3.1Notes: Relations and Functions

LEARNING OBJECTIVES: Students will be able to:

1) **determine the domain and range of a relation or function in set notation, tables of values, and graphs.**
2) **determine if a relation is a function using sets, tables and graphs (for graphs using the vertical line test).**

Key Vocabulary

Domain

-All your x values

-input

Range

-All your y values

-output

Example 1: Find the domain and range for each set of ordered pairs below.

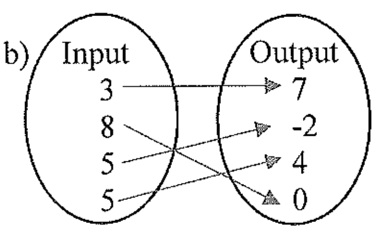
a)

x	1	2	3	4	5
y	11	12	13	13	13

Domain: {1, 2, 3, 4, 5}

Range: {11, 12, 13}

b)



Domain: {3, 8, 5}

Range: {-2, 0, 4, 7}

You try!

c) $\{(7,-1),(6,5),(-3,2),(0,5)\}$

Domain: {7, 6, -3, 0}

Range: {-1, 5, 2}

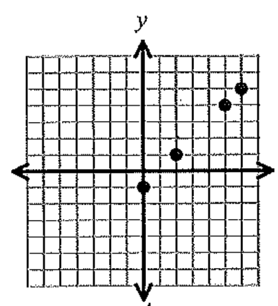
d)

3	Red
7	Blue
-2	Green
5	Green
-4	Green

Domain: {3, 7, -2, 5, -4}

Range: {Red, Blue, Green}

e)



D: {0, 2, 5, 6}

R: {-1, 1, 4, 5}

Key Vocabulary

Relation

a set of ordered pairs

Function

Your input goes to only one output.

Vertical Line Test

A relation is a function if, for any vertical line drawn, only touches the relation at most 1 time.

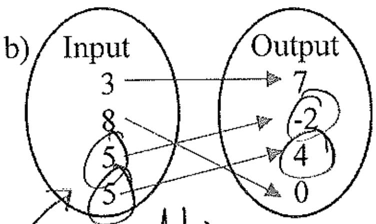
Example 2: For each relation, is it a function?

a)

x	1	2	3	4	5
y	11	12	13	13	13

yes

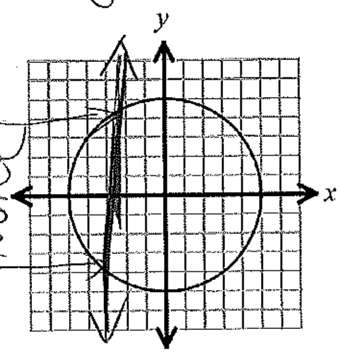
b)



Two outputs

No

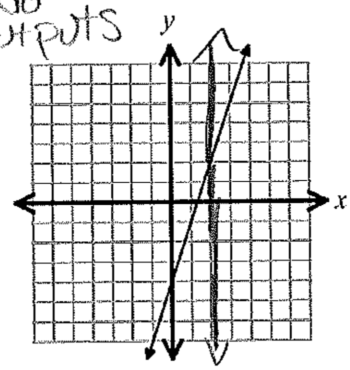
c)



Crosses twice

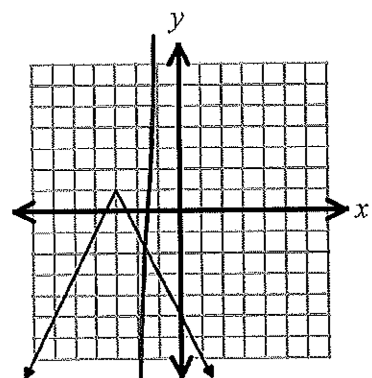
No

d)



yes

e)



yes

You try!

f) $\{(7,-1),(6,5),(-3,2),(0,5)\}$

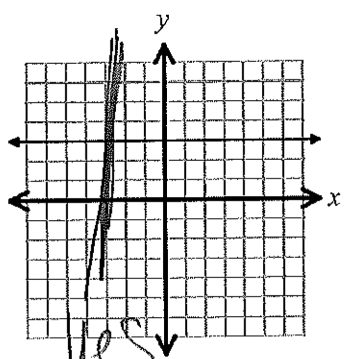
yes.

g)

3	Red
7	Blue
-2	Green
5	Green
-4	Green

yes

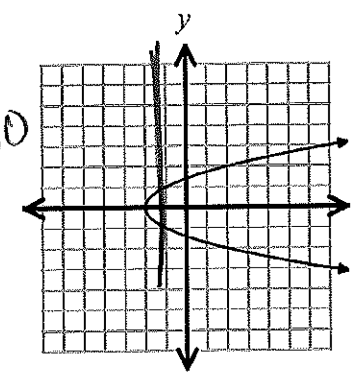
h)



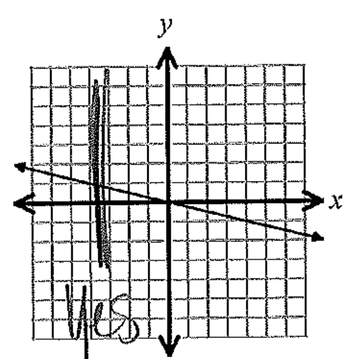
yes

i)

No



j)



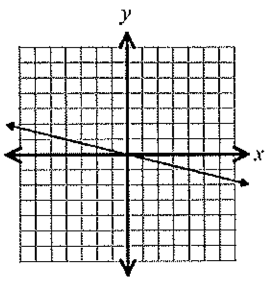
yes

Example 3: Terry goes to the carnival, and the amount of money he spends can be modeled by a function in terms of how many rides he purchases. Each ride costs $2.

- Which of the following best models the domain of this function (# of rides)?
  - A) 0, 1, 2, 3, …
  - B) all real numbers
  - C) 1, 2, 3, …
  - D) 0, 2, 4, 6, …
- Which of the following best models the range of this function ($ spent)?
  - A) 0, 1, 2, 3, …
  - B) all real numbers
  - C) 1, 2, 3, …
  - D) 0, 2, 4, 6, …

Example 4: Find the domain and range for each function shown below.

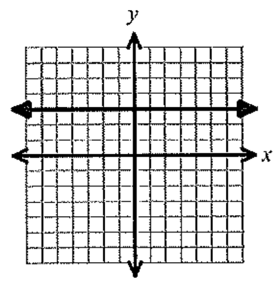
a)



D: All real #'s ℝ

R: ℝ

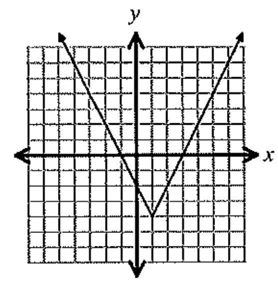
b)



D: ℝ

R: {3}

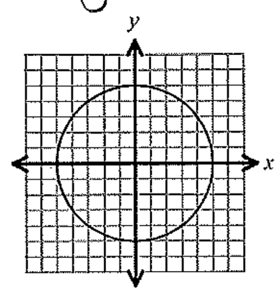
c)



D: ℝ

R: $y \geq -4$

Challenge: Example 5: Multiple choice: What is the domain for the relation shown?



A) $x \geq -5$
B) all real numbers
C) $x \leq 5$
D) $-5 \leq x \leq 5$

3.2 Notes: Linear Functions (Do as the same day as the 3.1 Notes)

LEARNING OBJECTIVES: Students will be able to:

1) **evaluate function values for inputs in their domains**
2) **write a linear function given a table of values**

Key Vocabulary

Function Notation

f(x) = output = y
↑
input

f(x) means plug in 2 for input

Example 1: Given $f(x) = -6x + 2$, find each value.

a) $f(3)$

$-6(3)+2$

$-18+2$

$f(3) = -16$

b) $f(-1)$

$-6(-1)+2$

$6+2$

$f(-1) = 8$

You try!

Example 2: Given $g(a) = 2(a-3) + 7$, find each value.

a) $g(5)$

$2(5-3)+7$

$2(2)+7$

$4+7 = g(5) = 11$

b) $g(-4)$

$2(-4-3)+7$

$2(-7)+7$

$-14+7$

$g(-4) = -7$

Key Vocabulary

Linear Function

- Makes a line (no curves)

$y = mx + b$

Rate of Change

- Slope

Example 3: Write a linear function for the data shown in each table.

a) ← x=0 y-int

x	-1	0	1	2
$f(x)$	17	32	47	62

① find Slope

$\frac{32-17}{0+1} = \frac{15}{1} = 15$

② plug into (h,k) form

$y = 15(x+1)+17$

$y = 15x + 15 + 17$

$y = 15x + 32$

b)

x	$g(x)$
1	5
2	2
3	-1
4	-4

$\frac{2-5}{2-1} = \frac{-3}{1} = -3$

$y = -3(x-1)+5$

$y = -3x + 3 + 5$

$y = -3x + 8$

You try! Example 4: Write a linear function for the data shown in each table.

a)

x	0	1	2	3
$h(x)$	1	-5	-11	-17

$\frac{-5-1}{1-0} = \frac{-6}{1} = -6$

$y = -6x + 1$

b)

x	$d(x)$
1	6
2	8.5
3	11
4	13.5

$\frac{8.5-6}{2-1} = \frac{2.5}{1} = 2.5$

$y = 2.5(x-1)+6$

$y = 2.5x - 2.5 + 6$

$y = 2.5x + 3.5$

Concept Summary Assess

CONCEPT SUMMARY Linear Function Representations

WORDS Linear functions are represented by words, rules, tables, or graphs. Function notation tells us the name of a function and the input variable.

ALGEBRA $f(x) = -2x + 1$

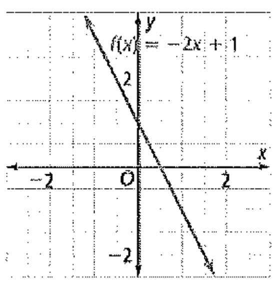
"f of x"

TABLE

x	−2	−1	0	1	2
$f(x)$	5	3	1	−1	−3

The table shows the domain and range of the function.

GRAPH The graph of the function $f(x) = -2x + 1$ is the graph of the linear equation $y = -2x + 1$.



3.4 Notes: Arithmetic Sequences

LEARNING OBJECTIVES: Students will be able to:
1) determine if a sequence is arithmetic and find the common difference when appropriate.
2) write explicit and recursive formulas for arithmetic sequences in sequence and function notation.

Warm-up:

1) If $f(x) = (3x + 7)^2$, then $f(1) = ?$

A. 10
B. 16
C. 58
D. 79
E. 100

$(3(1)+7)^2$

$(3+7)^2$

$10^2 = 100$

2) Given $f = cd^3$, $f = 450$, and $d = 10$, what is c ?

F. 0.45
G. 4.5
H. 15
J. 45
K. 150

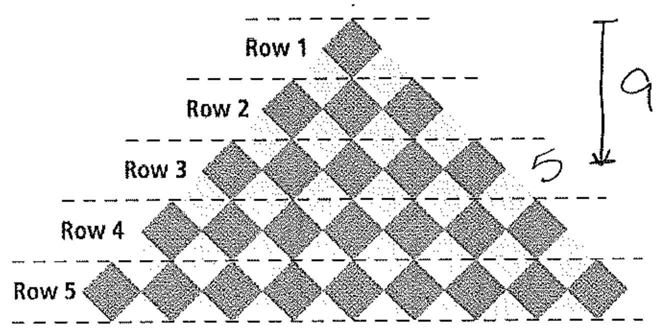
$cd^3 = 450$

$c10^3 = 450$

$c(1000) = 450$

$c = .45$

Exploration: A fashion designer is creating a patterned fabric as shown.



A. There are 5 shaded squares in row 3. The *total* number of shaded squares up to and including row 3 is 9. Fill in the table for the remaining rows.

Row Number	1	2	3	4	5
Number of Shaded Squares in the Row	1	3	5	7	9
Total Number of Shaded Squares	1	4	9	16	25

B. What number patterns do you see in the rows of tables?

- Odd #'s — Row #squared

Key Vocabulary

Sequence

An ordered list of #'s

Term of a Sequence

An item in the list

Arithmetic Sequences

The pattern for the next # comes from + or −

Common Difference (d)

The amount added (or subtracted) to get each next term in the sequence

Example 1: Determine whether or not each sequence is an arithmetic sequence. If it is, then identify the common difference.

a) 93, 86, 79, 72, 65, …

yes ; 7

c) -10, -12, -14, -15, -18, …

No

You try!

d) -5, -3, -1, 1, 3, …

yes; 2

f) 4.9, 3.8, 2.7, 1.6, 0.5, …

yes; 1.1

Key Vocabulary

Recursive:

A formal with a starting term (a_1) that described how to get a new term by using the previos term.

Recursive Formula for Arithmetic Sequence

Using Sequence Notation
$a_1 = ?$
$a_n = a_{n-1} + \boldsymbol{d}$

Example 2: Write a recursive formula for each arithmetic sequence.

a) $-9, -6, -3, 0, 3, \ldots$

$a_1 = -9$ $a_n = a_{n-1} + 3$

$a_2 = a_1 + d$

$-6 = -9 + d$

$3 = d$

b) $47, 39, 31, 23, 15, \ldots$

$a_1 = 47$ $a_n = a_{n-1} - 8$

$a_2 = a_1 + d$

$39 = 47 + d$

$-8 = d$

You try!

c) $81, 85, 89, 93, 97, \ldots$

$a_1 = 81$

$85 = 81 + d$

$4 = d$

$a_n = a_{n-1} + 4$

d) $-3, -5, -7, -9, -11, \ldots$

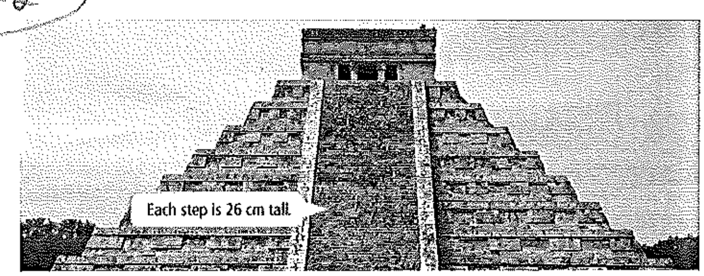
$a_1 = -3$

$-5 = -3 + d$

$-2 = d$

$a_n = a_{n+1} - 2$

Example 3: Write a recursive formula for the height above the ground of the *n*th step of the pyramid shown, if each step is 26 cm tall.



$a_1 = 26$

$a_n = a_{n-1} + 26$

Key Vocabulary

Explicit

A formula to find the n^{th} term w/o knowing previous terms

Explicit Formula for Arithmetic Sequence

$$\boxed{a_n = dn + a_0}$$

a_0 ← term before a_1

d ↑ Common difference

$a_0 = a_1 - d$

Example 4: Write an explicit formula for each arithmetic sequence.

a) $-9, -6, -3, 0, 3, \ldots$

+3 +3 +3 +3

$d = 3$

$a_0 = -9 - 3 = -12$

$\boxed{a_n = 3n - 12}$

b) $47, 39, 31, 23, 15, \ldots$

-8 -8

$d = -8$

$a_0 = 47 + 8 = 55$

$\boxed{a_n = -8n + 55}$

You try!

c) $81, 85, 89, 93, 97, \ldots$

+4 +4

$d = 4$

$a_0 = 81 - 4 = 77$

$\boxed{a_n = 4n + 77}$

d) $-3, -5, -7, -9, -11, \ldots$

-2 -2

$d = -2$

$a_0 = -3 + 2 = -1$

$\boxed{a_n = -2n - 1}$

Example 5: The cost of renting a bicycle is given in the table. Represent the cost as both a recursive and an explicit formula.

Number of days rented	1	2	3	4
Rental cost	26	38	50	62

12 12

Recursive Formula:

$a_1 = 26$

$a_n = a_{n-1} + 12$

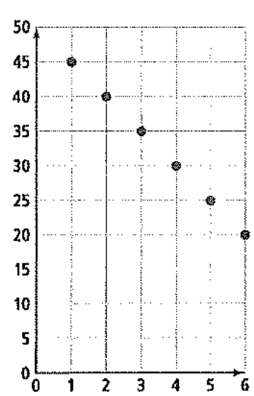
Explicit Formula:

$d = 12$

$a_0 = 26 - 12 = 14$

$\boxed{a_n = 12n + 14}$

Example 6: Which of the follow formulas represent the arithmetic sequence shown? Select all that apply.



A) $a_n = 45 - 5n$

B) $a_n = 50 - 5n$

C) $a_1 = 45;\ a_n = a_{n-1} - 5$

D) $a_1 = 50;\ a_n = a_{n-1} - 5$

$a_1 = 45$

$d = -5$

$a_n = -5n + 50$

Example 7: For each recursive formula, write the explicit formula.

a) $a_1 = 17;\ a_n = a_{n-1} + 4$ $= d$

$a_0 = a_1 - d$

$a_0 = 17 - 4$

$a_0 = 13$

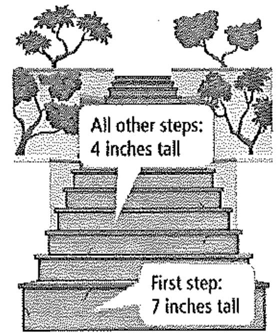
$\boxed{a_n = 4n + 13}$

b) $a_1 = -3;\ a_n = a_{n-1} - 2$

$a_0 = -3 + 2 = -1$

$\boxed{a_n = -2n - 1}$

You try! Example 8: The recursive formula for the height above the ground of the *n*th step of the stairs shown is $a_1 = 7;\ a_n = a_{n-1} + 4$. Write the explicit formula to find the height above the ground of the *n*th step.



$d = 4$

$a_0 = 7 - 4 = 3$

$\boxed{a_n = 4n + 3}$

Example 9: Write the recursive formula for each explicit formula given.

a) $a_n = -5 + 3n$

$d = 3$

$a_0 = -5 = a_1 - d$

$a_1 - 3 = -5$

$a_1 = -2$

$\boxed{a_1 = -2 \quad a_n = a_{n-1} + 3}$

b) $a_n = 11 + 40n$

$d = 40$

$a_0 = 11$

$11 = a_1 - 40$

$51 = a_1$

$\boxed{a_1 = 51 \quad a_n = a_{n-1} + 40}$

Arithmetic Sequence Formulas can be written in both sequence notation and function notation.

Recursive Formulas

Using Sequence Notation	Using Function Notation
$a_1 = ?$ $a_n = a_{n-1} + \boldsymbol{d}$	$f(1) = ?$ $f(n) = f(n-1) + \boldsymbol{d}$

Explicit Formulas

Using Sequence Notation	Using Function Notation
$a_n = dn + a_0$	$f(n) = dn + f(0)$

Example 10: Given the arithmetic sequence shown below, write each requested formula.

7, 2, -3, -8, -13, …

$-5 \quad -5$

Recursive (sequence notation)

$a_1 = 7 \quad a_n = a_{n-1} - 5$

Recursive (function notation)

$f(1) = 7$

$f(n) = f(n-1) - 5$

Explicit (sequence notation)

$a_n = -5n + 12$

$a_0 = 7 + 5 = 12$

Explicit (function notation)

$f(n) = -5n + 12$

3.5 Notes: Scatter Plots and Lines of Fit

26 & 27

LEARNING OBJECTIVES: Students will be able to:
1) **make and interpret the correlation (positive, negative or none) of scatterplots**
2) **determine a line of best fit for a scatterplot by selecting two points and finding the equation of the line**
3) **use a line of best fit to make a prediction for y given a value of x**
4) **find the average rate of change for a function given two coordinate points**

Warm-up:

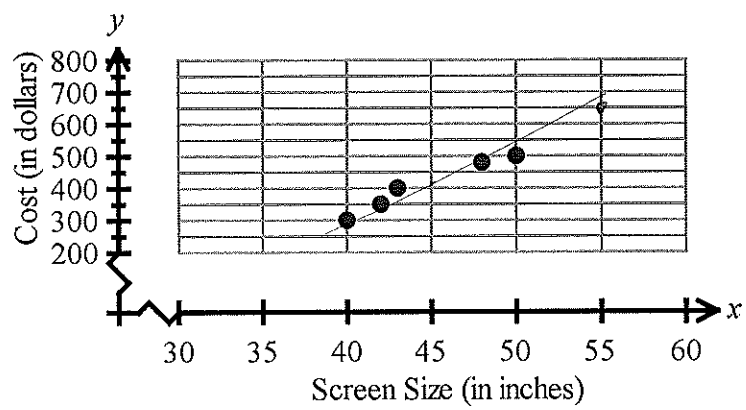
1) Students studying motion observed a cart rolling at a constant rate along a straight line. The table below gives the distance, d feet, the cart was from a reference point at 1-second intervals from $t = 0$ seconds to $t = 5$ seconds.

t	0	1	2	3	4	5
d	14	20	26	32	38	44

Which of the following equations represents this relationship between d and t?

~~**A.** $d = t + 14$~~
~~**B.** $d = 6t + 8$~~
C. $d = 6t + 14$
D. $d = 14t + 6$
E. $d = 34t$

Explore: Nicholas plotted data points to represent the relationship between screen size and the cost of television sets. Everything else is the same about the televisions sets, except for the screen size.



A) As the size of the television set increases, what happens to the cost?

goes up, increases

B) Using the data given, predict the cost of a television set that has a screen size of 55 inches.

\$650 - \$700

C) Using the data given, predict the cost of a television set that has a screen size of 45inches.

\$450

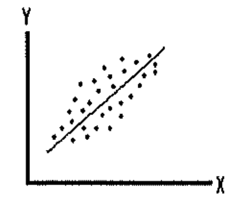
Key Vocabulary

Scatter Plot

A collection of ordered pairs that compares 2 sets of Data.

Positive Correlation

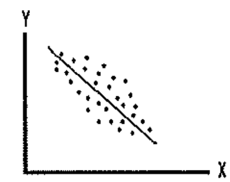
As x increases, y increases. The pattern is generally linear with an uphill pattern.



Positive Correlation

Negative Correlation

As x increases, y decreases. The pattern is generally linear with a downhill pattern.

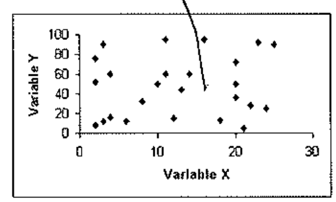


Negative Correlation

No Correlation

As x increases, y does not consistently increase or decrease.

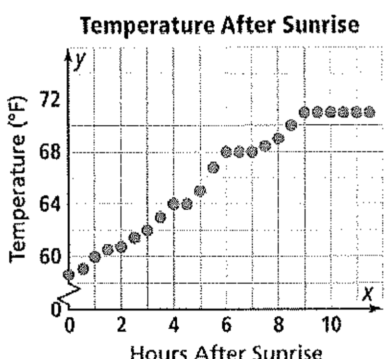
There is no pattern.



No Correlation

Example 1: For each scatter plot or set of data, describe the type of correlation.

a)

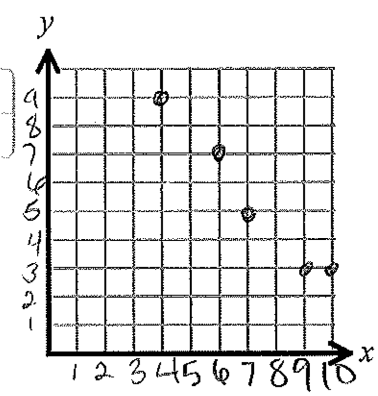


Positive Correlation

b)

x	4	6	7	9	10
y	9	7	5	3	3

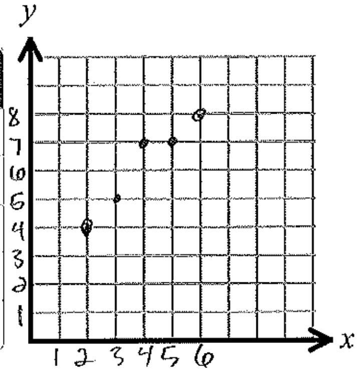
neg. Correlation



You Try!

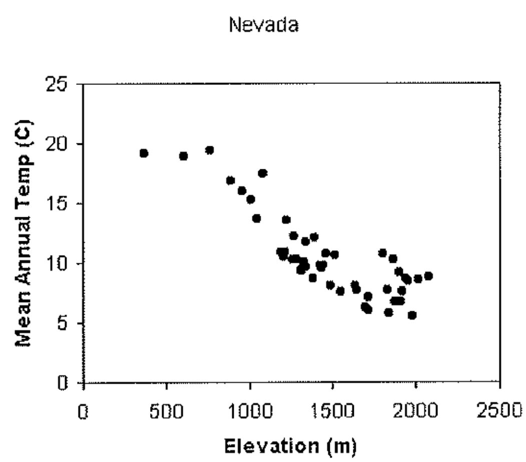
c)

x	y
2	4
3	5
4	7
5	7
6	8



pos. corr.

d)



neg. Corr.

Key Vocabulary

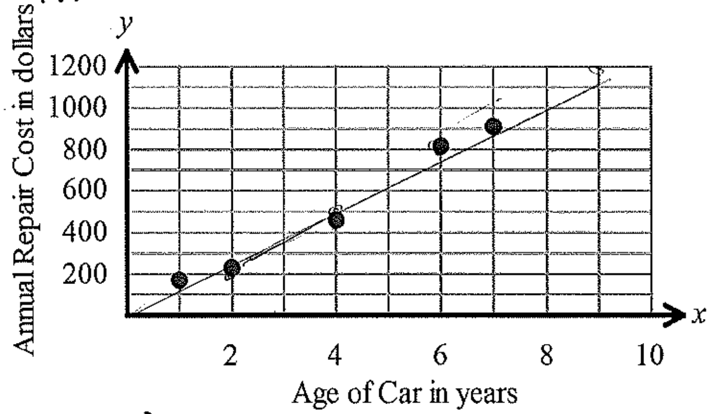
Line of Best Fit (or Trend Line)

- straight line ($y=mx+b$ typically) that goes "through" the data

$\frac{10}{8}=\frac{5}{4}$

- As many points above the line as below the line provides a better fit.

Example 2: Consider the scatter plot shown comparing the ages of cars and their annual cost of repairs.



a) Describe the correlation.

Positive

b) Draw a line of best fit (also called a trend line.) What is the equation of your line of best fit?

$y=\frac{5}{4}x$ (Answers will vary)

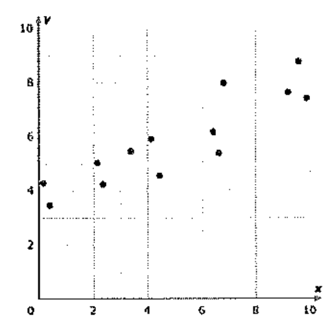
c) Use your trend line to predict the cost of repairs for a car that is 3 years old.

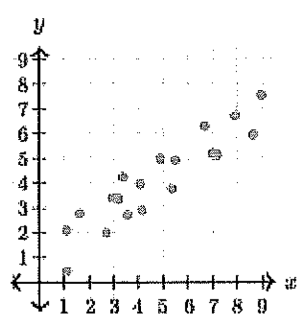
~ \$400

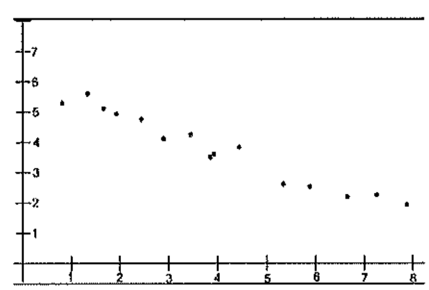
d) Use your trend line to predict the cost of repairs for a car that is 8 years old.

~ \$1000

Example 3: Match each scatter plot to the equation that could be a line of best fit for that scatter plot.

A) 

B) 

C) 

I) $y=-\frac{1}{3}x+6$

C

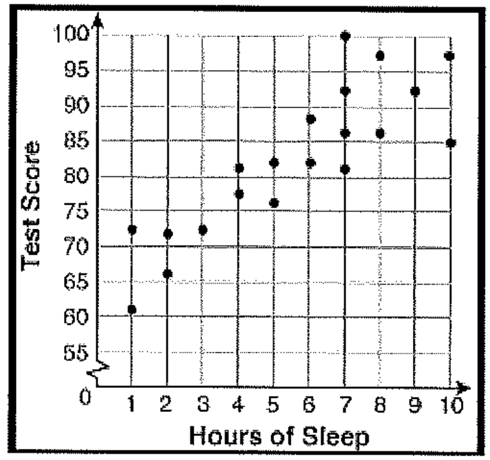
II) $y=\frac{1}{2}x+3.8$

A

III) $y=x+1$

B

Examples 4 – 5: Given the scatter plot shown, which shows the relationship between the number of hours of sleep (h) and the test score (T) for 20 students.



4) Which equation below would be a good trend line for the data?

A) $T = 2h - 65$
B) $T = 3h + 63$
C) $T = -1h + 64$
D) $T = 1h - 67$

5) Use your chosen trend line to predict the score for a student who gets 3.5 hours of sleep.

$3(3.5)+63$

73.5

Key Vocabulary

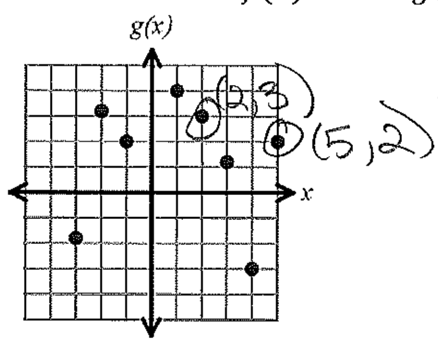
Average Rate of Change

Slope

Example 6: Find the average rate of change from $x = 2$ to $x = 5$ for $f(x)$ and for $g(x)$.

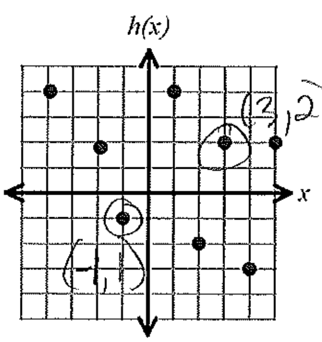
x	$f(x)$
1	-3
2	5
4	7
5	10
7	4

$\frac{10-5}{5-2} = \boxed{\frac{5}{3}}$



$\frac{2-3}{5-2} = \boxed{\frac{-1}{3}}$

You try! Example 7: Find the average rate of change from $x = -1$ to $x = 3$ for $h(x)$ and $b(x)$.



$\frac{2-1}{3+1} = \boxed{\frac{1}{4}}$

x	$b(x)$
-1	4
1	-3
2	0
3	-7
5	2

$\frac{-7-4}{3+1} = \boxed{\frac{-11}{4}}$