$

You can also use special △s 1-1-$\sqrt{2}$ or 1-$\sqrt{3}$-2.

③ Use equal cofunctions of complements $\lfloor$ $<$ 4.3

↳ what are complements?

-> If in degrees use $90°$; if in radians use $\frac{\pi}{2}$

Ex | a.) $\sin 46°$ b.) $\cot \frac{\pi}{12}$

$\sin 46 = \cos(90-46)$

$= \cos 44°$

$\cot \frac{\pi}{12} = \tan\left(\frac{\pi}{2} - \frac{\pi}{12}\right)$

$= \tan\left(\frac{6\pi}{12} - \frac{\pi}{12}\right)$

$= \tan \frac{5\pi}{12}$

This is because opp/Adj switch

④ Apps. try in HW

extra: How to solve $\Rightarrow \sin\theta = 0.2777$?

$\theta = \sin^{-1} 0.2777$

Chapter 4.2 Trigonometric Functions: The Unit Circle

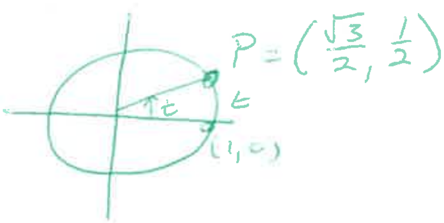
① Use a unit circle to define trigonometric functions of real numbers.

$\sin t = y$ $\qquad$ $\csc t = \frac{1}{y},\ y \neq 0$

$\cos t = x$ $\qquad$ $\sec t = \frac{1}{x},\ x \neq 0$

$\tan t = \frac{y}{x},\ x \neq 0$ $\qquad$ $\cot t = \frac{x}{y},\ y \neq 0$

$\boxed{\text{Ex}}$



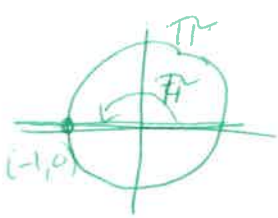
Use this figure to find the values of the trigonometric functions at t.

$\sin t = y = \frac{1}{2}$ $\qquad$ $\csc t = \frac{1}{y} = \frac{1}{\frac{1}{2}} = 2$

$\cos t = x = \frac{\sqrt{3}}{2}$ $\qquad$ $\sec t = \frac{1}{x} = \frac{1}{\frac{\sqrt{3}}{2}} = 1 \cdot \frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}}\frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

$\tan t = \frac{y}{x} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$ $\qquad$ $\cot t = \frac{x}{y} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{\sqrt{3}}{\cancel{2}} \cdot \frac{\cancel{2}}{1} = \sqrt{3}$

$\boxed{\text{Ex}}$

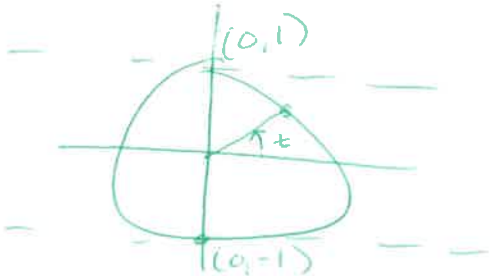


$\sin \pi = 0$ $\qquad$ $\csc \pi = \frac{1}{y} = \frac{1}{0}$ undefined

$\cos \pi = -1$ $\qquad$ $\sec \pi = \frac{1}{x} = \frac{1}{-1} = -1$

$\tan \pi = \frac{0}{-1} = 0$ $\qquad$ $\cot \pi = \frac{-1}{0}$ undefined

② Recognize the domain & range of sine & Cosine functions



$y = \sin t$

range — $\angle$s radian measure

Same for $x = \cos t$.

D: $(-\infty, \infty)$

R: $[-1, 1]$

③ find the exact value of the trig. functions at $\frac{\pi}{4}$

$\sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$\csc \frac{\pi}{4} = \frac{1}{\frac{\sqrt{2}}{2}} = 1 \cdot \frac{2}{\sqrt{2}} = \frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\cancel{2}\sqrt{2}}{\cancel{2}} = \sqrt{2}$

$\cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$\sec \frac{\pi}{4} = \frac{1}{\frac{\sqrt{2}}{2}} = \sqrt{2}$

$\tan \frac{\pi}{4} = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = 1$

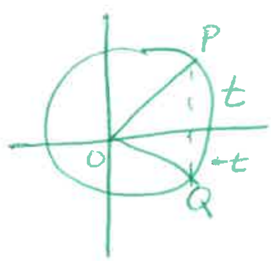
$\cot \frac{\pi}{4} = 1$

Ch 4.2 Cont'd Trig Func. : The Unit circle

(4) Use even & odd trigonometric functions.

$f(-t) = f(t)$ is even

Remember $f(-t) = -f(t)$ is odd



$P: (\cos t, \sin t)$

$Q: (\cos(-t), \sin(-t))$

since:

$\cos t = \cos(-t)$ it's even

&

$\sin t = -\sin t$ it's odd

Even & odd Functions

Even: $\cos(-t) = \cos t$ & $\sec(-t) = \sec t$

Odd: $\sin(-t) = -\sin t$ & $\csc(-t) = -\csc t$

$\tan(-t) = -\tan t$ & $\cot(-t) = -\cot t$

Ex | Find each Trig Function:

a.) $\sec(-\frac{\pi}{4})$

$\sec(-\frac{\pi}{4}) = \sec(\frac{\pi}{4}) = \sqrt{2}$

b.) $\sin(-\frac{\pi}{4})$

$\sin(-\frac{\pi}{4}) = -\sin\frac{\pi}{4}$

$= -\frac{\sqrt{2}}{2}$

(5) Recognize & use fundamental Identities

Reciprocal Identities:

$\sin t = \frac{1}{\csc t}$ $\qquad$ $\csc t = \frac{1}{\sin t}$

$\cos t = \frac{1}{\sec t}$ $\qquad$ $\sec t = \frac{1}{\cos t}$

$\tan t = \frac{1}{\cot t}$ $\qquad$ $\cot t = \frac{1}{\tan t}$

Quotient Identities:

$\tan t = \frac{\sin t}{\cos t}$ $\qquad$ $\cot t = \frac{\cos t}{\sin t}$

Ex Given $\sin t = \frac{2}{3}$ & $\cos t = \frac{\sqrt{5}}{3}$, find other 4 trig functions

$$\tan t = \frac{\sin t}{\cos t} = \frac{\frac{2}{3}}{\frac{\sqrt{5}}{3}} = \frac{2}{\sqrt{5}} = \boxed{\frac{2\sqrt{5}}{5}}$$

$$\csc t = \frac{1}{\sin t} = \frac{1}{\frac{2}{3}} = \boxed{\frac{3}{2}}$$

$$\sec t = \frac{1}{\cos t} = \frac{1}{\frac{\sqrt{5}}{3}} = \frac{3}{\sqrt{5}} = \boxed{\frac{3\sqrt{5}}{5}}$$

$$\cot t = \frac{\cos t}{\sin t} = \frac{\frac{\sqrt{5}}{3}}{\frac{2}{3}} = \boxed{\frac{\sqrt{5}}{2}}$$

→ Other Relationships: Pythagorean Identities

we know: $x^2 + y^2 = 1$ $\qquad$ by dividing by x^2 or y^2

$\sin^2 t + \cos^2 t = 1$ $\qquad$ $1 + \tan^2 t = \sec^2 t$ $\qquad$ $1 + \cot^2 t = \csc^2 t$

Ex) Given that $\sin t = \frac{1}{2}$ and $0 \le t < \frac{\pi}{2}$, find the value of $\cos t$ using a trig. function.

$$\sin^2 t + \cos^2 t = 1$$

$$\left(\tfrac{1}{2}\right)^2 + \cos^2 t = 1$$

$$\tfrac{1}{4} + \cos^2 t = 1$$

$$\cos^2 t = 1 - \tfrac{1}{4} = \tfrac{3}{4}$$

$$\sqrt{\cos^2 t} = \sqrt{\tfrac{3}{4}}$$

$\cos t = \frac{\sqrt{3}}{2}$ and since $0 \le t < \frac{\pi}{2}$, $\cos t$ is positive

(6) Use periodic properties

Defn of Periodic Function (periodic — repeats itself)

$$f(t+p) = f(t)$$

p — period of f

Periodic Properties of the Sine & Cosine functions & tangent

$\sin(t+2\pi) = \sin t$ and $\cos(t+2\pi) = \cos t$

$\therefore$ both have a period of 2π

Note on graph cos goes +, −, −, + & then repeats

~~Period~~ $\tan(t+\pi) = \tan t$ & $\cot(t+\pi) = \cot t$

$\therefore$ tan has a period of π

on graph tan goes +, −, +, − ~~...~~

Ex) Find the value of each trig func.

a.) $\cot \frac{5\pi}{4}$

$\cot\left(\frac{\pi}{4} + \pi\right) = \cot \frac{\pi}{4} = 1$

b.) $\cos\left(-\frac{9\pi}{4}\right)$

$\cos\left(-\frac{9\pi}{4} + 4\pi\right) = \cos \frac{7\pi}{4} = \frac{\sqrt{2}}{2}$

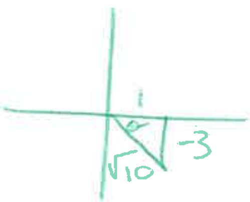
Note: On calc if radians use radians if ° use °

Ch 4.4 Trig. Functions of Any Angle

(1) Use the definitions of trig functions of any angle

→ Refer to book

Ex Let $P = (1, -3)$ be a pt on the terminal side of θ, find all 6 trig functions.



$\sin\theta = \frac{-3}{\sqrt{10}} = \frac{-3\sqrt{10}}{10}$ $\qquad$ $\csc\theta = \frac{\sqrt{10}}{-3} = -\frac{\sqrt{10}}{3}$

$\cos\theta = \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10}$ $\qquad$ $\sec\theta = \frac{\sqrt{10}}{1} = \sqrt{10}$

$\tan\theta = -3$ $\qquad$ $\cot\theta = -\frac{1}{3}$

Ex Evaluate, if possible, the cosine function & the cosecant function at the following 4 quadrantal ∠s:

a.) $\theta = 0° = 0$
$P: (1, 0)$
$\cos\theta = 1$
$\csc\theta = \frac{1}{0} = \text{und.}$

b.) $\theta = 90° = \pi/2$
$P(0, 1)$
$\cos\theta = 0$
$\csc\theta = \frac{1}{1} = 1$

c.) $\theta = 180° = \pi$
$P(-1, 0)$
$\cos\theta = -1$
$\csc\theta = \frac{1}{0} = \text{und.}$

d.) $\theta = 270° = \frac{3\pi}{2}$
$P(0, -1)$
$\cos\theta = 0$
$\csc\theta = \frac{-1}{1} = -1$

(2) Use the signs of the trig functions

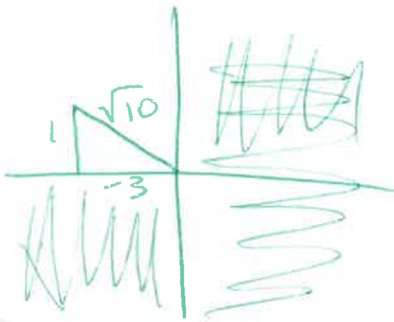
II	I
Sin & csc Pos	All func Pos.
III	**IV**
tan & cot pos	cos & sec pos.

A	Smart	Trig	Class ◡̈
All pos. in QI	sin & reciprocal csc pos.	tan & rec, cot pos	cos. & rec, sec pos.

Ex. If $\sin\theta < 0$ and $\cos\theta < 0$, name quadrant in which $\angle\theta$ lies

$\therefore$ Q $\underline{IV}$

Ex Given $\tan\theta = -\frac{1}{3}$ and $\cos\theta < 0$, find $\sin\theta$ & $\sec\theta$



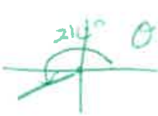
$\sin\theta = \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10}$

$\sec\theta = \frac{\sqrt{10}}{-3} = -\frac{\sqrt{10}}{3}$

③ Find reference angles

Ex Find the reference $\angle$, θ', for each of the following $\angle$s.

a.) $\theta = 210°$



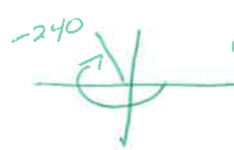
$\theta' = 210 - 180$

$\theta' = 30°$

b.) $\theta = \frac{7\pi}{4}$

$\theta' = \frac{\pi}{4}$

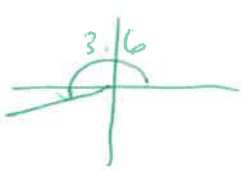
c.) $\theta = -240°$



$\theta' = 240 - 180$

$\theta' = 60°$

d.) $\theta = 3.6$ ← between $\pi = 3.14$ & $\frac{3\pi}{2} \approx 4.71$



$\theta' = 3.6 - \pi \approx 0.46$

e.) $\theta = 665°$

$665 - 360 = 305°$

$\theta' = 360 - 305 = 55°$

④ Use reference ∠s to evaluate Trig Func. $\boxed{C4.4}$

Ex) a.) $\sin 300°$

$\theta' = 360 - 300 = 60°$

$\sin 60° = -\frac{\sqrt{3}}{2}$

neg. since sine is neg in Q:IV

b.) $\tan \frac{5\pi}{4}$

$\theta' = \frac{5\pi}{4} - \frac{4\pi}{4} = \frac{\pi}{4}$

$\tan \frac{\pi}{4} = 1$

pos bc tan pos in QIII

c.) $\sec(-\frac{\pi}{6})$

$\theta' = \frac{\pi}{6}$

$\sec \frac{\pi}{6} = \frac{2\sqrt{3}}{3}$

pos bc sec pos in QIV

d.) $\cos \frac{17\pi}{6}$

$\theta' = \frac{17\pi}{6} - 2\pi = \frac{17\pi}{6} - \frac{12\pi}{6}$

$= \frac{5\pi}{6}$

$\cos \frac{5\pi}{6}$ QII so negative

$\theta' = \pi - \frac{5\pi}{6} = \frac{\pi}{6}$

$\cos \pi/6 = -\frac{\sqrt{3}}{2}$

e.) $\sin(-\frac{22\pi}{3})$

$-\frac{22\pi}{3} + 8\pi = \frac{2\pi}{3}$ in QII so pos

$\theta' = \pi - \frac{2\pi}{3} = \frac{\pi}{3}$

$\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$