5.0 Operations with Polynomials

Examples: Simplify the following expressions.

1) $-2+3(t^2-6t+2)+(5t^2-t-8)-(6t+1)$

2) $20-\frac{1}{2}(-6t^2+8t-3)-3(t^2-6t+5)+(t-3)$

3) $Subtract:\ 5z^2-z+3\ from\ 4z^2+9k-12$

4) $Subtract:\ -3k^2+6k\ from\ 5k^2-k-4$

5) $-2(x+3)(3x^2-2x+4)$

6) $5(x^2+4)(-3x^2-8x-1)$

7) $(x-5)(x+2)(3x-1)$

8) $(2x-1)(x+4)(x-1)$

9) $2x(x-7)^2(x+3)$

10) $5x^2(x+3)^2(2x-3)$

11)Write a polynomial to represent the volume of a cylinder with a radius of $(3x-2)$ inches and a height of $(x+5)$ inches. Leave your answer in terms of pi.

12) Write a polynomial to represent the volume of a cone with a radius of $(x+6)$ inches and a height of $(2x-1)$ inches. Leave your answer in terms of pi. (hint: volume of a cone is one third the base times the height)

5.1 Notes: Factoring and Solving Polynomials

13) You are designing a marble planter for a city park. You want the length of the planter to be six times the height and the width to be three times the height. The sides should be one foot thick. Because the planter will be on the sidewalk, it does not need a bottom. What should the outer dimensions of the planter be if it is to hold 4 cubic feet of dirt?

5.3 Reading: The Rational Zeros Theorem

Taken from http://www.sparknotes.com/math/algebra2/polynomials/section4.rhtml

Italicized portions have been added by Ault.

Roots of a Polynomial

A root or zero of a function is a number that, when plugged in for the variable, makes the function equal to zero. Thus, the roots of a polynomial $f(x)$ are values of x such that $f(x) = 0$.

The Rational Zeros Theorem

The Rational Zeros Theorem states:

> If $f(x)$ is a polynomial with integer coefficients and if $\frac{p}{q}$ is a zero of $f(x)$, then p is a factor of the constant term of $f(x)$ and q is a factor of the **leading coefficient** of $f(x)$.

We can use the Rational Zeros Theorem to find [*a list of all the possible*] rational zeros of a polynomial.

Here are the steps:

1. Arrange the polynomial in descending order
2. Write down all the factors of the constant term. These are all the possible values of p .
3. Write down all the factors of the leading coefficient. These are all the possible values of q .
4. Write down all the possible values of $\frac{p}{q}$. Remember that since factors can be negative, $\frac{p}{q}$ and $-\frac{p}{q}$ must both be included. Simplify each value and cross out any duplicates.
5. Use synthetic division to determine the values of $\frac{p}{q}$ for which $f(\frac{p}{q}) = 0$. These are all the rational roots of $f(x)$.

Example: Find all the rational zeros of $f(x)$ given below.

$$f(x) = \underset{\underset{q}{\downarrow}}{2}x^4 + x^3 - 19x^2 - 9x + \underset{\underset{p}{\downarrow}}{9}$$

Factors of constant term (p): ±1 , ±3 , ±9 .

Factors of leading coefficient (q): ±1 , ±2 .

Possible values of $\frac{p}{q}$: $\pm\frac{1}{1}, \pm\frac{1}{2}, \pm\frac{3}{1}, \pm\frac{3}{2}, \pm\frac{9}{1}, \pm\frac{9}{2}$.

These can be simplified to: $\pm 1, \pm\frac{1}{2}, \pm 3, \pm\frac{3}{2}, \pm 9, \pm\frac{9}{2}$.

Use synthetic division to try *some of the possible* factors:

$$\begin{array}{r|rrrrr} 1 & 2 & 1 & -19 & -9 & 9 \\ & & 2 & 3 & -16 & -25 \\ \hline & 2 & 3 & -16 & -25 & -16 \end{array}$$

Remainder = -16. Not a zero.

$$\begin{array}{r|rrrrr} -1 & 2 & 1 & -19 & -9 & 9 \\ & & -2 & 1 & 18 & -9 \\ \hline & 2 & -1 & -18 & 9 & 0 \end{array}$$

Remainder = 0. Is a zero.

$$\begin{array}{r|rrrrr} 1/2 & 2 & 1 & -19 & -9 & 9 \\ & & 1 & 1 & -9 & -9 \\ \hline & 2 & 2 & -18 & -18 & 0 \end{array}$$

Remainder = 0. Is a zero.

$$\begin{array}{r|rrrrr} -1/2 & 2 & 1 & -19 & -9 & 9 \\ & & -1 & 0 & 19/2 & -1/4 \\ \hline & 2 & 0 & -19 & 1/2 & 35/4 \end{array}$$

Remainder = 35/4. Not a zero.

etc.

$$\begin{array}{r|rrrrr} 3 & 2 & 1 & -19 & -9 & 9 \\ & & 6 & 21 & 6 & -9 \\ \hline & 2 & 7 & 2 & -3 & 0 \end{array}$$

Remainder = 0. Is a zero.

$$\begin{array}{r|rrrrr} -3 & 2 & 1 & -19 & -9 & 9 \\ & & -6 & 15 & 12 & -9 \\ \hline & 2 & -5 & -4 & 3 & 0 \end{array}$$

Remainder = 0. Is a zero

By the Fundamental Theorem of Algebra, this degree 4 polynomial has four roots, and so no other possible values need to be checked. Thus, the rational roots of $f(x)$ are $x = -1$, $\frac{1}{2}$, -3, and 3. *An easier way to solve this problem is, after finding one rational root, attempt to factor the declining polynomial to find the other roots.*

5.4 Notes: Exploring Key Features of Polynomials

Polynomial
Function Families

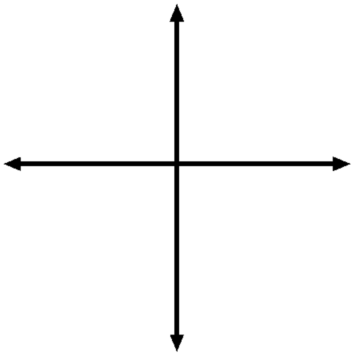
Linear	**Quadratic**
Cubic	**Quartic**

End Behavior

If we know the degree and leading coefficient of a polynomial, then we can get a general idea of what the graph would look like. End behavior describes what happens to the range as x goes to infinity and negative infinity.

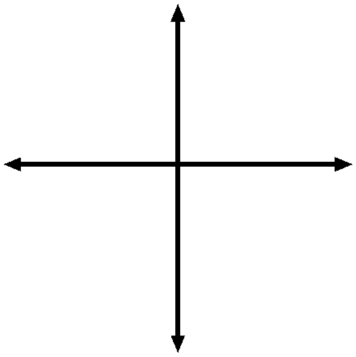
Degree:

Leading Coefficient:



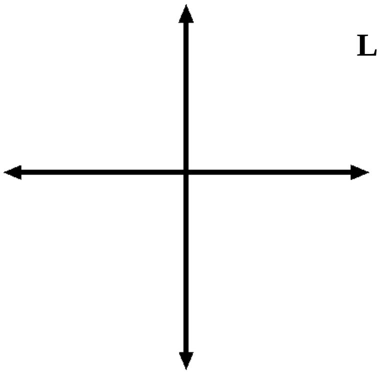
Degree:

Leading Coefficient:



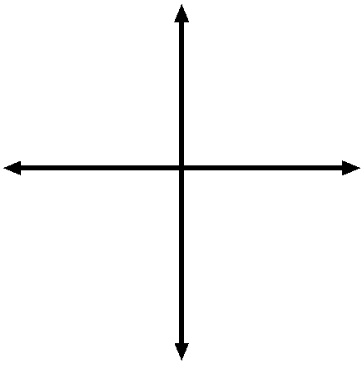
Degree:

Leading Coefficient:



Degree:

Leading Coefficient:



Examples: Describe the end behavior of the graph of the polynomial functions:

1) $f(x) = -2x^2 + 5x^4 - 3x + 7$

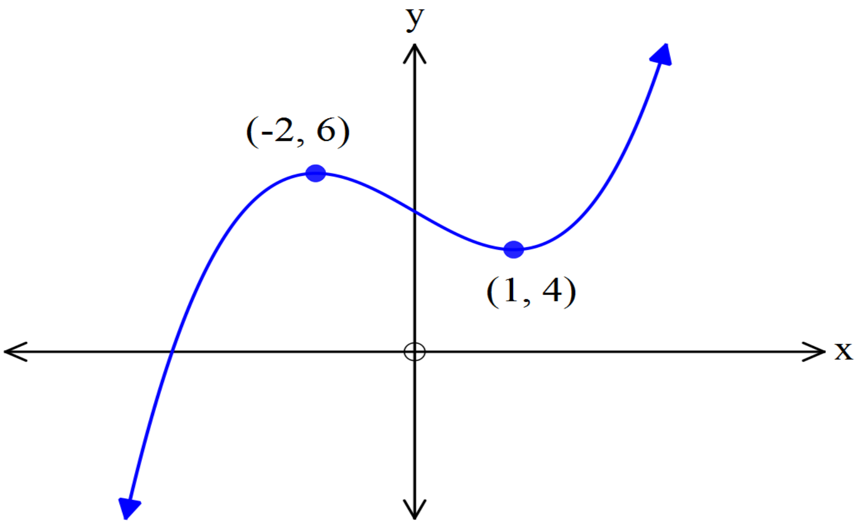
2) $f(x) = (5x - 3 - 8x^5 + 12x^2)(3x^4 - 1)$

3) Create a function that has the following end behavior:

$as\ x \to -\infty, f(x) \to -\infty \quad and \quad as\ x \to \infty, f(x) \to \infty$

Increasing and Decreasing

Is the function graphed below increasing, decreasing or both? Explain.

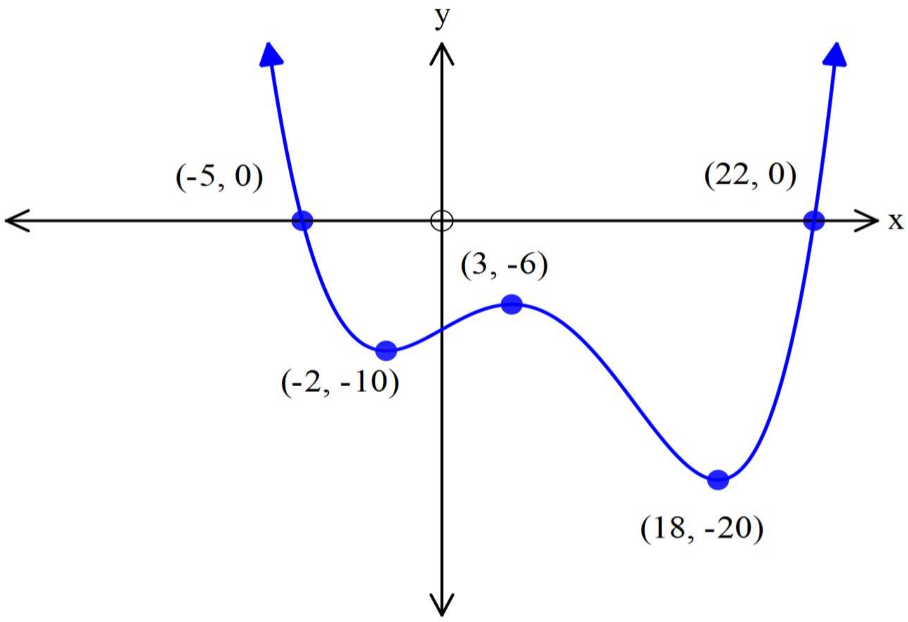


We describe the intervals where a function is either increasing or decreasing using the ___- values in interval notation.

Highlight where the function is increasing. Describe the sections in interval notation.

Highlight where the function is decreasing. Describe the section in interval notation.

Example: Describe the end behavior of the following function as well as the intervals where the function is increasing and decreasing.



End Behavior:

Increasing:

Decreasing:

For which x-values is the function positive?

For which x-values if the function negative?

Key Features

Example: Identify the key features of the function graphed below.

x-intercepts:

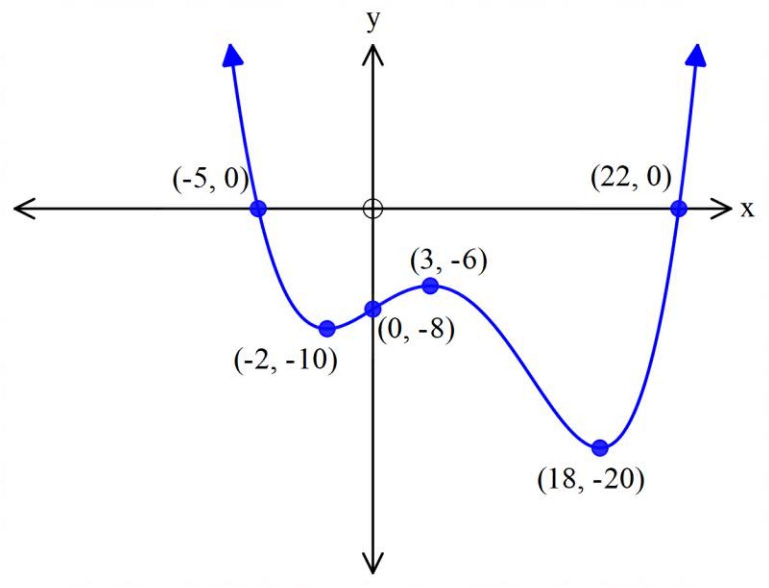
y-intercept:

Local (relative) Maximum:

Absolute Maximum:

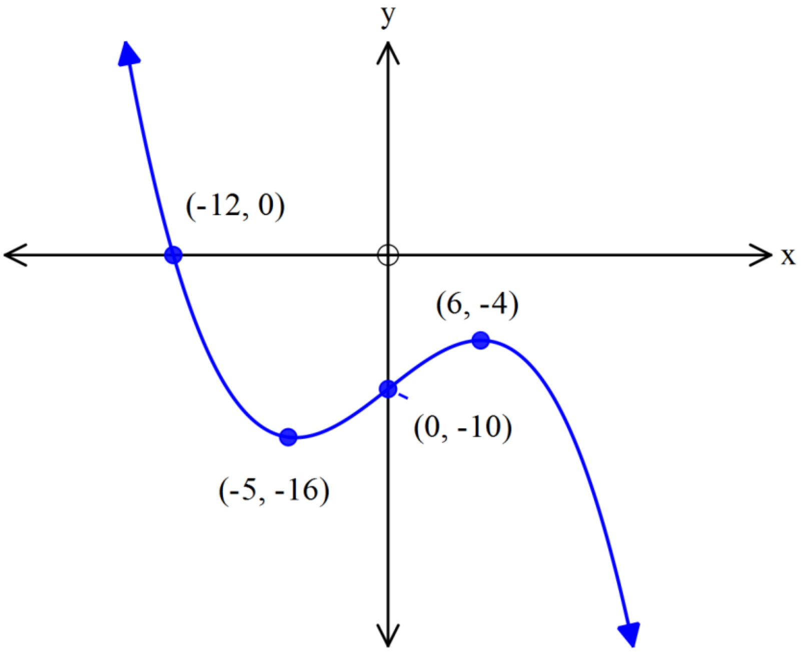
Local (relative) Minimum:

Absolute Minimum:



Example: Identify all the key features of the function graphed below.

Odd or Even Degree?



Positive or Negative Coefficient?

x-intercepts:

y-intercept:

Domain:

Range:

Local Max:

Max:

Local Min:

Min:

End Behavior:

Increasing:

Decreasing:

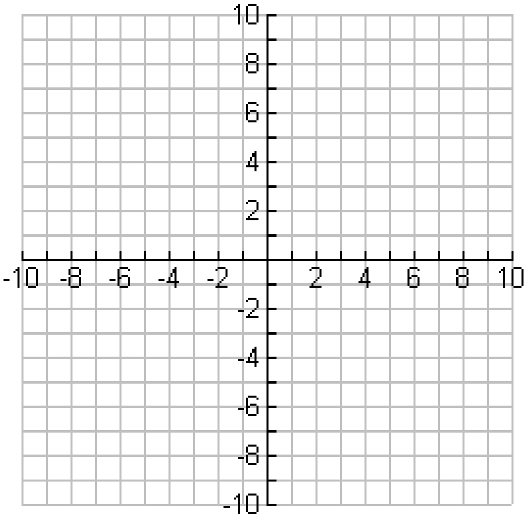
Positive:

Negative:

5.5 Notes: Exploring Key Features of Polynomials Part 2

Cubic Functions: Positive Leading Coefficient

Example: $f\ (x) = x^3 + 2x^2 - 5x - 6$



Odd or Even Degree?

x-intercepts:

Domain:

y-intercept:

Range:

End Behavior:

Local Max:

Local Min:

Max:

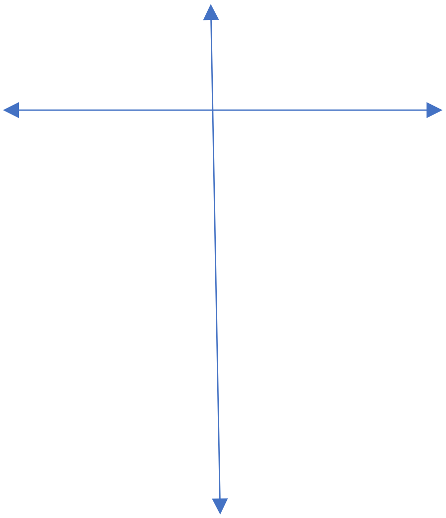
Min:

Increasing:

Decreasing:

Quartic Functions: Negative Leading Coefficient

Example: $f\,(x) = -x^4 + 8x^2 - 16$



Odd or Even Degree?

x-intercepts:

Domain:

y-intercept:

Range:

End Behavior:

Local Max:

Local Min:

Max:

Min:

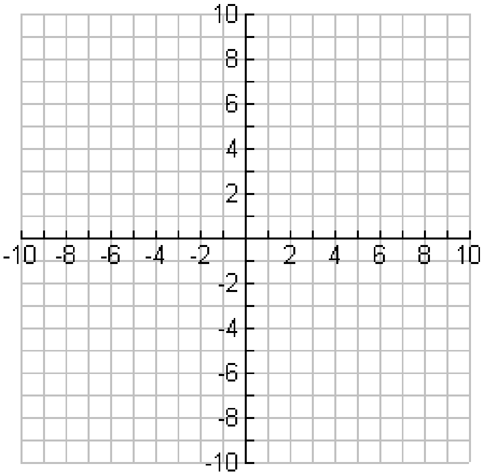
Increasing:

Decreasing:

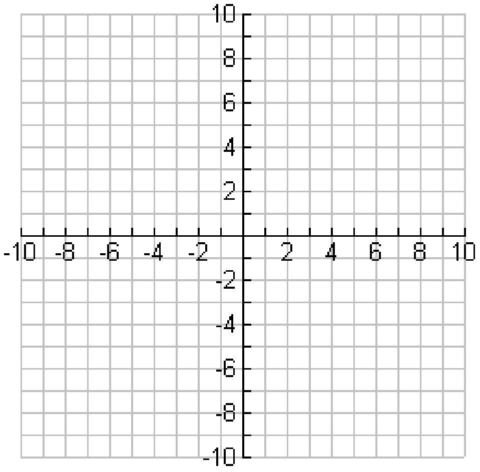
Graphing Cubic Functions in *h*, *k* form

$$y = a(x - h)^3 + k$$

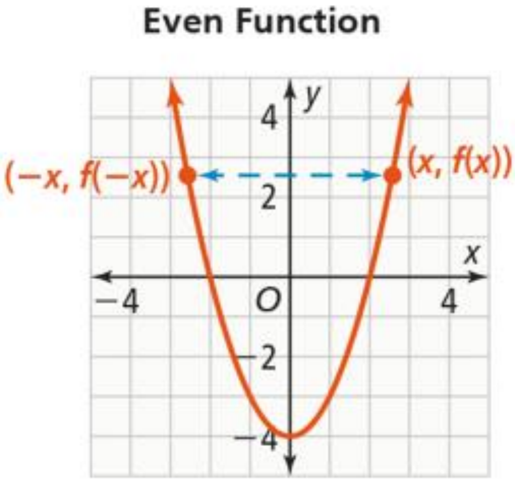
Example 1: $y = (x - 3)^3 - 1$

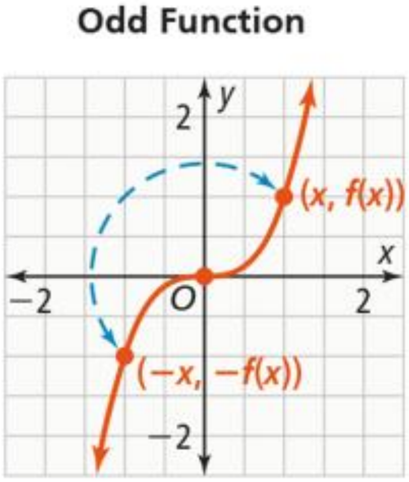


Example 2: $y = -\frac{1}{2}(x + 1)^3 - 4$



Even Functions & Odd Functions





For all x in the domain,
$f(x) = f(-x)$

For all x in the domain,
$f(-x) = -f(x)$

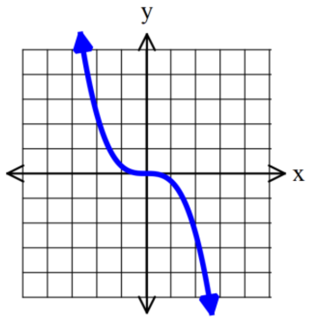
Example: Is $f(x)$ odd, even, or neither?

(a) $f(x) = 4x^4 + 5$

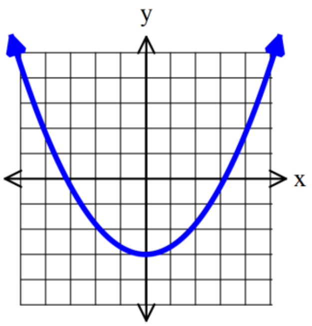
(b) $f(x) = 2x^3 + 3x$

Example: Is $f(x)$ odd, even, or neither?

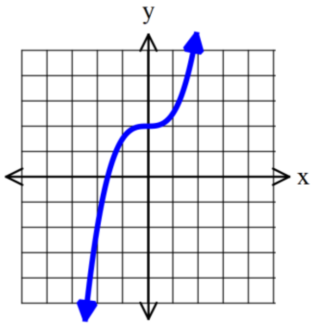
(a)



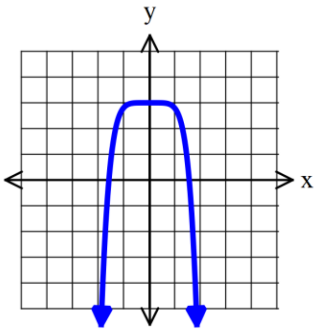
(b)



(c)



(d)

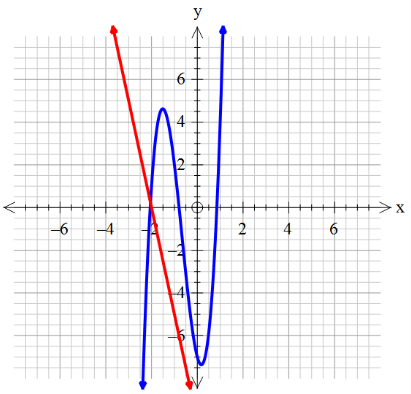


5.6 Notes: Writing Polynomial Functions/Models/Systems

5) According to data from the U.S. Census Bureau for the period 2000-2007, the number of male students enrolled in high school in the United States can be approximated by the function $M(x) = -0.004x^3 + 0.037x^2 + .049x + 8.11$ where x is the number of years since 2000 and $M(x)$ is the number of male students in the millions. The number of female students enrolled in high school in the United States can be approximated by the function $F(x) = -0.006x^3 + 0.029x^2 + 0.165x + 7.67$ where x is the number of years since 2000 and $F(x)$ is the number of female students in millions. Estimate the total number of students enrolled in high school in the United States in 2007.

Reflect: Explain how you can use the given information to estimate how many more male high school students than female high school students there were in the United States in 2007.

Systems



What are the solution(s) to the following system?

Why are they the solutions?