6.0 Notes: Review of Radicals and Proportions

Ratio	**A** relationship **of two numbers. Ratios can be written as a** fraction**, in words, or with a** colon**.**	**Sample:** 2:3 2 girls to 3 boys $\frac{2}{3}$
Proportion	**Two or more** ratios **set** equal **to each other.**	**Sample:** $\frac{2}{3} = \frac{4}{6} = \frac{10}{15}$ (×2, ×5)
Solving Proportions	**Steps for Solving Proportions:** 1) Cross-multiply 2) Set the products from step 1 equal to each to each other. 3) Solve for the variable.	

For #1 – 4, solve each proportion for the variable.

1) $\frac{4}{9} = \frac{x}{7}$

$\frac{9x}{9} = \frac{28}{9}$

$\boxed{x = \frac{28}{9} \text{ or } 3.\overline{1}}$

2) $\frac{(b-2)}{6} = \frac{3}{5}$

when there is an expression, you MUST ☆DISTRIBUTE

put parentheses around it

$5(b-2) = 18$

$5b - 10 = 18$

$+10 \quad +10$

$\frac{5b}{5} = \frac{28}{5}$

$\boxed{b = \frac{28}{5} \text{ or } 5.6}$

You Try #3 – 4!

3) $\frac{5}{d} = \frac{4}{3}$ $\boxed{3.75}$ or $\boxed{\frac{15}{4}}$

4) $\frac{11}{2} = \frac{8}{3+y}$ $\boxed{-\frac{17}{11}}$ or $\boxed{-1.\overline{54}}$

5) Solve the proportion for h: $\frac{(4+h)}{3} = \frac{(h-2)}{10}$ $\boxed{\text{Distribute 2 times}}$

$10(4+h) = 3(h-2)$

$40 + 10h = 3h - 6$
$-3h \quad -3h$

$40 + 7h = -6$
$-40 \quad -40$

$7h = -46$

$\frac{\cancel{7}h}{\cancel{7}} = \frac{-46}{7}$

$\boxed{h = -\frac{46}{7}}$ or $\boxed{-6.57}$

Simplifying Radicals	$\sqrt{100} = 10$ perfect squares $\sqrt{9} = 3$ $\sqrt{12} = \sqrt{4} \cdot \sqrt{3} = 2\sqrt{3}$ 2 6 2 3 use factor tree to help find any perfect squares	**Sample:** Simplify by taking out any perfect squares pairs $\sqrt{\text{leftovers}}$

$\boxed{\text{pairs = perfect square}}$

For #6 – 9, simplify each radical.

6) $\sqrt{54}$

2 27

3 9 ← perfect square

3 3

$3\sqrt{2 \cdot 3} = \boxed{3\sqrt{6}}$

7) $-7\sqrt{300}$

30 10

5 6 2 5

2 3

$2 \cdot 2 \cdot 3 \cdot 5 \cdot 5$

$-7 \cdot 2 \cdot 5\sqrt{3} = \boxed{-70\sqrt{3}}$

You Try #8 – 9!

8) $\sqrt{90}$

$3\sqrt{10}$

9) $5\sqrt{48}$

$20\sqrt{3}$

Dividing Radicals	☆ Simplify fraction first ☆ simplify the radicals $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ ☆ rationalize the denominator	**Sample:** $\sqrt{\frac{18}{2}} = \sqrt{\frac{9}{1}} = \sqrt{9} = 3$ $\sqrt{\frac{9}{4}} = \frac{\sqrt{9}}{\sqrt{4}} = \frac{3}{2}$

For #10 – 15, simplify each radical.

10) $\sqrt{\frac{16}{25}} = \frac{\sqrt{16}}{\sqrt{25}} = \boxed{\frac{4}{5}}$

11) $\frac{\sqrt{4}}{\sqrt{49}} = \boxed{\frac{2}{7}}$

12) $\frac{\sqrt{50}}{\sqrt{18}} = \sqrt{\frac{50 \div 2}{18 \div 2}} = \sqrt{\frac{25}{9}} = \boxed{\frac{5}{3}}$

You Try #13 – 15!

13) $\sqrt{\frac{100}{49}}$ $\boxed{\frac{10}{7}}$

14) $\frac{\sqrt{121}}{\sqrt{64}}$ $\boxed{\frac{11}{8}}$

15) $\frac{\sqrt{3}}{\sqrt{12}}$ $\boxed{\frac{1}{2}}$

Rationalizing Radicals	Not simplest form if there is a $\sqrt{\ }$ in the denominator ☆ Multiply by a useful 1 ☆ Simplify if possible	**Sample:** my useful 1 $\frac{1}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{1\sqrt{5}}{\sqrt{25}} = \boxed{\frac{\sqrt{5}}{5}}$ choose the same number makes a perfect square

For #16 – 23, simplify each radical. Rationalize, if needed.

16) $\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \boxed{\frac{\sqrt{3}}{3}}$
pair = perfect square

17) $\frac{4}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \boxed{\frac{4\sqrt{5}}{5}}$
pair

18) $\frac{6}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{6\sqrt{2}}{2}$ $\div 2$ $\boxed{3\sqrt{2}}$

19) $\frac{10}{\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}} = \frac{10\sqrt{10}}{10}$ $\boxed{\sqrt{10}}$

You Try #20 – 23!

20) $\frac{1}{\sqrt{13}}$ $\boxed{\frac{\sqrt{13}}{13}}$

21) $\frac{7}{\sqrt{6}}$ $\boxed{\frac{7\sqrt{6}}{6}}$

22) $\frac{18}{\sqrt{3}}$ $\boxed{6\sqrt{3}}$

23) $\frac{14}{\sqrt{14}}$ $\boxed{\sqrt{14}}$

6.1 Notes: Dilations and Scale Factor

Objectives:

- **Students will be able to classify dilations as enlargements or reductions.**
- **Students will be able to find the scale factor of dilation.**

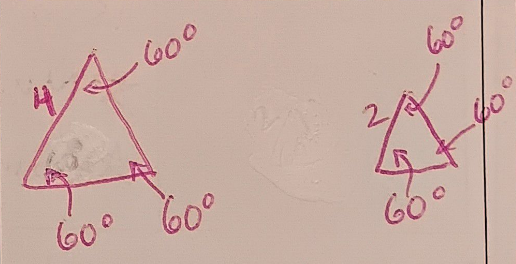
Exploration #1: Use the following link to explore dilations:
https://www.geogebra.org/m/waP9naNC
Follow the directions below.

- Click on the slider to make the *scale factor* = 2. What do you notice about the size of the sides of the image and preimage?

  Corent the length of the sides

- Move the slider of the scale factor to ½. What do you notice?

- Make a **conjecture** ("guess") about what scale factor tells you about the image of a dilation.

Dilation (centered at the origin)	**If a figure is dilated, then the image has the same** angles **as the pre-image, but the** sides **can be different.**	
Scale Factor (k) of a Dilation	**The scale factor of a dilation is a** ratio **that is the multiplier for the sides of the pre-image to get the lengths of the sides of the image. We use k for scale factor.**	Scale Factor: $k = \frac{\text{image}}{\text{pre-image}}$ Reduction: when k is a fraction $0 < k < 1$ Enlargement: when $k > 1$

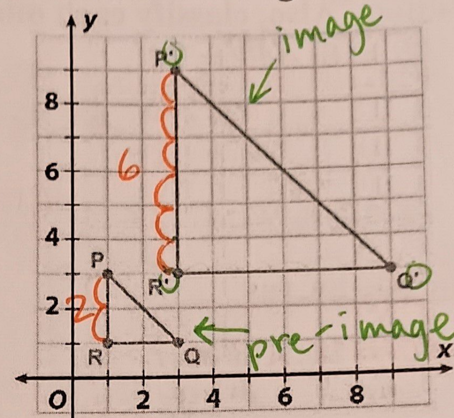
For #1 – 2, use the diagram of the dilation shown, which is centered at the origin.

1) Is this dilation a *reduction* or an *enlargement?*

enlargement

2) Find the scale factor of the dilation: $\frac{\text{image}}{\text{pre-image}}$

$\frac{P'R'}{PR}$ $\frac{6 \div 2}{2 \div 2} = 3$



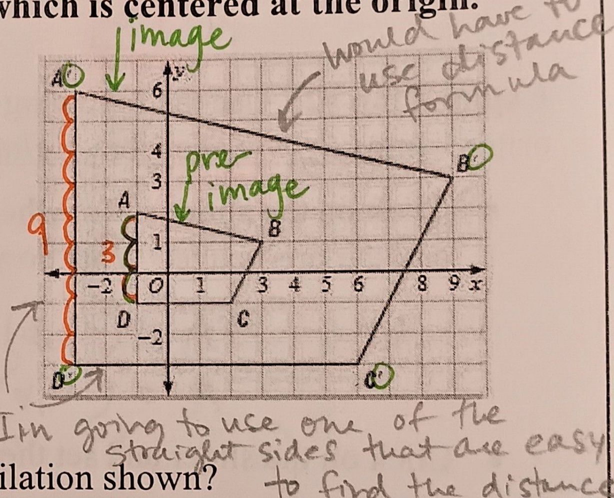
You try #3 – 4! Use the diagram of the dilation shown, which is centered at the origin.

3) Is this dilation a *reduction* or an *enlargement?*

enlargement

4) Find the scale factor of the dilation: $\frac{\text{image}}{\text{pre-image}}$

$\frac{A'D'}{AD}$ $\frac{9}{3} = 3$



5) Multiple choice. Which statement below is true for the dilation shown?

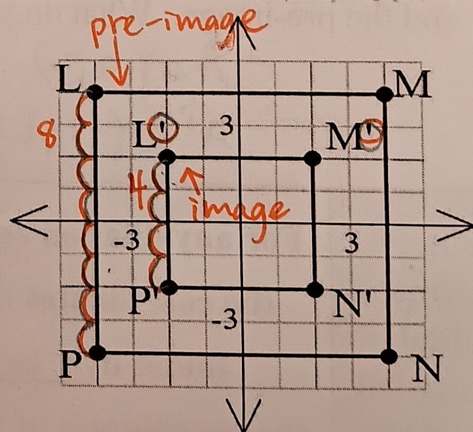
A) It is an ~~enlargement~~; the scale factor is $k = 2$.

B) It is an enlargement; the scale factor is $k = \frac{1}{2}$.

C) It is a reduction; the scale factor is $k = 2$.

D) It is a reduction; the scale factor is $k = \frac{1}{2}$.

$\frac{4}{8} = \frac{1}{2}$



You try #6!

6) Multiple choice. A figure is dilated. Which scale factor below shows a reduction?

A) $k = 6$ B) $k = \frac{7}{6}$ C) $k = \frac{1}{2}$ D) $k = -3$

why not?

What would this do?

For #7 – 10, ΔABC is dilated. Given the lengths below, find the scale factor of the dilation. Also, classify each dilation as a *reduction* or an *enlargement*.

7) $AB = 8; A'B' = 32$
pre image   image

$k = \frac{32}{8} = \boxed{4}$

enlargement

8) $BC = 24; B'C' = 9$
pre image   image

$k = \frac{9 \div 3}{24 \div 3} = \boxed{\frac{3}{8}}$ reduction

You Try #9-10!

9) $AC = 14; A'B' = 7$
pre   image

$k = \frac{7}{14} = \boxed{\frac{1}{2}}$ reduction

10) $AB = 18; A'B' = 12$
pre   image

$k = \frac{12 \div 6}{18 \div 6} = \boxed{\frac{2}{3}}$ reduction

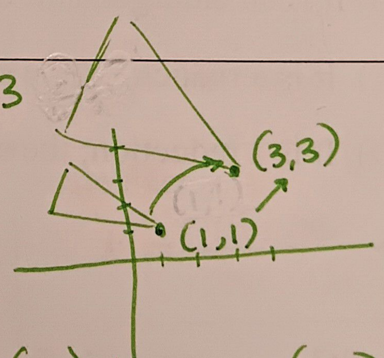
Exploration #2: Use the following link to explore the coordinates of a dilation centered at the origin. https://www.geogebra.org/m/d6HBmDNZ

- Click on the slider and set the scale factor to 2. Compare the coordinates of the image and the pre-image. What do you notice?

  $A = (-2,3)$ $B = (2,2)$ $\times 2$
  $A' = (-4,6)$ $B' = (4,4)$

- Click on the slider and set the scale factor to $\frac{1}{2}$. Compare the coordinates of the image and the pre-image. What do you notice?

  $A = (-2,3)$ $B = (2,2)$ $\times \frac{1}{2}$
  $A' = (-1,1.5)$ $B' = (1,1)$

Coordinates of a dilation centered at the origin	**For any dilation centered at the origin, the coordinates of the image can be found by** multiplying **the coordinates of the pre-image by the** scale factor.	k=3  (1,1) becomes (3,3)

For #11 – 14, find the coordinates of the image of ΔABC after a dilation with the given scale factor. Given: $A(-6,4); B(3,-2); C(-1,0)$

11) $k = 2$

$(-6,4)$
$\times 2 \ \times 2$

$A' = (-12, 8)$

$B' = (6, -4)$

$C' = (-2, 0)$

12) $k = \frac{1}{3}$

$(-6,4)$
$\times \frac{1}{3} \ \times \frac{1}{3}$

$A' = (-2, \frac{4}{3})$

$B' = (\frac{3}{3}, -\frac{2}{3}) = (1, -\frac{2}{3})$

$C' = (-\frac{1}{3}, 0)$

You try #13-14!

13) $k = \frac{1}{2}$

$(-6,4)$
$\times \frac{1}{2} \ \times \frac{1}{2}$

$A' = (-3, 2)$

$B' = (\frac{3}{2}, -\frac{2}{2}) = (\frac{3}{2}, -1)$

$C' = (-\frac{1}{2}, 0)$

14) $k = -5$

$A' = (30, -20)$

$B' = (-15, 10)$

$C' = (5, 0)$

6.2 Day 1 Notes: Similar Figures

Objectives:

- **Students will be able to identify similar figures.**
- **Students will be able to find the scale factor between similar figures.**

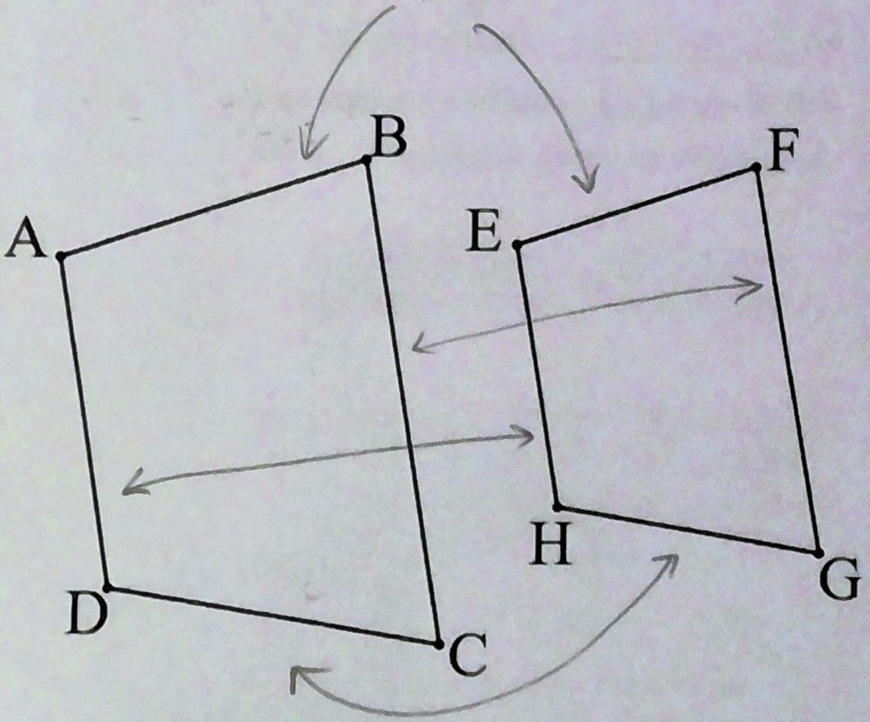
Exploration #1: Use the following link to explore similar figures: https://www.geogebra.org/m/mVYrt5u9

- Click on the slider to *change the size of* the image of the similar figures.
- As you adjust the size of the image, what do you notice about the angles?

- Compare the ratios made by corresponding sides on the right side of the screen. What do you notice?

- In the space below, draw what you learned in the exploration. Specifically, the ratios.

AB times $k = EF$

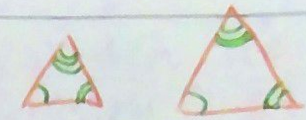
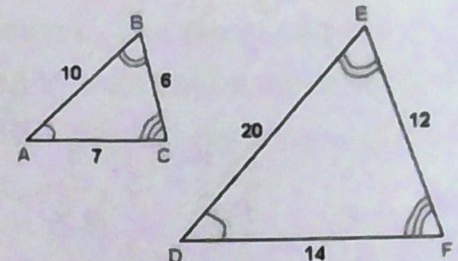


$\frac{EF}{AB} = k$

$\frac{FG}{BC} = k$ $\frac{\text{image}}{\text{pre-image}}$

$\frac{EH}{AD} = k$

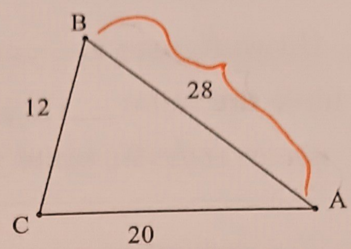
$\frac{HG}{DC} = k$

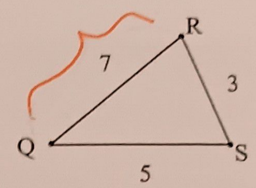
Similar Figures **~**	**If two figures are similar, then they have the same** ratios **, but not** ***necessarily*** **the same** length **.**	 like a dilation
Corresponding Angles of Similar Figures	**With similar figures, the corresponding angles are** congruent**.**	**Given: $\Delta ABC \sim \Delta DEF$.** **A *similarity statement* correlates the congruent angles and proportional sides.**
Corresponding Sides of Similar Figures	**The corresponding sides of similar figures are** proportional**.** (2 fractions that are equal) Note that each ratio is equivalent to the scale factor.	
Scale Factor of Similar Figures	**Use the simplified** ratio **of two corresponding** sides**, in the order of triangles given or requested.** $\frac{\text{1st named shape}}{\text{2nd named shape}}$ or $\frac{\text{small shape}}{\text{large shape}}$ We could think of similar figures as the result of a dilation, but we **DO NOT** know which is the image and which is the pre-image.	**Angles:** $\angle A \cong \angle D$ $\angle B \cong \angle E$ $\angle C \cong \angle F$ **Sides:** $\frac{DE}{AB} = \frac{EF}{BC} = \frac{FD}{CA} = k$ **Scale Factor of ΔABC to ΔDEF:** $\frac{AB}{DE} = k$ **Scale Factor of ΔDEF to ΔABC:** $\frac{DE}{AB} = k$

For #1 – 4: Given that $\triangle ABC \sim \triangle QRS$.

1) Write a proportion comparing the sides of the two triangles.

$$\frac{AB}{QR} = \frac{BC}{RS} = \frac{AC}{QS}$$





2) What is the scale factor of $\triangle ABC$ to $\triangle QRS$? $\dfrac{\text{1st named }\Delta}{\text{2nd named }\Delta}$

only need 1 side to find k

$$\frac{AB}{QR} = \frac{28}{7} = 4 \qquad \boxed{k=4}$$

3) What is the scale factor of $\triangle QRS$ to $\triangle ABC$? $\dfrac{\text{1st named }\Delta}{\text{2nd named }\Delta}$

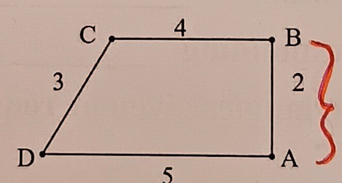
$$\frac{QR}{AB} = \frac{7}{28} = \frac{1}{4} \qquad \boxed{k=\frac{1}{4}}$$

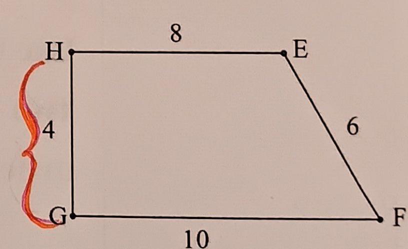
4) Complete the statement: $\angle B \cong$ $\angle R$.

You Try #5 – 8! Given that ABCD ~ GHEF.

5) Write a proportion comparing the sides of the quadrilaterals.

$$\frac{AB}{GH} = \frac{BC}{HE} = \frac{CD}{EF} = \frac{AD}{GF}$$





6) What is the scale factor of ABCD to GHEF? $\dfrac{\text{1st named quad}}{\text{2nd named quad}}$

$$\frac{AB}{GH} = \frac{2}{4} = \frac{1}{2}$$

7) What is the scale factor of GHEF to ABCD? $\dfrac{\text{1st named quad}}{\text{2nd named quad}}$

$$\frac{GH}{AB} = \frac{4}{2} = 2$$

8) Complete the statement: $\angle C \cong$ $\angle E$.

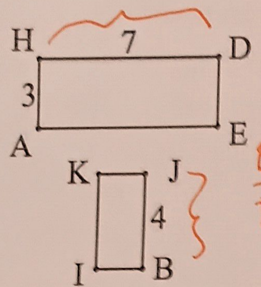
ABCD ~ GHEF

corresponding angles

For #9 – 10, find the scale factor (small figure to large figure) for each set of similar figures below.

9) Given: AEDH ~ KIBJ $\frac{\text{small figure}}{\text{large figure}}$

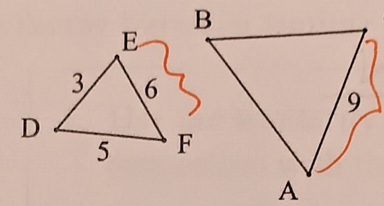
large Small



$\frac{BJ}{DH} = \boxed{\frac{4}{7}}$

You try! 10) Given: ΔDEF~ΔBAC $\frac{\text{small figure}}{\text{large figure}}$

Small large



$\frac{AC}{EF} = \frac{9 \div 3}{6 \div 3} = \boxed{\frac{3}{2}}$

For #11 – 12: Find the scale factor of each pair of similar figures. Use the requested order.

11a) large: small

DEF : ABC

$\frac{DE}{AB} = \frac{9}{6} = \boxed{\frac{3}{2}}$

11b) small: large

ABC DEF

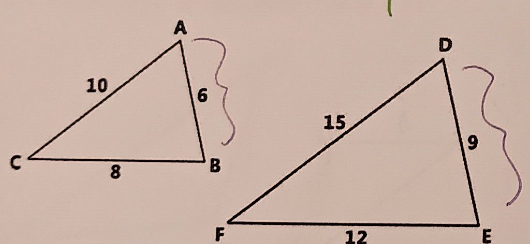
$\frac{AB}{DE} = \frac{6}{9} = \boxed{\frac{2}{3}}$

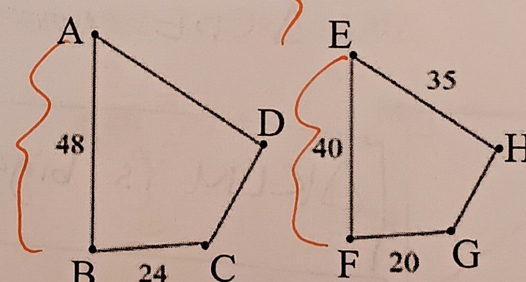
12a) ABCD to EFGH

$\frac{AB}{EF} = \frac{48 \div 8}{40 \div 8} = \boxed{\frac{6}{5}}$

12b) EFGH to ABCD

$\frac{EF}{AB} = \frac{40}{48} = \boxed{\frac{5}{6}}$





You Try #13 – 15!

13) Which statements below are **TRUE**, given that Δ**PQR**~Δ**HKG**? Select all that apply.

A) ∠P ≅ ∠H

B) ∠G ≅ ∠Q Not corresponding angles

C) $\frac{PQ}{HK} = \frac{QR}{KG} = \frac{PR}{HG}$

D) PQ ≅ HK

Sides aren't congruent in similar triangles

14) Given that $\Delta \mathbf{WXY} \sim \Delta \mathbf{DFE}$, then complete each statement below.

$\angle Y \cong$ <u>$\angle E$</u>

b. $\frac{WX}{DF} = \frac{?}{DE}$ WY

c. $\angle D \cong$ <u>$\angle W$</u>

15) Given that $\Delta \boldsymbol{CDE} \sim \Delta \boldsymbol{KLM}$, and the scale factor of ΔCDE *to* ΔKLM is $\frac{3}{7}$.

a. Find the scale factor of ΔKLM *to* ΔCDE. $\frac{7}{3}$

b. Which triangle is larger? How do you know?

First mentioned triangle is on top.

ΔKLM corresponds to 7

ΔCDE corresponds to 3

$\boxed{\Delta KLM \text{ is bigger}}$

6.2 Day 2 Notes: Similar Figures

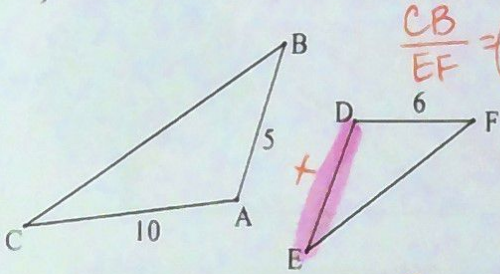
Objectives:

- **Students will be able to identify similar figures.**
- **Students will be able to solve problems involving similar figures.**
- **Students will be able to find the scale factor between similar figures.**

Finding sides with similar figures	**If two figures are similar, then the sides are** proportional**.**	1. Use the similarity statement to write a proportion with the sides. 2. Substitute values from the diagram. 3. Cross-multiply to solve.
Finding angles with similar figures	**If two figures are similar, then the corresponding angles are** congruent**.**	1. Use the similarity statement to decide which angles are congruent. 2. Write an equation setting the corresponding congruent angles equal. 3. Solve using algebra.

For #1 – 4: Given each pair of similar figures, find the requested side.

1) Find DE if $\Delta CBA \sim \Delta EFD$.



$\frac{CB}{EF} = \frac{BA}{FD} = \frac{CA}{ED}$

use these 2 for the proportion

I'm given both of these lengths

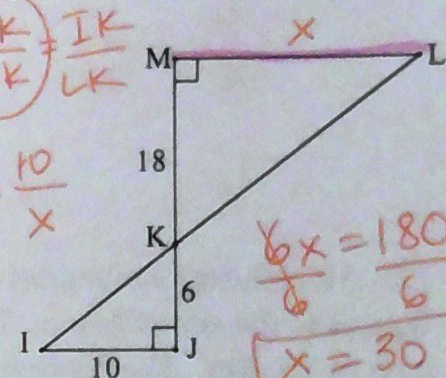
$\frac{5}{6} = \frac{10}{x}$

$\frac{5x}{5} = \frac{60}{5}$

$\boxed{x = 12}$

DE=12

2) Find ML if $\Delta IJK \sim \Delta LMK$.



$\frac{IJ}{LM} = \frac{JK}{MK} = \frac{IK}{LK}$

$\frac{6}{18} = \frac{10}{x}$

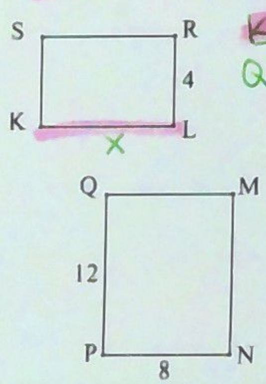
$\frac{6x}{6} = \frac{180}{6}$

$\boxed{x = 30}$

ML=30

You try #3 – 4!

3) Find KL if $KLRS \sim QPNM$.



$\frac{KL}{QP} = \frac{LR}{PN} = \frac{RS}{NM} = \frac{KS}{QM}$

proportion

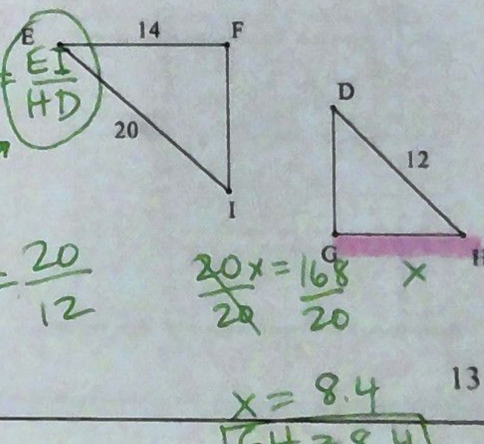
$\frac{x}{12} = \frac{4}{8}$

$\frac{8x}{8} = \frac{48}{8}$

$x = 6$

$\boxed{KL = 6}$

4) Find GH if $\Delta EFI \sim \Delta HGD$.



$\frac{EF}{HG} = \frac{FI}{GD} = \frac{EI}{HD}$

$\frac{14}{x} = \frac{20}{12}$

$\frac{20x}{20} = \frac{168}{20}$

$x = 8.4$

$\boxed{GH = 8.4}$

For #5 – 7: Given that $JMLK \sim ADCB$.

5) Find $m\angle M$.

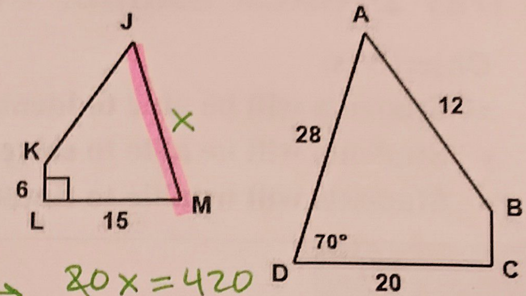
$\angle M \cong \angle D$

$\boxed{\angle M = 70^\circ}$

6) Find $m\angle C$.

$\angle C \cong \angle L$

$\boxed{\angle C = 90^\circ}$



7) Find the length of JM.

$\frac{JM}{AD} = \frac{ML}{DC} = \frac{LK}{CB} = \frac{JK}{AB}$

$\frac{x}{28} = \frac{15}{20} \rightarrow \frac{\cancel{20}x = 420}{\cancel{20} \quad 20}$

$\boxed{x = 21}$

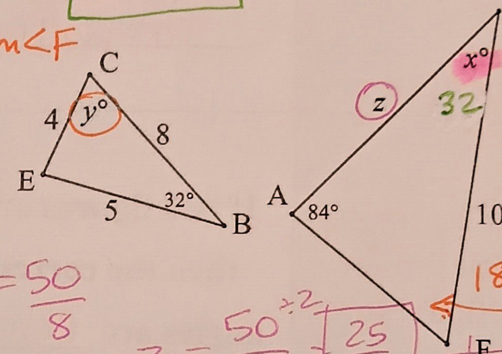
You Try #8 – 10! Given that $\Delta BCE \sim \Delta DFA$.

8) Find x. $= m\angle D = m\angle B$

$\boxed{x = 32}$

9) Find y. $= m\angle C = m\angle F$

$\boxed{y = 64}$



$\begin{array}{r} 84 \\ +32 \\ \hline 116 \end{array}$

$180 - 116 = 64$

10) Find z. $= AD$

$\frac{BC}{DF} = \frac{CE}{FA} = \frac{BE}{DA}$

$\frac{8}{10} = \frac{5}{z} \rightarrow \frac{\cancel{8}z = 50}{\cancel{8} \quad 8}$

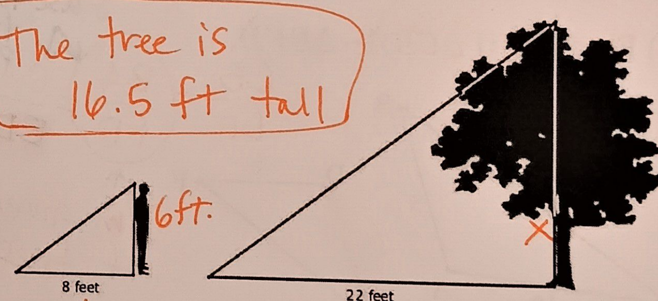
$z = \frac{50^{\div 2}}{8_{\div 2}} = \boxed{\frac{25}{4}}$ or $\boxed{6.25}$

11) A man who is 6 feet tall casts a shadow that is 8 feet long. At the same time, a tree's shadow is 22 feet long. Assuming the triangles shown below are similar, find the height of the tree.

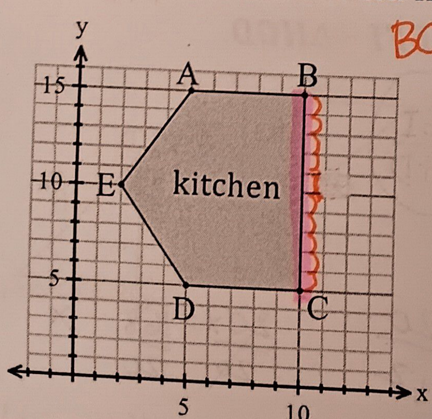
$\frac{6}{8} = \frac{x}{22}$ $\quad \frac{\cancel{8}x = 132}{\cancel{8} \quad 8}$

$x = 16.5$

The tree is 16.5 ft tall



12) An architect's blueprint of a home being constructed is a scaled drawing with all similar figures to the actual home. The ratio of drawings on the blueprint to the home being built is 1 inch to 2.1 feet. The diagram below shows the blueprint of the kitchen. Find the length of the longest wall in the kitchen if each square on the grid represents one inch.



BC =

10 inches $\rightarrow$ $\boxed{21 \text{ feet}}$

$\begin{array}{r} 10 \\ \times 2.1 \\ \hline 21 \end{array}$

6.3 Notes: Proving Similar Triangles

Objectives:

- **Students will be able to decide if triangles are similar.**
- **Students will be able to solve problems using similar triangles.**

Explorations: Use the following links to explore similar triangles: Follow the directions below.

- **#1:** https://www.geogebra.org/m/DfZCQQAa What happens when two triangles have proportional sides?

all angles are the same

they are similar Δs

- **#2:** https://www.geogebra.org/m/Ksvpuvds What happens when two triangles have two pairs of congruent angles?

all angles are $\cong$

All sides are proportional

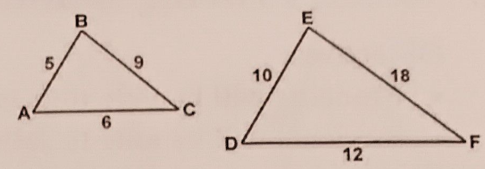
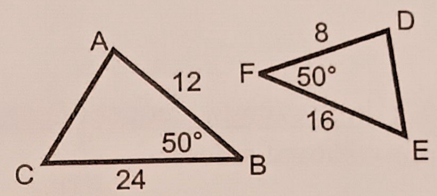
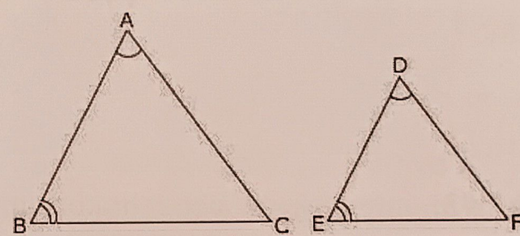
They are similar Δs

- **#3:** https://www.geogebra.org/m/jE9AKzZp What happens when two triangles have two pairs of proportional sides and one pair of congruent included angles?

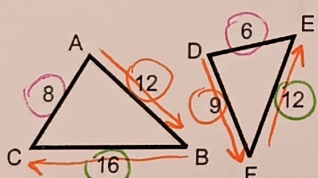
all $\angle$s are $\cong$

all sides are proportional

They are similar Δs

SSS~ Postulate	**If two triangles have three pairs of** proportional **sides, then the triangles are similar.**	
SAS~ Theorem	**If two sides in one triangle are** proportional **to two sides in another triangle, and if their included angles are** congruent**, then the triangles are similar.**	
AA~ Postulate	**If two angles in one triangle are** congruent **to two angles in another triangle, then the triangles are similar.**	

For #1 – 8: Are the pair of triangles shown similar? If so, by what postulate or theorem? Choose from SSS ~, SAS ~, or AA ~.

1) 

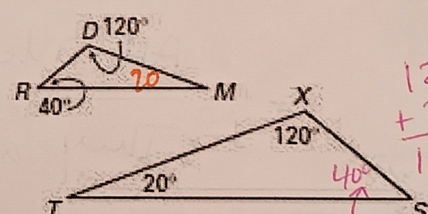
No angles

$\frac{AB}{DF} = \frac{BC}{FE} = \frac{AC}{DE}$ $\boxed{SSS}$

Are all sides proportional?

$\frac{12}{9} = \frac{16}{12} = \frac{8}{6}$ $k = \frac{4}{3}$

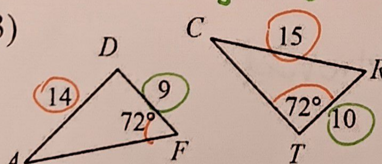
yes! → $\frac{4}{3} = \frac{4}{3} = \frac{4}{3}$

2) 

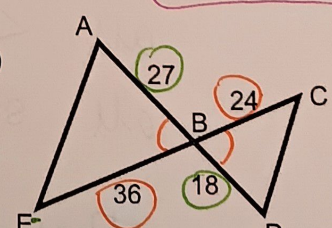
$120 + 20 = 140$

$180 - 140 = 40°$

Yes by AA

3) 

No – the angle is not the included angle.
We don't know if they are similar

4) 

$\triangle ABE \sim \triangle DBC$?

$\frac{AB}{DB} = \frac{BE}{BC} = \frac{AE}{DC}$?

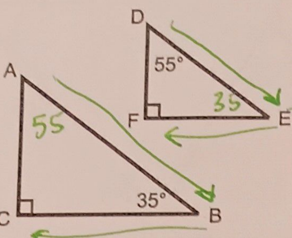
$\frac{27}{18} = \frac{36}{24}$

$\frac{3}{2} = \frac{3}{2}$

Yes by SAS

You try #5 – 8!

5) $\triangle ABC \sim \triangle DEF$?

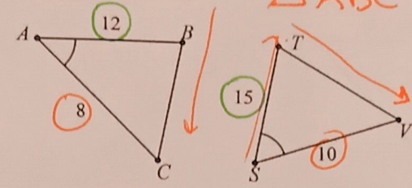


$\boxed{\text{yes by AA}}$

angles

90
+55
145
+35
180

6) $\triangle ABC \sim \triangle STV$?



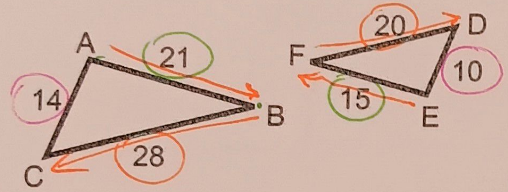
$$\frac{AB}{ST} = \frac{BC}{TV} = \frac{AC}{SV}$$

$$\frac{12}{15} = \frac{8}{10}$$

$$\frac{4}{5} = \frac{4}{5}$$

$\boxed{\text{yes by SAS}}$

7)



$\triangle ABC \sim \triangle EFD$

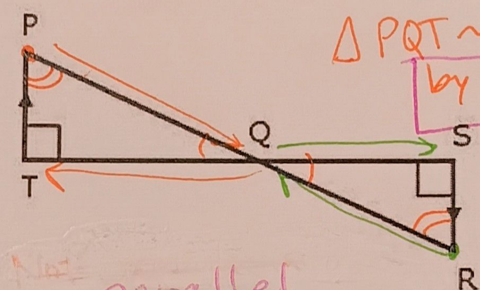
$$\frac{AB}{EF} = \frac{BC}{FD} = \frac{AC}{ED}$$

$$\frac{21}{15} = \frac{28}{20} = \frac{14}{10}$$

$$\frac{7}{5} = \frac{7}{5} = \frac{7}{5}$$

$\boxed{\text{yes by SSS}}$

8)



$\triangle PQT \sim \triangle RQS$

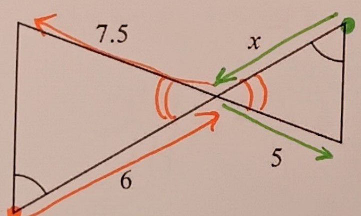
$\boxed{\text{by AA}}$

$\boxed{\text{yes}}$

parallel lines with transversal

For #9 – 14: Find the value of the variable.

9)



by AA, we have similar $\triangle$s

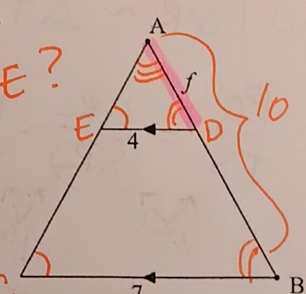
$$\frac{6}{7.5} = \frac{x}{5}$$

$$\frac{\cancel{7.5}x}{\cancel{7.5}} = \frac{30}{7.5}$$

$\boxed{x = 4}$

10) If $AB = 10$, find f.

AD



$\triangle ABC \sim \triangle ADE$?

yes by AA

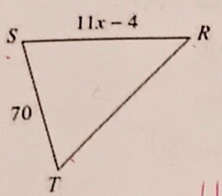
$$\frac{AB}{AD} = \frac{BC}{DE} = \frac{AC}{AE}$$

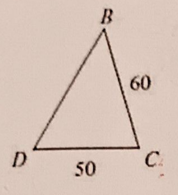
$$\frac{10}{f} = \frac{7}{4}$$

$$\frac{\cancel{7}f}{\cancel{7}} = \frac{40}{7}$$

$\boxed{f = \frac{40}{7}}$ or $\boxed{5.71\ldots}$

11) Given that $\Delta RST \sim \Delta DCB$, find x.





$\frac{RS}{DC} = \frac{ST}{CB} = \frac{RT}{DB}$

$\frac{11x-4}{50} = \frac{70}{60}$

$60(11x-4) = 3500$

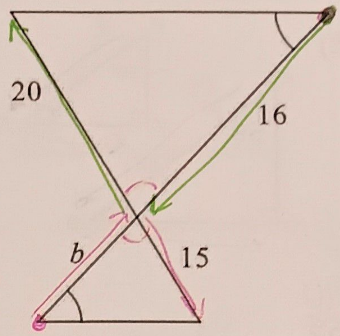
$660x - 240 = 3500$
$\quad +240 \quad +240$

$\frac{660x}{660} = \frac{3740}{660}$

$\boxed{x = 5.\overline{6}}$

You try #12! 12) Find b.

Similar by AA



$\frac{b}{16} = \frac{15}{20}$

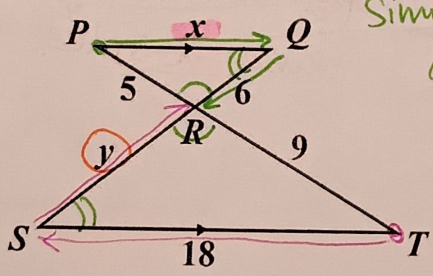
$\frac{20b}{20} = \frac{240}{20}$

$\boxed{b = 12}$

You try #13 – 14!

13) Find the missing variables.

Similar by AA



$\Delta PQR \sim \Delta TSR$

$\frac{PQ}{TS} = \frac{QR}{SR} = \frac{PR}{TR}$

2nd 1st

$\frac{6}{y} = \frac{5}{9}$

$\frac{5y}{5} = \frac{54}{5}$

$\boxed{y = 10.8}$

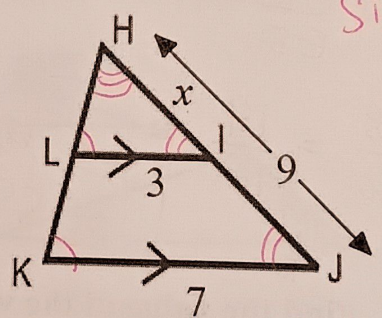
$\frac{x}{18} = \frac{6}{10.8}$

$\frac{10.8x}{10.8} = \frac{108}{10.8}$

$\boxed{x = 10}$

14) Find x.

Similar by AA



$\Delta HJK \sim \Delta HIL$

$\frac{HJ}{HI} = \frac{JK}{IL} = \frac{HK}{HL}$

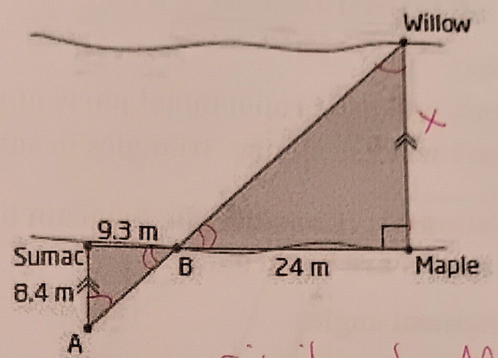
$\frac{9}{x} = \frac{7}{3}$

$\frac{7x}{7} = \frac{27}{7}$

$\boxed{x = \frac{27}{7}}$ or $\boxed{3.86}$

15) To determine the width of a river, Naomi finds a willow tree and a maple tree that are directly across from each other on opposite shores. Using a third tree on the shoreline, Naomi plants two stakes, A and B, and measures the distances shown.

Find the width of the river using the information that Naomi found.



Similar by AA

$$\frac{x}{8.4} = \frac{24}{9.3}$$

$$\frac{\cancel{9.3}x}{\cancel{9.3}} = \frac{20.16}{9.3}$$

$$x = 2.17$$

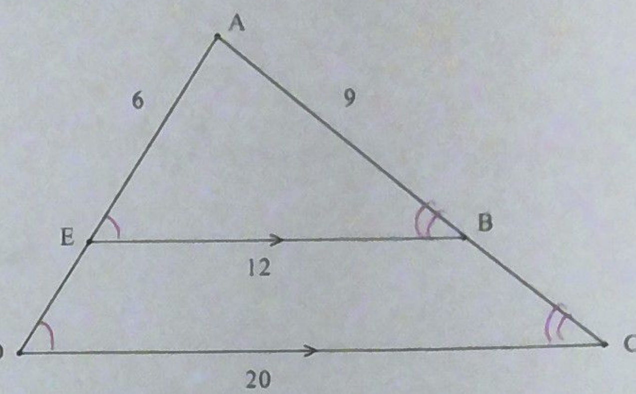
$$\boxed{x = 2.17}$$

6.4 Notes: Proportional Parts

Objectives:

- **Students will use proportional parts of triangles to solve problems.**
- **Students will use similar triangles to solve problems.**

Guided Exploration #1: **Consider the diagram below.** Reminder: If two lines are // (parallel), then corresponding angles are congruent.



a) Mark any congruent angles.

b) Which triangles are similar in the diagram?

small big

$\triangle ABE \sim \triangle ACD$

c) Find the scale factor (small to big).

just need 1 side to find k

$\frac{EB}{DC} = \frac{12}{20} = \frac{3}{5}$

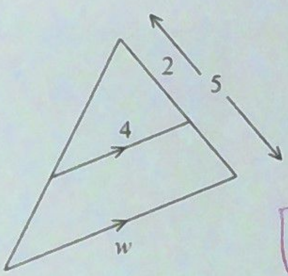
d) Use a proportion to find the lengths of AD and AC.

$\frac{AB}{AC} \rightarrow \frac{9}{x} = \frac{3}{5}$ ×3 ×3 $x = 15$ $\boxed{AC = 15}$

$\frac{AE}{AD} \Rightarrow \frac{3}{5} = \frac{6}{x}$ ×2 ×2 $x = 10$ $\boxed{AD = 10}$

For #1 – 4: Find the missing variable.

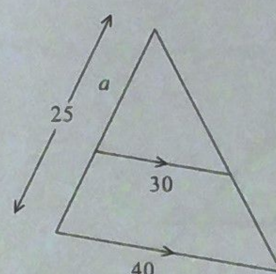
1)



$\frac{2}{5} = \frac{4}{W}$ ×2 ×2

$\boxed{W = 10}$

2)



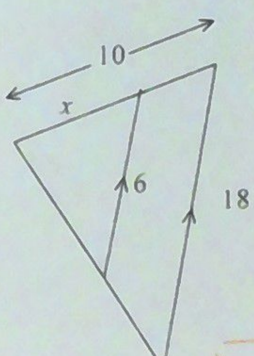
$\frac{a}{25} = \frac{30}{40}$

$\frac{\cancel{40}a}{\cancel{40}} = \frac{750}{40}$

$\boxed{a = 18.75}$

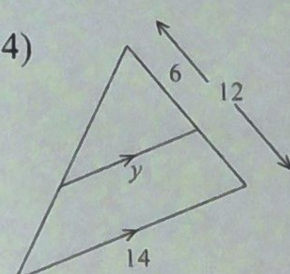
You try #3 - 4!

3)



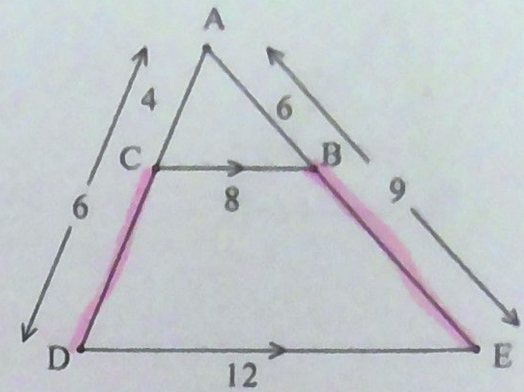
$\boxed{x = 33}$

4)



$\boxed{Y = 7}$

Guided Exploration #2: Consider the diagram below, which has two similar triangles by AA~.



a) Find the scale factor of ΔABC to ΔAED.

$k = \frac{AB}{AE} \quad \frac{6 \div 3}{9 \div 3} = \frac{2}{3}$

b) Find the lengths of CD and BE.

$CD = 6 - 4 = 2$

$BE = 9 - 6 = 3$

c) Find the ratio of $\frac{AC}{CD}$ and $\frac{AB}{BE}$. What do you notice?

$\frac{AC}{CD} \quad \frac{4}{2} = \boxed{2} \qquad \frac{AB}{BE} \quad \frac{6}{3} = \boxed{2}$

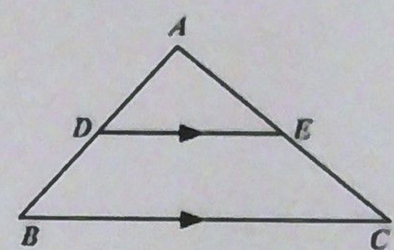
ratio of the parts of the same side

d) Find the ratio of $\frac{BC}{DE}$? Is it the same as your answer for part c? For part a?

No Yes

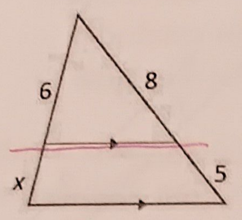
$\frac{8}{12} = \frac{2}{3}$

these are not parts of the same side

Triangle Proportionality Theorem	**If a triangle has two sides intersected by a line parallel to the 3rd side of the triangle, then the intersected sides are split into** proportional parts.	 If $DE \parallel BC$, then $\frac{AD}{DB} = \frac{AE}{EC}$. Note: The parallel sides are *not* proportional to these parts. The parallel sides are proportional to the *scale factor* of the similar triangles, which is not the same ratio.

For #5 – 11: Find each missing variable.

5)

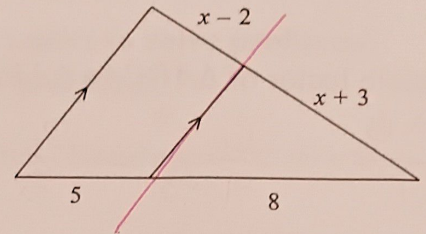


$\frac{6}{x} = \frac{8}{5}$

$\frac{8x}{8} = \frac{30}{8}$

$x = 3.75$

6)



$\frac{x-2}{x+3} = \frac{5}{8}$

$8(x-2) = 5(x+3)$

$8x - 16 = 5x + 15$

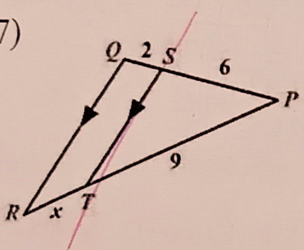
$-5x \quad +16 \quad -5x \quad +16$

$\frac{3x}{3} = \frac{31}{3}$

$\boxed{x = \frac{31}{3}}$

You try #7 - 8!

7)

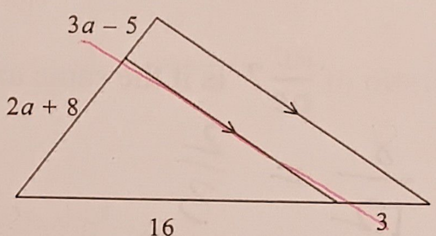


$\frac{2}{6} = \frac{x}{9}$

$6x = 18$

$\boxed{x = 3}$

8)



$\frac{3a-5}{2a+8} = \frac{3}{16}$

$3(2a+8) = 16(3a-5)$

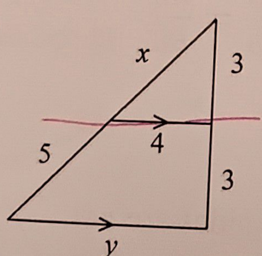
$6a + 24 = 48a - 80$

$-6a + 80 \quad -6a + 80$

$\frac{104}{42} = \frac{42a}{42}$

$\boxed{a = 2.48}$

9)



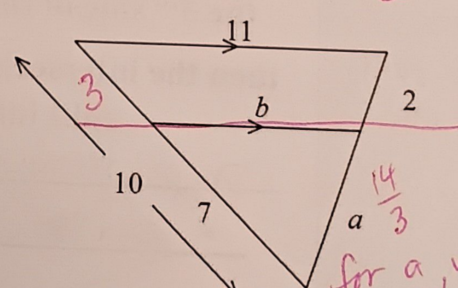
$\frac{x}{5} = \frac{3}{3}$

$\frac{3x}{3} = \frac{15}{3}$

$x = 5$

$x = 5$

10)



for a, we can use parts

$\frac{3}{7} = \frac{2}{a}$

$\frac{3a}{3} = \frac{14}{3}$

$a = \frac{14}{3}$

b → need k

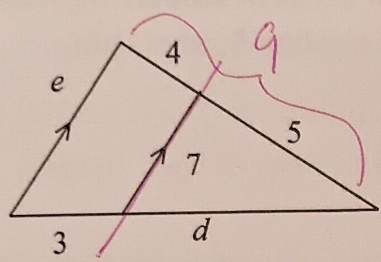
$\frac{7}{10} = \frac{b}{11}$

$\frac{10b}{10} = \frac{77}{10}$

$\boxed{b = 7.7}$

You try #11!

11) Find d and e.



k

parts

$\frac{4}{5} = \frac{3}{d}$

$\frac{4d}{4} = \frac{15}{4}$

$\boxed{d = \frac{15}{4}}$

$\frac{5}{9} = \frac{7}{e}$

$\frac{5e}{5} = \frac{63}{5}$

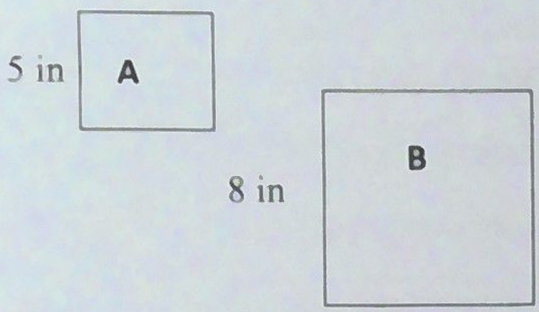
$\boxed{e = \frac{63}{5}}$

6.5 Notes: Perimeter & Area of Similar Polygons

Objectives:

- **Students will be able to write ratios for perimeters and areas of similar figures.**
- **Students will be able to find the area and perimeter of similar figures using proportions.**

Exploration: Consider the squares below. Find the perimeter and area of each square.



	Square A	Square B
Perimeter	5+5+5+5 = 20	8+8+8+8 = 32
Area	5×5 = 25	8×8 = 64

1) What is the scale factor (A to B)?

$\frac{5}{8}$

2) What is the ratio of the perimeters (A to B)? Make sure you reduce your ratio.

$\frac{20 \div 4}{32 \div 4} = \frac{5}{8}$

3) What is the ratio of the areas (A to B)?

$\frac{25}{64}$ can't reduce

4) What do you notice about your answers from #1 and #2?

$$\textit{scale factor} = \frac{5}{8}$$

$$\textit{ratio of perimeters} = \frac{5}{8}$$

5) Is there a relationship between your answers for #1 and #3?

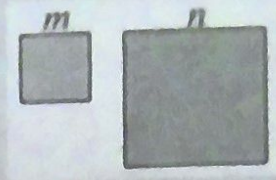
$$\textit{scale factor} = \frac{5}{8}$$

$$\textit{ratio of areas} = \frac{5^2}{8^2} \text{ or } \left(\frac{5}{8}\right)^2$$

Reminder: Scale factor is the ratio of two corresponding sides of similar figures.

$$\textit{scale factor}: \frac{side}{side}$$

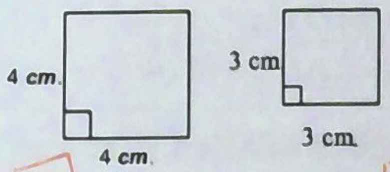
Given that two similar figures have a scale factor of $\frac{m}{n}$, then...



Ratio of Perimeters of Similar Figures	If the scale factor of two similar figures is $\frac{m}{n}$, then the ratio of their perimeters is also $\underline{\frac{m}{n}}$.	Ratio of perimeters = *scale factor* $\frac{perimeter}{perimeter} = \frac{m}{n} = \textit{scale factor}$
Ratio of Areas of Similar Figures	If the ratio of areas of two similar figures is $\frac{m}{n}$, then the ratio of their areas is $\left(\frac{m}{n}\right)^2$.	Ratio of areas = $(\textit{scale factor})^2$ $\frac{area}{area} = \left(\frac{m}{n}\right)^2 = (\textit{scale factor})^2$

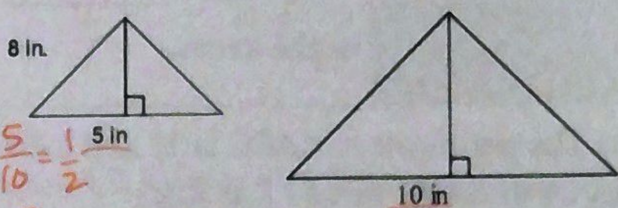
For #1 a-b: For each pair of similar figures, find the ratio of the perimeter and the ratio of the area (small to big).

a)



P: $\boxed{\frac{3}{4}}$ A: $\left(\frac{3}{4}\right)^2 = \boxed{\frac{9}{16}}$

b)



$K = \frac{5}{10} = \frac{1}{2}$

P: $\boxed{\frac{1}{2}}$ A: $\left(\frac{1}{2}\right)^2 = \boxed{\frac{1}{4}}$

For #2 a-d: Given that $\Delta ABC \sim \Delta EFG$. Find the request ratios.

a) scale factor (ΔABC to ΔEFG)

$k = \frac{6}{2} = 3$

b) ratio of perimeters (ΔABC to ΔEFG)

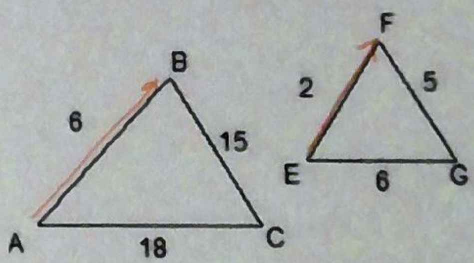
3

c) scale factor (ΔEFG to ΔABC)

$\frac{2}{6} = \frac{1}{3}$

d) ratio of areas (ΔEFG to ΔABC)

$\left(\frac{1}{3}\right)^2 = \frac{1}{9}$



You try #3 a-c! Given that two similar figures have corresponding sides of $6in$ and $7in$.

a) Find the scale factor (big to small).

$\frac{7}{6}$

b) Find the ratio of the perimeters (big to small).

$\frac{7}{6}$

c) Find the ratio of the areas (big to small).

$\left(\frac{7}{6}\right)^2 = \frac{49}{36}$

Missing Triangle Parts?!?! *** You CAN find them! ***		
To find a missing perimeter	**Set up a proportion with the scale factor equal to the perimeters.**	**Ratio of perimeters = *scale factor*** $\frac{m}{n} = \frac{perimeter}{perimeter}$
To find a missing area	**Set up a proportion with the scale factor** Squared **equal to the areas.**	**Ratio of areas = $(scale\ factor)^2$** $\left(\frac{m}{n}\right)^2 = \frac{area}{area}$

4) The perimeter of ΔABC is 12 and $\Delta ABC \sim \Delta XYZ$. Find the perimeter of ΔXYZ if the scale factor of ΔABC to ΔXYZ is 2:3.

$\frac{\Delta ABC}{\Delta XYZ} \quad \frac{2}{3} = \frac{12}{x}$ ×6 ×6

$x = 18$

or cross-multiply

$\frac{2x}{2} = \frac{36}{2}$

$x = 18$

You try #5! The perimeter of ΔXYZ is 27 and $\Delta ABC \sim \Delta XYZ$. Find the perimeter of ΔABC if the scale factor of ΔABC to ΔXYZ is 5:4.

$\frac{5}{4} = \frac{x}{27}$

$\frac{4x}{4} = \frac{135}{4}$

$x = 33.75$

6) Two rectangles are similar, and their scale factor is 3:5. What is the area of the smaller rectangle if the larger rectangle has an area of 50 m²? $k = \frac{3}{5}$ $\frac{\text{small}}{\text{big}}$

$$\frac{x}{50} = \left(\frac{3}{5}\right)^2$$

$$\frac{x}{50} = \frac{9}{25} \quad \times 2$$

$x = 18$

You try #7! Two similar figures have corresponding sides of lengths 9 inches and 11 inches. The smaller figure has an area of 35 inches squared. Find the area of the larger figure.

$k = \frac{9}{11}$ $\frac{35}{x}$

$$\left(\frac{9}{11}\right)^2 = \frac{35}{x}$$

$$\frac{81}{121} = \frac{35}{x}$$

$$\frac{81x}{81} = \frac{4235}{81}$$

$$\boxed{x = 52.28}$$

$k = \frac{2}{1}$ or 2

8) Two triangles are similar and the ratio of each pair of corresponding sides is 2: 1. Which statement regarding the two triangles is <u>not true</u>? Assume that each ratio represents the larger triangle first.

A. Their areas have a ratio of 4: 1 $\left(\frac{2}{1}\right)^2 = \frac{4}{1}$

B. The scale factor is a ratio of 2: 1

C. Their perimeters have a ratio of 2: 1

~~D.~~ Their corresponding angles have a ratio of 2: 1

they are ≅

1:1

For #9 a-b: Two similar figures have areas of 25 mm^2 and 36 mm^2. Find the requested ratios.

a) Scale factor (small to big) $k^2 = \frac{25}{36}$

$$k = \sqrt{\frac{25}{36}} = \boxed{\frac{5}{6}}$$

b) Ratio of perimeters (big to small)

$$\boxed{\frac{6}{5}}$$

Challenge Problems! These are problems that could prepare you for a *bonus* on the test.

For #10 a-c: Two similar figures have areas of 8 mm^2 and 18 mm^2. Find the requested ratios.

a) Scale factor (small to big)

$k^2 = \frac{8}{18}$

$k = \sqrt{\frac{8}{18}} = \frac{\sqrt{4}\sqrt{2}}{\sqrt{9}\cdot\sqrt{2}} = \frac{2\cancel{\sqrt{2}}}{3\cancel{\sqrt{2}}} = 1$ $\boxed{\frac{2}{3}}$

b) Ratio of perimeters (big to small)

$\boxed{\frac{3}{2}}$ same as k

c) If the perimeter of the smaller figure is 24 *mm*, then find the perimeter of the larger figure.

$\frac{\text{smaller}}{\text{larger}}$ $\frac{24}{x} = \frac{2}{3}$ ×12 ×12

$x = 36$

You Try #11! The ratio of the areas of square A (small) to square B (big) is $\frac{16}{25}$. If square B has one side of 10 cm, then what is the length of a side of square A?

$k = \sqrt{\frac{16}{25}} = \frac{4}{5}$ $\frac{A}{B}$

A. 4 *cm* B. 8 *cm*

C. 10 *cm* D. 64 *cm*

$\frac{4}{5} = \frac{x}{10}$ ×2 ×2

$x = 8$