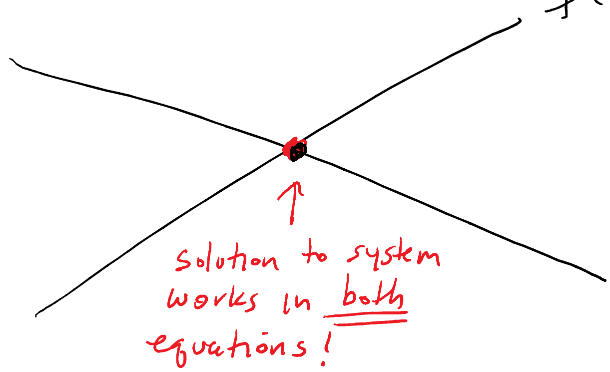
4 – 1 Notes: Solving Systems of Equations by Graphing

Learning Objectives: 1) Solve a system of equations by graphing and check your solutions by substitution, 2) identify a system with infinite or no solutions by their graphs, slopes and *y*-intercepts.



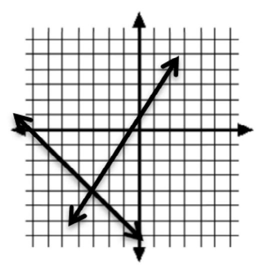
Key Vocabulary

System of Linear Equations:

Solution of a System of Linear Equations:

Example 1) What is the solution to the system shown?

(3, −4)



Example 2: Consider the system. $\begin{cases} y = -2x - 1 \\ x + y = 2 \end{cases}$

a) Is $(-3, 5)$ a solution for the system? How do you know?

check in both

$5 = -2(-3) - 1$
$5 = 6 - 1$
$5 = 5$ yes

$-3 + 5 = 2$
$2 = 2$
yes!

b) Is $(-1, 2)$ a solution for the system? How do you know?

$2 = -2(-1) - 1$
$2 = 2 - 1$
$2 = 1$
No

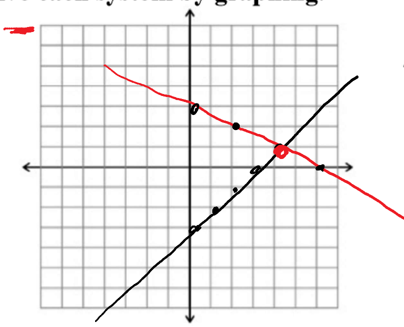
$-1 + 2 = 2$
$1 = 2$
no.

Must work in both, doesn't even work for one

Examples #3 – 8: Solve each system by graphing.

3) $\begin{cases} y = -\frac{1}{2}x + 3 \\ y = x - 3 \end{cases}$

(4, 1)



4) $\begin{cases} x - y = -2 \\ y = 2x + 1 \end{cases}$

$x - y = -2$

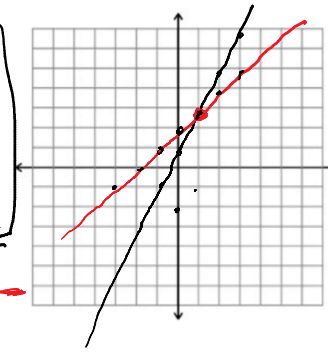
$-x \qquad -x$

$-y = -x - 2$

$y = x + 2$

$y = 2x + 1$

$\boxed{(1, 3)}$



Check your solution by using substitution with each equation:

$1 = -\frac{1}{2}(4) + 3$

$1 = -2 + 3$

$1 = 1$ ✓

$1 = 4 - 3$

$1 = 1$ ✓

$1 - 3 = -2$

$-2 = -2$ ✓

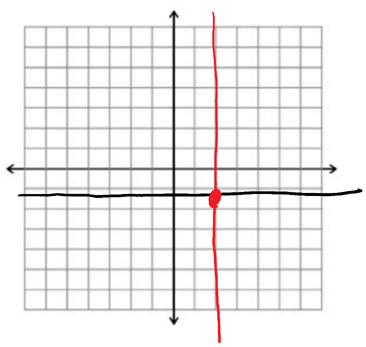
$3 = 2(1) + 1$

$3 = 2 + 1$

$3 = 3$ ✓

5) $\begin{cases} y = -1 \\ x = 2 \end{cases}$

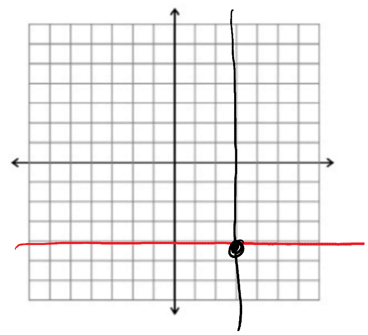
(2, −1)



You try #6 – 8!

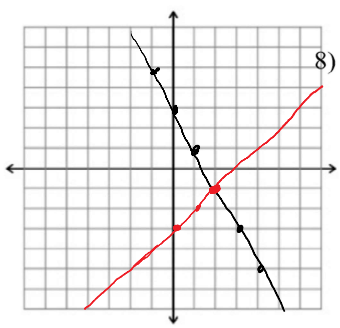
6) $\begin{cases} x = 3 \\ y = -4 \end{cases}$

(3, −4)



7) $\begin{cases} y = -2x + 3 \\ y = x - 3 \end{cases}$

(2, −1)



8) $\begin{cases} -4x + 4y = 4 \\ y = 3x + 5 \end{cases}$

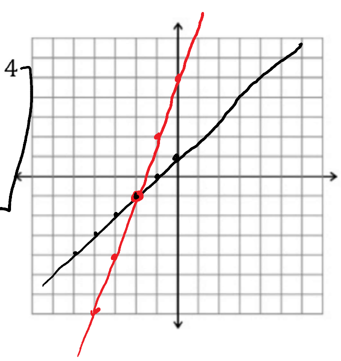
$-4x + 4y = 4$

$4y = 4x + 4$

$y = x + 1$

$y = 3x + 5$

(−2, −1)

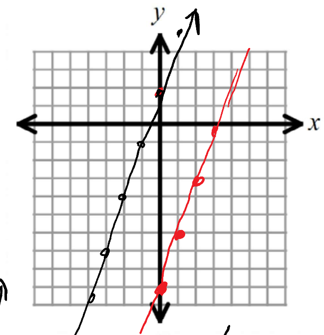


Example 9: Solve the system by graphing: $\begin{cases} y = 3x - 2 \\ 3x - y = 9 \end{cases}$

What do you think the solution is?

No solution.
The lines are parallel
So they will never
Intersect. Same slope, different y-int!

$3x - y = 9$
$-y = -3x + 9$
$y = 3x - 9$



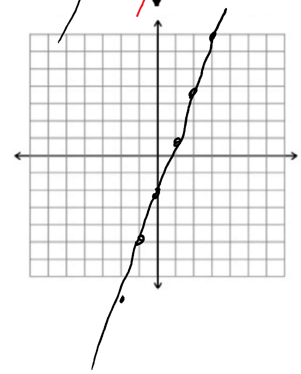
Example 10: Solve the system by graphing: $\begin{cases} y = 3x - 2 \\ 5y = 15x - 10 \end{cases}$

What do you think the solution is?

$\frac{5y}{5} = \frac{15x}{5} - \frac{10}{5}$

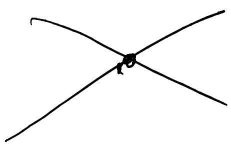
$y = 3x - 2$ same line

Infinite solutions
since they are the same line.

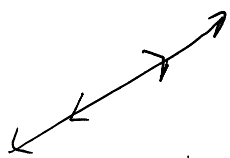


A system of linear equations can have …

One Solution

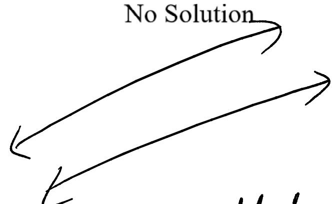


Infinitely Many Solutions



same line
same slope, same y-int

No Solution



parallel lines
same slope different y-int

Example 11: A system of two linear equations has infinitely many solutions. What must be true about the equations? Choose all that apply.

a) They are perpendicular.
b) They are the same line.
c) They have the same *y*-intercept.
d) They are parallel.

Example 12: Which of the following equations will have no solution with $6x + 2y = 8$? Use slopes and y-intercepts to help you decide!

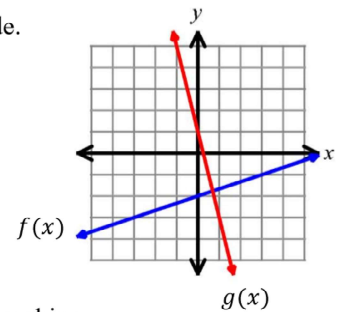
a) $y = -3x + 4$
b) $y = 6x - 5$
c) $y = -3x - 1$
d) $y = 6x + 8$

same slope;
different y-int

$6x + 2y = 8$
$-6x \quad\quad -6x$
$2y = -6x + 8$
$y = -3x + 4$

Example 13: The functions $f(x)$ and $g(x)$ are graphed to the side. Approximate the value of x when $f(x) = g(x)$?

$\approx (.7, -1.7)$ but how can we really tell?



Reflection: Describe some disadvantages to solving systems by graphing.

When you have non-integer solutions it is hard to read them off the graph!

Example 14: A coffee shop sells teas for \$4 each and coffees for \$5 each. If the coffee shop sold 9 drinks for a total of \$40, how many of each type of drink were sold?

$x+y=9$

a) Write two equations to model this situation.

$4x + 5y = 40$

$x + y = 9$

b) Solve the system by graphing.

Teas = x

Coffees = y

Easiest to use intercepts here!

$4(0)+5y=40$

$y\text{-int}=8$

$4x+5(0)=40$

$x\text{-int}=10$

$0+y=9$

$y \text{ int}=9$

$x+0=9$

$x\text{-int}=9$

$x=5$

$y=4$

but check to make sure by plugging in!

$4x+5y=40$

$4(5)+5(4)=40$

$20+20=40$ ✓

$x+y=9$

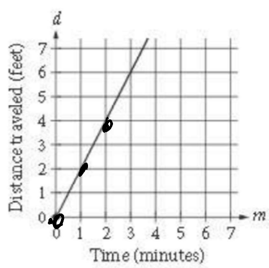
$5+4=9$ ✓

4 – 2 Notes: Solving Systems of Equations by Substitution

Learning Objectives: 1) Solve a system of equations by substitution, 2) identify what a system with infinite or no solutions looks like when using substitution, 3) model situations using a system.

Warm-up:

1) The graph shows the distance traveled *d*, in feet, by a product on a conveyer belt *m* minutes after the product is placed on the best. Which of the following equations correctly relates *d* and *m*?



A) $d = 2m$

B) $d = \frac{1}{2}m$

C) $d = m + 2$

D) $d = 2m + 2$

y-int = "d-int" = 0

slope = $\frac{2}{1}$ = 2

y = "d"
x = "m"

y = 2x + 0
d = 2m + 0

Key Vocabulary

Substitution:

If you know the value of one variable in a system, you can find the value of the other variable by ___________________________ the known value into one of the equations.

Example 1: Solve the system by using substitution. Check your solution.

$$\begin{cases} 8x + 3y = 25 \\ x = 2 \end{cases}$$

Sometimes you don't know a specific numerical value but you do have an equation with one variable isolated. You can substitute the **expression** into the other equation!

For Examples #2 – 7, solve each system.

2) $\begin{cases} y = -2x + 4 \\ x + y = -5 \end{cases}$

$x + (-2x + 4) = -5$
$-x + 4 = -5$
$-x = -9$
$x = 9$

$y = -2x + 4$
$y = -2(9) + 4$
$y = -18 + 4 = -14$

$\boxed{(9, -14)}$

3) $\begin{cases} 3x - 2y = 5 \\ y = 2x - 1 \end{cases}$

$3x - 2(2x - 1) = 5$
$3x - 4x + 2 = 5$
$-x + 2 = 5$
$-x = +3$
$x = -3$

Now find y
$3x - 2y = 5$
$3(-3) - 2y = 5$
$-9 - 2y = 5$
$-2y = 14$
$y = -7$

$\boxed{(-3, -7)}$

4) $\begin{cases} 6x - 3y = 12 \\ x = 2y + 2 \end{cases}$

$6(2y + 2) - 3y = 12$
$12y + 12 - 3y = 12$
$9y + 12 = 12$
$9y = 0$
$y = 0$

now x
$6x - 3(0) = 12$
$6x = 12$
$x = 2$

$\boxed{(2, 0)}$

check in your head!

5) $\begin{cases} y = x + 12 \\ y = -x - 2 \end{cases}$

$x + 12 = -x - 2$
$+x \quad\quad +x$
$2x + 12 = -2$
$-12 \quad -12$
$2x = -14$
$x = -7$

Now y
$y = -7 + 12$
$y = 5$

$\boxed{(-7, 5)}$

You try #6 – 7!

6) $\begin{cases} y = x + 1 \\ 2x + 2y = 10 \end{cases}$

$2x + 2(x + 1) = 10$
$2x + 2x + 2 = 10$
$4x + 2 = 10$
$4x = 8$
$x = 2$

$y = x + 1$
$y = 2 + 1$
$y = 3$

$\boxed{(2, 3)}$

7) $\begin{cases} x = y + 12 \\ x = y - 2 \end{cases}$

$y + 12 = y - 2$
$-y \quad\quad -y$
$12 = -2$

No solution!

Example 8: For the system $\begin{cases} 2x - 3y = 9 \\ 3x = 6y + 3 \end{cases}$ what is the most efficient way to isolate one variable? Solve the system.

Since multiples of 3 will be easy!

$\frac{3x}{3} = \frac{6y}{3} + \frac{3}{3}$

$\boxed{x = 2y + 1}$ Plug into other

$2(2y + 1) - 3y = 9$
$4y + 2 - 3y = 9$
$y + 2 = 9$
$y = 7$

$2x - 3y = 9$
$2x - 3(7) = 9$
$2x - 21 = 9$
$2x = 30$
$x = 15$

$\boxed{(15, 7)}$

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Example 9: Solve the system (by substitution so you see how it looks!): $\begin{cases} 5x - y = 8 \\ y = 5x - 8 \end{cases}$

$5x - (5x-8) = 8$

$0x + 8 = 8$ TRUE

Infinite solutions!

For Examples 10 – 12: Write a system of equations to model each situation, and then solve.

10) Lindsey and Gretchen work at two different hair salons and pay different amounts for their station. Lindsey pays $140 for rent, and $10 per customer that she works on that month. Gretchen only pays $100 for rent, but has to pay $18 per customer. How many customers would it take for them to pay the same amount?

L: $y = 140 + 10x$

G: $y = 100 + 18x$

$140 + 10x = 100 + 18x$

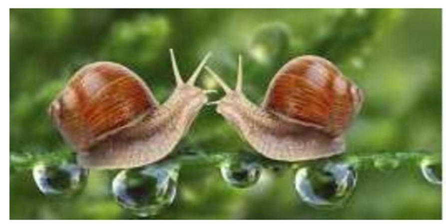
$-10x \qquad -10x$

$140 = 100 + 8x$

$40 = 8x$

$5 = x$

11) Two snails are moving along a branch. (They have a very exciting life!) Snail #1 starts at a position of 15 cm from the start of the branch and moves at 3 cm/min. Snail #2 starts at a position of 9 cm from the start of the branch and moves at 4 cm/min. After how many minutes will they be at the same position? What is their position at that time? NOTE: You have been asked 2 questions!



#1 $y = 15 + 3m$

#2 $y = 9 + 4m$

$15 + 3m = 9 + 4m$

$-3m \qquad -3m$

$15 = 9 + m$

$6 = m$

$y = 15 + 3(6)$

$y = 15 + 18$

$y = 33$

In 6 min they will be at 33 cm

12) Two numbers have a sum of 16. The larger number is one more than two times the smaller number. Find each number.

$x + y = 16$

$y = 1 + 2x$

$x + (1 + 2x) = 16$

$3x + 1 = 16$

$3x = 15$

$x = 5$

$y = 1 + 2(5)$

$y = 11$

5 & 11

4 – 3 Notes, Day 1: Solving Systems by Using Elimination

Learning Objectives: 1) Solve a system of equations by elimination by adding two equations, 2) identify situations when elimination is easier than substitution (or vice versa).

Warm-up:

1)

$$3x + x + x + x - 3 - 2 = 7 + x + x$$

In the equation above, what is the value of x?

A) $-\frac{5}{7}$

B) 1

C) $\frac{12}{7}$

D) 3

$6x - 5 = 7 + 2x$

$6x = 12 + 2x$

$4x = 12$

$x = 3$

2)

$$g(x) = 2x - 1$$
$$h(x) = 1 - g(x)$$

The functions g and h are defined above. What is the value of $h(0)$?

A) -2

B) 0

C) 1

D) 2

$h(0) = 1 - g(0)$

$g(0) = 2(0) - 1 = -1$

$h(0) = 1 - (-1)$

$= 2$

Key Vocabulary

Elimination

The elimination method eliminates (gets rid of) one of the variables when you add the two equations together. Then you will have an equation that you can Solve.

Steps in solving an equation by elimination:
1) If needed, put both equations in standard form.
2) Find a variable that will be easiest to eliminate. (when you add)
3) Add vertically.
4) Solve for remaining variable.
5) Substitute that value into an equation to solve for other variable.

Example 1: Solve by elimination.

$3x + 2y = 4$
$5x - 2y = 12$

$8x + 0y = 16$
$x = 2$

step 5)

$3(2) + 2y = 4$
$6 + 2y = 4$
$2y = -2$
$y = -1$

$(2, -1)$

For #2 – 5: Solve each system by the elimination method.

2) $\begin{cases} x + 7y = 13 \\ x - 7y = 5 \end{cases}$

$2x = 18$
$x = 9$

$9 + 7y = 13$
$7y = 4$
$y = \frac{4}{7}$

$\boxed{(9, \frac{4}{7})}$

3) $\begin{cases} 3x + 4y = 4 \\ -4y = 16 + 2x \end{cases}$

$3x + 4y = 4$
$-2x - 4y = 16$

$x = 20$

$3(20) + 4y = 4$
$60 + 4y = 4$
$4y = -56$
$y = -14$

$\boxed{(20, -14)}$

You Try!

4) $\begin{cases} 4x - 2y = -2 \\ 3x + 2y = -12 \end{cases}$

$7x = -14$
$x = -2$

$4(-2) - 2y = -2$
$-8 - 2y = -2$
$-2y = 6$
$y = -3$

$\boxed{(-2, -3)}$

5) $\begin{cases} 4x - 2y = 7 \\ 2y = 8 + 4x \end{cases}$

$4x - 2y = 7$
$-4x + 2y = 8$

$0x + 0y = 15$

Oh, oh!

No solution

6) In the system, $\begin{cases} 3x + 2y = 13 \\ 5x + 2y = 15 \end{cases}$, can you add the two equations to solve for one of the variables? If so, solve the system. If not, what could you do to make a system you could solve?

subtract instead!

$-2x + 0y = -2$

$x = 1$

$3(1) + 2y = 13$

$3 + 2y = 13$

$2y = 10$

$y = 5$

$\boxed{(1,5)}$

multiplying one equation by -1 and adding would also work

Sometimes the original system does not have opposite terms. You can change any equation by multiplying it by a negative one (or any other number) to make opposite terms.

For #7 – 10, solve each system.

7) $5x - 3y = 19 \rightarrow 5x - 3y = 19$

$(5x + 4y = 5) \cdot -1 = -5x - 4y = -5$

$-7y = 14$

$y = -2$

$5x - 3y = 19$

$5x - 3(-2) = 19$

$5x + 6 = 19$

$5x = 13$

$x = 13/5$

$\boxed{(\frac{13}{5}, -2)}$

8) $x - 3y = 7$

$3y = -23 - x \rightarrow$

$x - 3y = 7$

$x + 3y = -23$

$2x = -16$

$x = -8$

$x - 3y = 7$

$-8 - 3y = 7$

$-3y = 15 \quad y = -5$

$\boxed{(-8, -5)}$

9) $x + y = 3 \rightarrow x + y = 3$

$(x + y = 5) \cdot -1 = -x - y = -5$

$0x + 0y = -2$

$0 = -2$

No solution!

10) $0.25x - 0.05y = 1$

$(0.25x + 0.1y = 2.5) \cdot -1 \rightarrow$

$.25x - .05y = 1$

$-.25x - .1y = -2.5$

$-.15y = -1.5$

$y = 10$

$x = 6$

OR

$100(.25x - .05y = 1)$

$25x - 5y = 100$

$100(.25x + .1y = 2.5)$

$25x + 10y = 250$

Now $25x - 5y = 100$

$(25x + 10y = 250) \cdot -1 =$

$25x - 5y = 100$

$-25x - 10y = -250$

$-15y = -150$

$y = 10$

11) The Spanish club sells food at sporting events. At the football game they charge \$3 for the popcorn and \$1 for the sodas. They made \$75 at the football game. At the track meet they sold the popcorn for \$2 and the sodas for \$1. They made \$55 at the track meet. How many bags of popcorn and sodas did they sell, if they sold the same at both games?

p = popcorn   s = soda

F \$ $3p + 1s = 75$ $\qquad 3p + 1s = 75$

T \$ $(2p + 1s = 55) -1 = \quad -2p - 1s = -55$

$p = 20$

$s = 15$

$\boxed{\begin{matrix} 20 & p \\ 15 & s \end{matrix}}$

Reflect: Explain how you know is better to use elimination versus substitution to solve a system of equations? When is substitution better then elimination?

elimination better when you have opposite terms (in standard form)

substitution better when one variable is solved in terms of another (or can be easily)

Example 12: Determine which method of solving is easiest for each system. Write "Elimination" or "Substitution." <u>**Do not solve the systems.**</u>

a) $\begin{cases} 4x - 3y = 9 \\ 7x + 3y = 2 \end{cases}$ b) $\begin{cases} y = 6x - 3 \\ x = 2y \end{cases}$

Elimination Substitution

Example 13: Solve the system below by using substitution. Then solve it again by using elimination. Which method do you prefer? Why?

$\begin{cases} 2x = 12 + 2y \\ 2x + 2y = 48 \end{cases} \rightarrow x = 6 + y$

$2(6+y) + 2y = 48$

$12 + 2y + 2y = 48$

$12 + 4y = 48$

$4y = 36$

$y = 9$

$2x + 2y = 48$

$2x + 2(9) = 48$

$2x + 18 = 48$

$2x = 30$

$x = 15$

(15, 9)

EASIER!

$\begin{cases} 2x = 12 + 2y \\ 2x + 2y = 48 \end{cases} \rightarrow$

$2x - 2y = 12$

$2x + 2y = 48$

$4x = 60$

$x = 15$

$2x + 2y = 48$

$2(15) + 2y = 48$

$30 + 2y = 48$

$2y = 9$

$y = 9$

(15, 9)

4 – 3 Notes, Day 2: More Solving Systems with Elimination

Learning Objectives: 1) Solve a system of equations by elimination by adding two equations after multiplying one or both equations by a constant, 2) identify situations when elimination is easier than substitution (or vice versa).

Warm-up: $+ -4a - 6b + 5c$

1) $(a + 2b + 3c) - (4a + 6b - 5c)$ is equivalent to:

A. $-4a - 8b - 2c$
B. $-4a - 4b + 8c$
C. $-3a + 8b - 2c$
D. $-3a - 4b - 2c$
E. $-3a - 4b + 8c$

2) As part of a lesson on motion, students observed a cart rolling at a constant rate along a straight line. As shown in the chart below, they recorded the distance, y feet, of the cart from a reference point at 1-second intervals from $t = 0$ seconds to $t = 5$ seconds.

t	0	1	2	3	4	5
y	14	19	24	29	34	39

Which of the following equations represents this data?

F. $y = t + 14$
G. ~~$y = 5t + 9$~~
H. $y = 5t + 14$
J. ~~$y = 14t + 5$~~
K. $y = 19t$

$m = \frac{19-14}{1-0} = 5$

y-int = 14

Key Vocabulary

Elimination with Multiplication – multiplying one or both equations by numbers to create opposite terms (before adding).

Often the equations in a system aren't easy to graph, aren't easy to use the substitution method, and won't eliminate a variable if you just add the equations. We can use multiplication with one or both equations so that one variable will eliminate.

Example 1: $\begin{cases} 3x - 4y = 10 \\ (x + 2y = 0)\,2 \end{cases} \rightarrow$

$3x - 4y = 10$
$2x + 4y = 0$
$5x = 10$
$x = 2$
$y = -1$

$(2, -1)$

Steps:
1) If needed, write each equation in standard form.
2) Decide which variable you would like to eliminate.
3) Multiply one or both equations by a constant in order to create opposite coefficients for the x and/or y terms.
4) Add vertically.
5) Solve for one variable.
6) Use substitution and solve for the other variable.

Solve the systems:

Reflect: how is 3) different than 1) and 2) ?

must multiply both equations

2) $\begin{cases} 2x = y + 10 \\ x + 2y = 5 \end{cases}$

$(2x - y = 10)\,2 \rightarrow 4x - 2y = 20$
$x + 2y = 5 \qquad x + 2y = 5$
$5x = 25$
$x = 5$
$y = 0$

$(5, 0)$

3) $\begin{cases} 2x - 7y = 20 \\ 5x + 8y = -1 \end{cases}$

Eliminate x

$(2x - 7y = 20)\,5 \rightarrow 10x - 35y = 100$
$(5x + 8y = -1)\,{-2} \quad -10x - 16y = 2$
$-51y = 102$
$y = -2$

Eliminate y

$(2x - 7y = 20)\,8 \rightarrow 16x - 56y = 160$
$(5x + 8y = -1)\,7 \rightarrow 35x + 56y = -7$
$51x = 153$
$x = 3$

solution $\boxed{3, -2}$

You Try 4!

4) $\begin{cases} (2x - 3y = 5)\,{-2} \\ 5x - 6y = 12 \end{cases}$

$-4x + 6y = -10$
$5x - 6y = 12$
$x = 2$
$y = -\frac{1}{3}$

$2x - 3y = 5$
$2(2) - 3y = 5$
$4 - 3y = 5$
$-3y = 1$
$y = -\frac{1}{3}$

You Try 5! What happened?

5) $\begin{cases} -6x + 2y = 12 \\ (-3x + y = 6)\,{-2} \end{cases}$

$-6x + 2y = 12$
$6x - 2y = -12$
$0x + 0y = 0$

infinite solutions!

Solve the systems:

6) $\begin{cases} 3x = 4y - 5 \\ -6x + 8y = 2 \end{cases}$

$(3x - 4y = -5)2 \rightarrow 6x - 8y = -10$

$-6x + 8y = 2 \qquad -6x + 8y = 2$

$0x + 0y = -8$

$0 = -8$

no solution

Reflect: Describe the difference between systems with no solution and those with infinitely many solutions. How can you tell which is which? Look at examples 5) and 6) if you need help!

NO solution

$0x + 0y = \#$

or $0 = \#$

INFINITE

$0x + 0y = 0$

or $0 = 0$

Example 7: A store sells guitars and basses. In one day, a total of 5 instruments were sold. If guitars sell for $200 each and basses sell for $150 each, and the total cost was $900, then write a system of equations to represent this situation. Use *x* for number of guitars sold and *y* for number of basses sold.

$(x + y = 5)(-150) \rightarrow -150x - 150y = -750$

$200x + 150y = 900 \longrightarrow 200x + 150y = 900$

$50x = 150$

$x = 3$

$y = 2$

Example 8: Susan is buying black and green olives from the olive bar for her party. She buys 4 lb of olives. Black olives cost $3.00 a pound. Green olives cost $5.00 a pound. She spends $15.50. Write a system of equations to represent this situation. Use *B* for number of pounds of black olives and *G* for number of pounds of green olives purchased.

$(B + G = 4)(-3) \rightarrow -3B - 3G = -12$

$3B + 5G = 15.50 \qquad 3B + 5G = 15.50$

$2G = 3.50$

$G = \frac{3.50}{2} = 1.75$

$B = 2.25$

Example 10: Josie owns a nail shop that charges \$12 for a manicure and \$20 for a pedicure. Her cousin owns a shop and charges \$16 for a manicure and \$30 for a pedicure. On Monday they compared how much they made. Josie made \$520 and her cousin made \$760. If they both sold the same number of pedicures and manicures, how many pedicures and manicures did they each sell? *NOTE: You will need to multiply BOTH equations by a constant on this one!*

m = mani
p = pedi

$(12m + 20p = 520)(-3) \rightarrow -36m - 60p = -1560$
$(16m + 30p = 760)(2) \quad 32m + 60p = 1520$
$-4m = -40$
$\boxed{\cancel{m = 10} \quad p = 20}$

$12m + 20p = 520$
$12(10) + 20p = 520$
$20p = 400 \quad p = 20$

Example 11: Which equation would make this system have an infinite number of solutions? Choose all that apply. $\{y = x + 2$ NONE!

same slope, same y-int!

A) $2y = 4x + 8$
$y = 2x + 4$
NO

B) $y - x = 3$
$y = x + 3$
NO

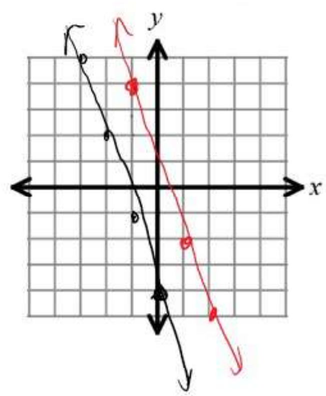
C) $5x - 5y = 10$
$-5y = -5x + 10$
$y = x - 2$
NO

D) $-4x + 4y = 12$
$4y = 4x + 12$
$y = x + 3$
NO

Example 12: What is the solution to the system below, where the first line is written as an equation, and the second line is given as a set of ordered pairs?

Line 1	Line 2
$y = -3x - 4$	x: -1, 1, 2; y: 4, -2, -5

x	y
-1	4
1	-2
2	-5



parallel.
No solution!

$2 < 8$ TRUE

$-2 < -5$ FALSE

4 – 4 Notes: Graphing Linear Inequalities

Learning Objectives:

1) Graph a linear inequality and identify multiple solutions and non-solutions,

Warm-up:

1) The equations below are linear equations of a system where $a, b,$ and c are positive integers.

$$ay + bx = c$$
$$ay + bx = c$$

Which statement below is true at least once for such a system of equations? Choose all that apply.

A) The two lines are parallel.
B) The system represents a single line.
C) The two lines intersect at one point.
D) There is no solution.
E) There are infinitely many solutions.
F) There is one solution.

2) Which statement below shows the solution of the inequality for y? $x - y < 3$

A) $y < x - 3$ 6
B) $y > x + 3$
C) $y < -x - 3$
D) $y > x - 3$ 8

$-x \quad -x$

$-y < -x + 3$

$\overline{-1} \quad \overline{-1} \; \overline{-1}$

$y > x - 3$

Key Vocabulary

Linear Inequality

$3x + 4y < 12$ examples

$y \geq mx + b$

inequality

Example 1: Which ordered pair is **not** a solution of $x - 3y \leq 6$? Choose all that apply.

A) (0,0)

$0 - 3(0) \leq 6$

$0 \leq 6$

TRUE

B) (6,-1)

$6 - 3(-1) \leq 6$

$6 + 3 \leq 6$

$9 \leq 6$

FALSE

C) $\left(4, -\frac{5}{3}\right)$

$4 - \frac{3}{1}\left(-\frac{5}{3}\right) \leq 6$

$4 \; \frac{+15}{3}$

$4 + 5 \leq 6$

FALSE

Graphing a linear inequality in two variables:

Remember

$x < 5$ $\quad$ $x \leq 5$

5 $\quad$ 5

DECISIONS

1) Dashed line or solid line

2) Shade up or down

$y \geq \; y >$ $\quad$ $y \leq \; y <$

Dashed	Solid
$y >$	$y \geq$
$y <$	$y \leq$

For examples 2 – 9: Graph each inequality.

$x < 5$ $x \leq 5$

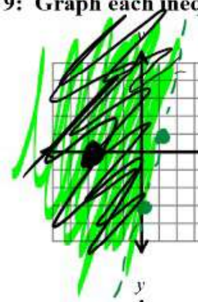
② Shade up or down $y \geq$ $y >$ $y \leq$ $y <$

$y <$ $y \leq$

For examples 2 – 9: Graph each inequality.

2) $y > 4x - 3$

$y = 4x - 3$

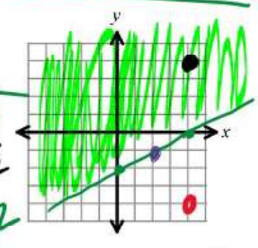


3) $x - 2y \leq 4$

$-x \quad -x$

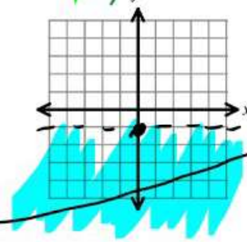
$\frac{-2y}{-2} \leq \frac{-x}{-2} + \frac{4}{-2}$

$y \geq \frac{1}{2}x - 2$



4) $\frac{5y}{5} < \frac{-5}{5}$

$y < -1$

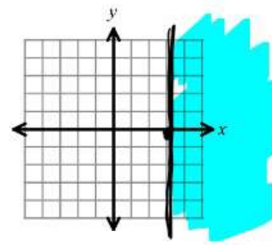


horiz

< or x

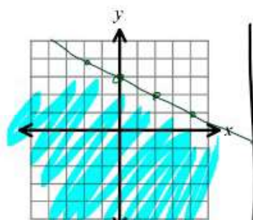
5) $\frac{-3x}{-3} \leq \frac{-9}{-3}$

$x \geq 3$



You try!

6) $y \leq -\frac{1}{2}x + 3$

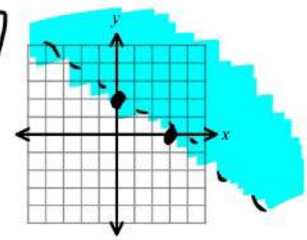


7) $2x + 3y > 6$

$-2x \quad -2x$

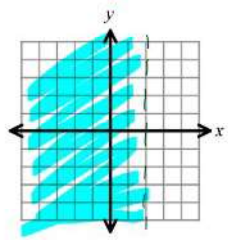
$\frac{3y}{3} > \frac{-2x}{3} + \frac{6}{3}$

$y > -\frac{2}{3}x + 2$



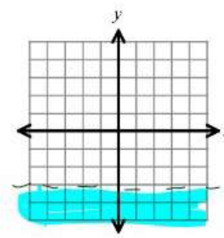
8) $2x < 4$

$x < 2$



9) $-4y > 12$

$y < -3$



For examples 10 – 13: Match each linear inequality to its graph below.

10) $y \geq 3x - 1$ ✓ B

11) $y \leq 3x - 1$ ✓ A

12) $-y > 3x - 1$ D

$\frac{-y}{-1} \quad \frac{3x}{-1} \quad \frac{-1}{-1}$

$y < -3x + 1$

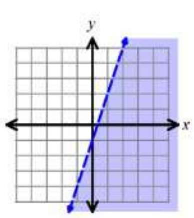
13) $-3(y - 1) \geq 9x$ C

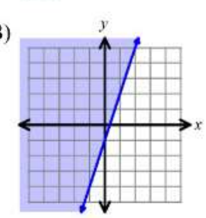
$-3y + 3 \geq 9x$

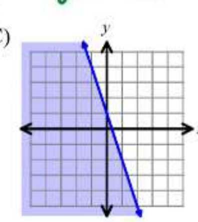
$-3 \quad -3$

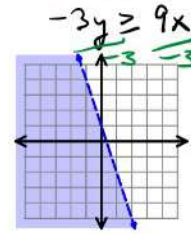
$\frac{-3y}{-3} \geq \frac{9x}{-3} - \frac{3}{-3}$

$y \leq -3x + 1$

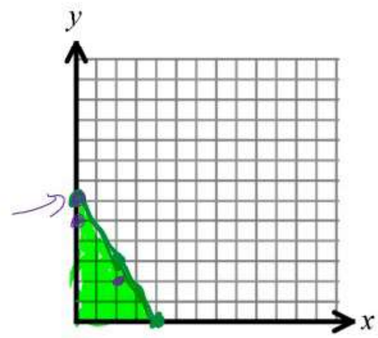
A) 

B) 

C) 

D) 

Example 14: You are going clothes shopping and can spend **at most** $\leq$ \$120 on clothes. It costs \$30 for a pair of pants and \$20 for a shirt. Let x represent the number of pants you can buy. Let y represent the number of shirts you can buy.



a.) Write and graph an inequality that describes the different number of pants and shirts you can buy.

$$\$30x + \$20y \leq \$120$$

$$-30x \qquad -30x$$

$$\frac{20y}{20} \leq \frac{-30x}{20} + \frac{120}{20}$$

$$\boxed{y \leq -\frac{3}{2}x + 6}$$

b.) Give three possible combinations of pants and shirts that you can buy.

WORKS	DOES NOT WORK
(0,6)	(3,4)
(0,5)	
(2,2)	

4 – 5 Notes: Systems of Linear Inequalities

Learning Objectives: 1) Graph a system of linear inequalities and identify multiple solutions and non-solutions, 2) Write and solve a system of linear inequalities from a word problem.

Warm-up:

1) Which of the following expressions has an even integer value for all integers a and c?

Counterexamples

a) $8a + 2ac$
b) $3a + 3c$ $\quad 3(1)+3(2) = 9$
c) $2a + c$ $\quad 2(1)+5=7$
d) $a + 2c$ $\quad 3+2(4)=11$
e) $ac + a^2$ $\quad 3\cdot 4+3^2 = 12+9=21$

even · even = even
2 · (anything) = even

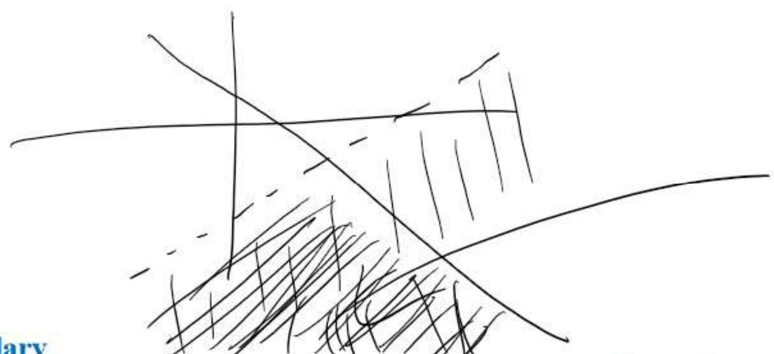
2) For what value of a would the following system of equations have an infinite number of solutions?

$$(2x - y = 8)\cdot -3$$
$$6x - 3y = 4a$$

$-6x+3y=-24$
$6x-3y=4a$
$0+0=-24+4a$
$-24+4a=0$
$a=6$

A. 2
B. 6
C. 8
D. 24
E. 32

$0+0=0$ Infinite solutions



Solution region!

Key Vocabulary

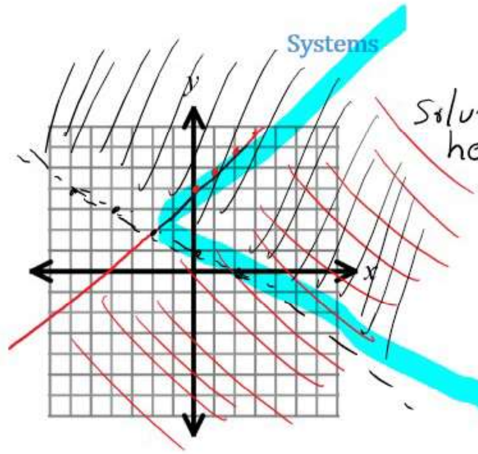
System of Linear Inequalities: A system of linear inequalities consists of two or more linear inequalities that have the same variables. The solution of a system of linear inequalities are all the ordered pairs that make all the inequalities in the system true.

Solution most of time is region!

Example 1) Solve the system of inequalities by graphing.

$$\begin{cases} y > -\frac{1}{2}x + 1 \\ y \le x + 4 \end{cases}$$

dashed. above - - -
solid below —



Reflect: Which of the following ordered pairs are solutions to the previous system? How can you tell?

(0, 0)	(2, 3)	(-4, 2)	(-2, 4)	(2, 0)	(5, 0)	(0, 4)
N	Y	N	N	N!	Y	Y

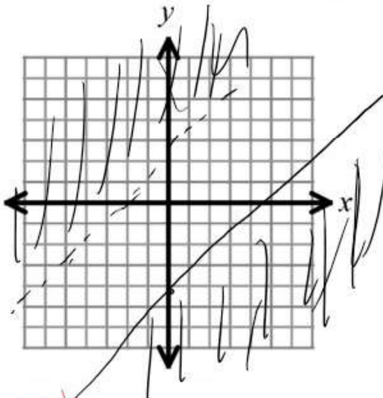
1) see if in solution area or 2) plug into both to test

For Examples 2 – 5: Graph each system of linear inequalities

2) $\begin{cases} y > x + 3 \\ -x + y \le -5 \end{cases}$

above
below $y \le x - 5$

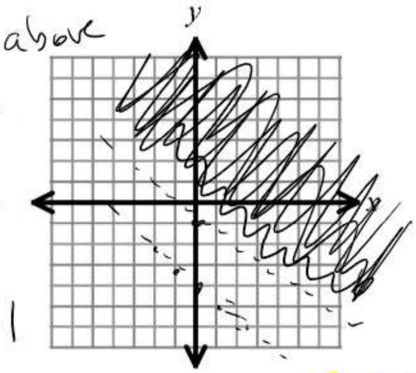
No solution
no overlap!



3) $\begin{cases} y > -x - 4 \\ y > -x - 1 \end{cases}$

above
above

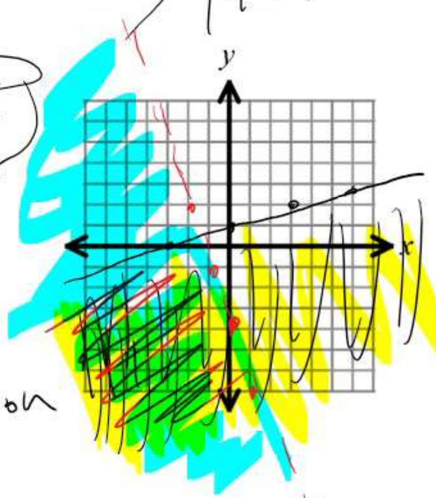
overlap is only above $y > -x - 1$



4) $\begin{cases} -\frac{1}{3}x + y \le 1 \\ y < -3x + 4 \end{cases}$

below

$y \le \frac{1}{3}x + 1$
below

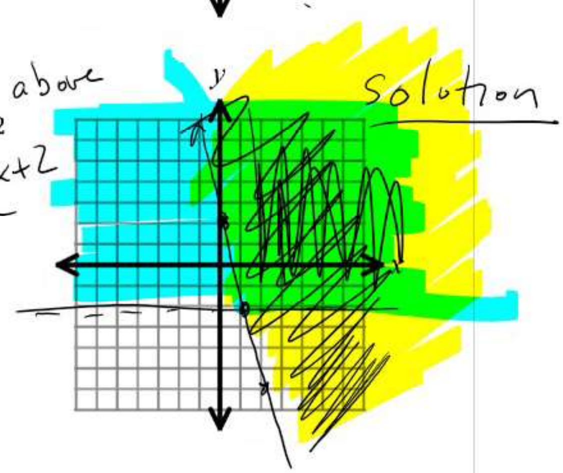


solution

5) $\begin{cases} y \ge -2 \\ 4x + y \ge 2 \end{cases}$

above

$y \ge -4x + 2$
above



For #6 – 7, graph the system of inequalities

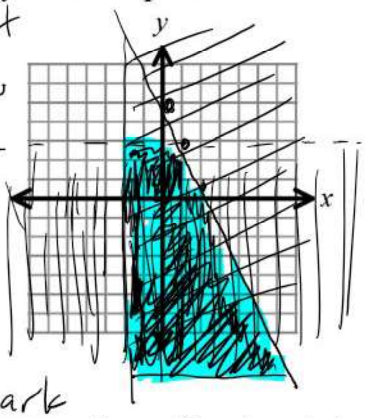
For #6 – 7, graph the system of inequalities.

6) $\begin{cases} x \geq -2 \\ 6x + 3y \leq 15 \\ y < 3 \end{cases}$ right / below

$3y \leq -6x + 15$

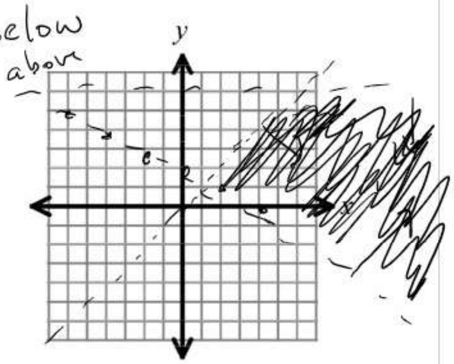
$y \leq -2x + 5$

below



Solution area dark

7) $\begin{cases} y \leq 6 \\ y > -\frac{1}{2}x + 2 \\ y < x \end{cases}$ below / above / below



Example 8: Given the system of inequalities shown below, determine all the points that are solutions to this system of inequalities.

$\begin{cases} x + y < 3 \\ 2x - y > 6 \end{cases}$

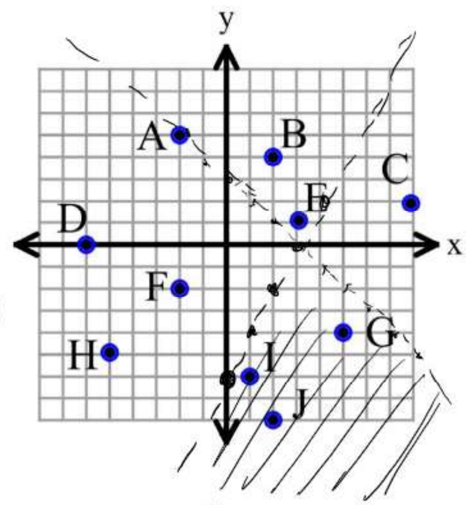
$y < -x + 3$ below

$y < 2x - 6$ below

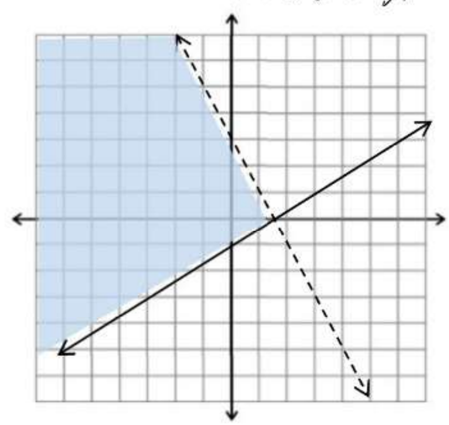
$-y > -2x + 6$

$y < 2x - 6$ switch sign!

I, J, G



Example 9: The solution set of a system of inequalities is shown in the graph as a shaded region. The equations of the boundaries are $y = \frac{2}{3}x - 1$ and $-2x - y = -3$. Write a system of linear inequalities that could represent the solution.



$\boxed{y \geq \frac{2}{3}x - 1 \quad \text{(shaded above)}}$

$-2x - y = -3$

$-y = 2x - 3$

$y = -2x + 3$

$\boxed{y < -2x + 3 \quad \text{(below, dotted)}}$

Example 10: Penelope is selling bracelets and earrings to make money for summer vacation. The bracelets cost \$2 and earrings cost \$3. She needs to make at least \$600. Penelope knows that she will sell more than 50 bracelets. Use x for # of bracelets and y for # of earrings.

a) Write a system of inequalities to represent this situation.

$2x+3y \geq 600$

$x > 50$ right

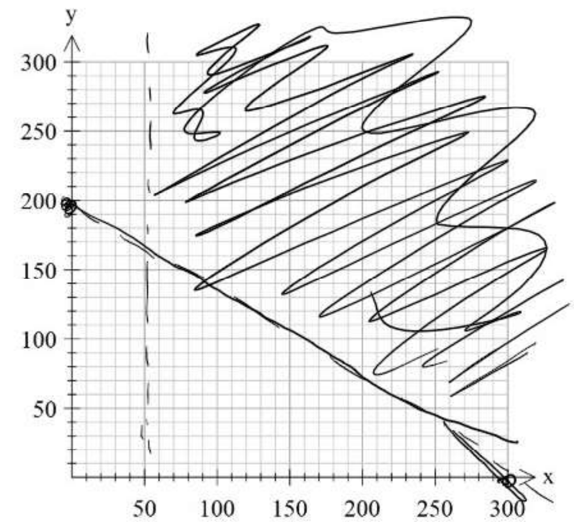
b) Graph the two inequalities and shade the intersection.

$3y \geq -2x+600$

$y \geq -\frac{2}{3}x+200$ above

note scale of graph!

go down 200, right 300



c) Name three combinations of bracelets and earrings that Penelope could see.

(150, 150) (200, 300) (300, 0)!

NOT (50, 200)

Example 11) If the system of inequalities $y \geq 2x+1$ and $y > \frac{1}{2}x - 1$ is graphed, which quadrant contains no solution to the system?

A) Quadrant II
B) Quadrant IV
C) Quadrant III
D) There are solutions in all four quadrants.