Exponents Day 1 Notes: Simplifying Exponential Expressions

Learning Objectives:
1) Multiply exponential expressions with one or more like bases
2) Raise exponential expressions to a power (power of a power/product)

Warm up:

1) Solve: $-\frac{6}{b} = \frac{9}{4}$

$-6 \cdot 4 = 9 \cdot b$
$-24 = 9b$
$b = -24/9 = -8/3$

$b = -\frac{8}{3}$

2) Solve: $\frac{3}{2x} = \frac{4}{x+1}$

$3(x+1) = 4(2x)$
$3x + 3 = 8x$
$3 = 5x$

$x = \frac{3}{5}$

Key Vocabulary:

Coefficient: multiplier in front of variable	$4x^3$
Base: what you are raising to a power	$4x^3$
Exponent: power	$4x^3$

What does x^4 mean? $x \cdot x \cdot x \cdot x$ What does x^3 mean? $x \cdot x \cdot x \cdot x$

Simplify the following expression: $x^4 \cdot x^3$. $\underbrace{x \cdot x \cdot x \cdot x}_{x^4} \cdot \underbrace{x \cdot x \cdot x}_{x^3} = x^7$

ERROR ALERT! How are these expressions different? -4^2 and $(-4)^2$

$-(4 \cdot 4)$ -16 $-4 \cdot -4$ $+16$

Helpful values to have memorized…

$1^2 = 1$
$2^2 = 4$
$3^2 = 9$
$4^2 = 16$
$5^2 = 25$
$6^2 = 36$
$7^2 = 49$
$8^2 = 64$
$9^2 = 81$
$10^2 = 100$

$11^2 = 121$
$12^2 = 144$
$13^2 = 169$
$14^2 = 196$
$15^2 = 225$

$1^3 = 1$
$2^3 = 8$
$3^3 = 27$
$4^3 = 64$
$5^3 = 125$
$6^3 = 216$
$7^3 = 343$
$8^3 = 512$
$9^3 = 729$
$10^3 = 1000$

Multiplying with the same base:

Keep the base, add the exponents

Examples: Simplify each of the following expressions. Leave in exponential form. Evaluate expressions with numerical bases and powers of 4 or lower.

1. $2^5 \cdot 2^8$ 2^{13}
2. $a^3 \cdot a$ a^4
3. $(-3)^2 \cdot (-3)^2$ $(-3)^4$ parentheses important!

You try #4 – 5!

4. $(-4)^3 \cdot (-4)^4$ $(-4)^7$
5. $y^4 \cdot y$ y^5
6. $-5z^7 \cdot 3z^9$ $-15z^{16}$
7. $(6x^2y^3)(4x^3y^6)$ $24x^5y^9$

You try! 8. $2y^2z^4 \cdot 3y^9z^6$ $6y^{11}z^{10}$

Simplifying a Power to Power

keep the base, multiply the exponents

Examples: Simplify each of the following expressions. Leave in exponential form.

9. $(x^2)^3$ x^6
10. $(y^5)^4$ y^{20}
11. $(2^3)^2$ $2^6 = 64$

You try!

12. $(a^3)^5$ a^{15}
13. $(6^2)^{11}$ 6^{22}
15. $(b^4)^2$ b^8

Simplifying Power of a Product

Distribute the power to each factor, then Use power of a power if necessary

Examples: Simplify each of the following expressions. Leave in exponential form.

16. $(xy)^3$ x^3y^3
17. $(3b^4)^5$ $3^5(b^4)^5$ $3^5b^{20} = 243b^{20}$
18. $(-4x)^2$ $(-4)^2x^2$ $16x^2$

You try!

19. $(5x^3)^3$

5^3x^9

$125x^9$

20. $(-2y^6)^4$

$(-2)^4(y^6)^4$

$\boxed{16y^{24}}$

21. $(-g^3h)^8$

$(-1)^8(g^3)^8h^8$

$1g^{24}h^8$

$\boxed{g^{24}h^8}$

Examples: Simplify each expression.

22. $(3xy^2)^3 \cdot x^2$

$3^3x^3y^6 \cdot x^2$

$27x^5y^6$

23. $(-vw^4)^2 \cdot 2vw^3$

$(-1)^2v^2w^8 \cdot 2 \cdot v \cdot w^3$

$2v^3w^{11}$

You try!

24) $(-4ab^2)^2 \cdot 8a^5b$

$(-4)^2a^2b^4 \cdot 8a^5b$

$16 \cdot a^2b^4 8a^5b$

$128a^7b^5$

25. $(4g^5h^2)^3 \cdot 2gh$

$4^3g^{15}h^6 \cdot 2gh$

$64g^{15}h^6 \cdot 2gh$

$\boxed{128g^{16}h^7}$

<u>SUMMARY Properties (so far):</u>

Product-of-Powers Property: For all nonzero numbers x and all integers *m* and *n*,

$$x^m \cdot x^n = x^{(m + n)}$$

Power-of-a-Power Property: For all nonzero number *x* and all integers *m* and *n*,

$$(x^m)^n = x^{(m \cdot n)}$$

Power-of-a-Product Property: For all nonzero numbers *x* and *y*, and any integer *n*,

$$(xy)^n = (x^n) \cdot (y^n)$$

Exponents Day 2 Notes: More Simplifying

Learning Objectives:
1) Divide exponential expressions with one or more like bases
2) Raise rational exponential expressions to a power (power of a quotient)
3) Raise expressions to the zero power
4) Rewrite negative exponents as positive exponents and/or apply exponential properties.

Warm up:

1) $(3x^3)^3$ is equivalent to: 3^3x^9

A. x
B. $9x^6$
C. $9x^9$
D. $27x^6$
E. $27x^9$

2) Which of the following is equivalent to the inequality $4x - 8 > 8x + 16$?

F. $x < -6$
G. $x > -6$
H. $x < -2$
J. $x > 2$
K. $x < 6$

$4x - 8 > 8x + 16$
$-4x \quad -4x$
$-8 > 4x + 16$
$-16 \quad -16$
$-24 > 4x$
$-6 > x$

What does $\frac{a^6}{a^2}$ mean? $\frac{a \cdot a \cdot a \cdot a \cdot a \cdot a}{a \cdot a} = a^4$

Dividing with the Same Base

keep the base, subtract the exponents (numerator − denominator)

For #1 – 6: Simplify each expression. For numerical bases to the power of 4 or less, evaluate the expression.

1. $\frac{6^{12}}{6^5}$

$\frac{\cancel{6} \cdot \cancel{6} \cdot \cancel{6} \cdot \cancel{6} \cdot \cancel{6} \cdot 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6}{6 \cdot 6 \cdot 6 \cdot 6 \cdot 6}$ $\boxed{= 6^7}$

2. $\frac{x^2 \cdot x^8}{x^4}$

$\frac{x^{10}}{x^4}$ $\boxed{x^6}$

3. $\frac{(-2)^7}{(-2)^4}$

$(-2)^3$ $\boxed{-8}$

You try!

4. $\frac{b^5}{b^2}$ b^3

5. $\frac{x^{10}}{x^3 \cdot x^2}$ $\frac{x^{10}}{x^5} = x^5$

6. $\frac{(-6)^7}{(-6)^3}$ $(-6)^4$

$= (-1)^4(6)^4$
$= 1 \cdot 1296$
$\boxed{1296}$

Challenge Problem: Simplify $\frac{2a^3}{a^5} \cdot \frac{a^6}{4a^2} = \frac{2a^9}{4a^6}$

$= \frac{1}{2} \cdot a^3 = \boxed{\frac{a^3}{2}}$

Explore: What does $\left(\frac{3}{4}\right)^2$ mean? $\frac{3}{4} \cdot \frac{3}{4} = \frac{3^2}{4^2} = \frac{9}{16}$

Taking the Power of a Quotient (fraction)

distribute the power to all factors

For #7 – 13: Simplify each expression. For numerical bases to the power of 4 or less, evaluate the expression.

7. $\left(\frac{2}{7}\right)^3$

$\frac{2^3}{7^3} = \frac{8}{343}$

8. $\left(\frac{r}{s}\right)^5$

$\frac{r^5}{s^5}$

9. $\left(\frac{-4}{w}\right)^3$

$\frac{(-4)^3}{w^3} = \frac{-64}{w^3}$

10. $\left(\frac{y^5}{y^3}\right)^2$

$= (y^2)^2 = y^4$

or

$= y^{10}/y^6 = y^4$

You try!

11. $\left(\frac{-5}{t}\right)^4$

$\frac{(-5)^4}{t^4} = \frac{624}{t^4}$

12. $\left(\frac{a}{b^3}\right)^9$

$\frac{a^9}{b^{27}}$

13. $\left(\frac{6}{5}\right)^2$

$\frac{6^2}{5^2} = \frac{36}{25}$

14. $\left(\frac{a^6}{a^4}\right)^5$

$= (a^2)^5$

$= a^{10}$

or $\frac{a^{30}}{a^{20}} = a^{10}$

Challenge Problems: Simplify each expression.

A. $\left(\frac{2y^7}{xy^5}\right)^3 = \left(\frac{2y^2}{x}\right)^3$

$= \frac{2^3 y^6}{x^6} = \boxed{\frac{8y^6}{x^6}}$

B. $\left(\frac{-3x^2}{w}\right)^2 \cdot \frac{2}{3w}$

$= \frac{(-3)^2 x^4}{w^2} \cdot \frac{2}{3w}$

$= \frac{18x^4}{3w^3} = \frac{6x^4}{w^3}$

Zero Exponent

FILL IN THIS TABLE

x^3	$1 \cdot x \cdot x \cdot x$
x^2	$1 \cdot x \cdot x$
x^1	$1 \cdot x$
x^0	1
x^{-1}	$1/x$
x^{-2}	$1/x^2$

divide by x

divide by x^2

Negative Exponents

$a^{-n} = \frac{1}{a^n}$

$\frac{1}{a^{-m}} = a^m$

For #15 – 26: Simplify each expression so that there are no negative or zero exponents in your answer.

15. a^0 $\quad 1$

16. $5x^0$ $\quad 5\cdot 1 = 5$

17. $-6\cdot(392{,}568{,}132.873)^0$ $\quad -6\cdot(1) = -6$

18. 5^{-1} $\quad \frac{1}{5^1} = \frac{1}{5}$

19. $\left(\frac{1}{3}\right)^{-1}$ $\quad \frac{1^{-1}}{3^{-1}} = \frac{3^1}{1^1} = 3$

20. $\frac{1}{7^{-2}}$ $\quad 1\cdot 7^2 = 49$

You try #21 – 26!

21. $(-24x^3 + 10x^4y^{10})^0$ $\quad \boxed{1}$

22. $-16b^0$ $\quad -16(1) = -16$

23. $2(7x^3)^0$ $\quad 2(1) = 2$

24. 14^{-1} $\quad \frac{1}{14}$

25. $\frac{1}{2^{-6}}$ $\quad 1\cdot 2^6 = 64$

26. 3^{-3} $\quad \frac{1}{3^3} = \frac{1}{27}$

For #27 – 32: Simplify each expression so that there are no negative or zero exponents in your answer.

27. $\frac{a^3}{a^{-4}}$ $\quad a^{3-(-4)}$ $\quad \boxed{a^7}$

28. $\frac{x^3y^{10}}{x^7y^{-3}}$ $\quad x^{3-7}\; y^{10-(-3)}$ $\quad x^{-4}\cdot y^{13}$ $\quad \boxed{\frac{y^{13}}{x^4}}$

29. $\frac{b^3 5^2}{b^3 5^4}$ $\quad b^{3-3}\cdot 5^{2-4}$ $\quad b^0\cdot 5^{-2} = 1\cdot\frac{1}{5^2}$ $\quad \boxed{=\frac{1}{25}}$

You try!

30. $\frac{2^5y^4}{2^3y^6}$ $\quad 2^2y^{-2}$ $\quad = \frac{4}{y^2}$

31. $\frac{x^5y^{-2}}{x^7y^{-5}}$ $\quad x^{-2}y^3$ $\quad \frac{y^3}{x^2}$

32. $\frac{x^{-3}y^{-3}}{xy^4}$ $\quad x^{-4}y^{-7} = \boxed{\frac{1}{x^4y^7}}$

Challenge Problems:

Simplify each expression.

C. $\frac{5a^{-2}}{a^5b}\cdot\frac{b^3}{25b^{-8}} = \frac{5a^{-2}b^3}{25a^5b^{-7}}$

$= \frac{1a^{-7}b^{10}}{5}$

$\boxed{= \frac{b^{10}}{5a^7}}$

D. $\left(\frac{3x^2y}{x^{-2}y^4}\right)^{-2}$

easiest to flip fraction first because of neg. exponent

$= \left(\frac{x^{-2}y^4}{3x^2y}\right)^2 = \left(\frac{x^{-4}y^3}{3}\right)^2$

$= \frac{x^{-8}y^6}{3^2} = \frac{y^6}{9x^8}$

Exponent Properties Summary

Product-of-Powers Property: For all nonzero numbers x and all integers *m* and *n*,

$$x^m \cdot x^n = x^{m+n}$$

Power-of-a-Power Property: For all nonzero number *x* and all integers *m* and *n*,

$$(x^m)^n = x^{m \cdot n}$$

Power-of-a-Product Property: For all nonzero numbers *x* and *y*, and any integer *n*,

$$(xy)^n = x^n y^n$$

Quotient-of-Powers: For all nonzero numbers *a* and any positive integers *m* and *n*, *m* > *n*,

$$\frac{a^m}{a^n} = a^{(m-n)}$$

Power-of-a-Quotient Property: For all real numbers *a* and *b*, *b*≠*0*, and a positive integer *m*,

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, b \neq 0$$

Zero Exponents: For all real numbers *a* such that $a \neq 0$, $a^0 = 1$

Negative Exponents: For all real numbers *a* such that $a \neq 0$, $a^{-1} = \frac{1}{a}$ and $\frac{1}{a^{-1}} = a$

$a^{-n} = \frac{1}{a^n}$ $\qquad$ $\frac{1}{a^{-m}} = a^m$

6.1: Solving Exponential Equations

Learning Objectives:
1) Solve exponential equations (where the variable is in the exponent!)

It is helpful to recognize perfect squares, cubes, fourths and fifths when solving exponential equations. Fill in the following table. We have done a few for you and shaded squares are "bonus".

n	n^2	n^3	n^4	n^5
1	1	1	1	1
2	4	8	16	32
3	9	27	81	243
4	16	64	256	1024
5	25	125		
6	36	216		
7	49			
8	64			
9	81			
10	106			
11	121			
12	144			
13	169			
14	196			
15	225			

Exploration #1: Which of the following expressions below are equivalent to 9^2? Choose all that apply.

A) $\mathbf{3^4}$ $= 81$ B) $\mathbf{81^1}$ C) $\left(\frac{1}{9}\right)^{-2}$ D) $\left(\frac{1}{3}\right)^{-4}$ E) $\left(\frac{1}{81}\right)^{-1}$ $= \left(\frac{81}{1}\right)^1 = 81$

$\left(\frac{9}{1}\right)^2 = 9^2$ $\left(\frac{3}{1}\right)^4 = 3^4 = 81$

Exploration #2: Rewrite the expression $\mathbf{64^5}$ in as many ways as you can thinking of by changing the base and the power. Hint: Use bases that go into 64 like 2, 4, ?

$64 = 8^2$
$64 = 4^3$
$64 = 2^6$

$\boxed{(8^2)^5 = 8^{10} \quad (4^3)^5 = 4^{15} \quad (2^6)^5 = 2^{30}}$

$64 = \frac{1}{8^{-2}}$ $\left(\frac{1}{8^{-2}}\right)^5 = \frac{1}{8^{-10}}$

$64 = \frac{1}{4^{-3}}$ $\left(\frac{1}{4^{-3}}\right)^5 = \frac{1}{4^{-15}}$

Exploration #3: Which of the following expressions are equivalent to $\mathbf{49^x}$? Choose all that apply.

A) $\mathbf{7^{2x}}$ $= (7^2)^x = 49^x$ B) $\left(\frac{1}{49}\right)^{-x}$ $= \left(\frac{49}{1}\right)^x$ C) $\mathbf{24.5^{2x}}$ D) $\left(\frac{1}{7}\right)^{-2x}$ $= \left(\frac{7}{1}\right)^{2x} = (7^2)^x = 49^x$

Solving Exponential Equations using the same base.

Example: $3^{5x} = 9^{x-4}$

$= (3^2)^{x-4}$
$3^{5x} = 3^{2x-8}$
$5x = 2x - 8$
$3x = -8$
$\boxed{x = -8/3}$

Step #1: Rewrite both equations using the same **base**.

Step #2: Set the exponents **equal** to each other. (after bases =)

Step #3: Solve the resulting equation for the variable.

Example 1: Solve the following Exponential Equations:

a.) $6^x = 36$
$6^x = 6^2$
$x = 2$

b.) $2^{x+5} = 8$
$2^{x+5} = 2^3$
$x + 5 = 3$
$\boxed{x = -2}$

c.) $2^x = 2^{3x-7}$
$x = 3x - 7$
$7 = 2x$
$x = 7/2$

You try!

d.) $5^x = 25$
$5^x = 5^2$
$x = 2$

e.) $11^{2x-4} = 121$
$11^{2x-4} = 11^2$
$2x - 4 = 2$
$2x = 6$
$x = 3$

f.) $6^{2x-9} = 216$
$6^{2x-9} = 6^3$
$2x - 9 = 3$
$2x = 12$
$x = 6$

Solving Multi-Step Exponential Equations:

Example: $2(3)^{5x} = 162$

Step #1: Isolate the base.

Step #2: Rewrite both equations using the same **base**.

Step #3: Set the exponents **equal** to each other.

Step #4: Solve the resulting equation for the variable.

Example 2: Solve the following Exponential Equations.

a.) $5(3)^x = 405$

$\frac{5(3)^x}{5} = \frac{405}{5}$

$3^x = 81$

$3^x = 3^4$

$x = 4$

b.) $7\left(\frac{1}{2}\right)^x = 56$

$\frac{7\left(\frac{1}{2}\right)^x}{7} = \frac{56}{7}$

$\left(\frac{1}{2}\right)^x = 8$

$\left(\frac{2}{1}\right)^{-x} = 2^3$

$x = -3$

You try!

c.) $4\left(\frac{1}{3}\right)^x = 108$

$\frac{4\left(\frac{1}{3}\right)^x}{4} = \frac{108}{4}$

$\left(\frac{1}{3}\right)^x = 27$

$3^{-x} = 3^3$

$x = -3$

d.) $\frac{1}{4}(2)^x = 8$

$4\left(\frac{1}{4}(2)^x\right) = 4(8)$

$2^x = 32$

$2^x = 2^5$

$x = 5$

Example 3: Describe and correct the error a student made when solving the equation $8^{x+3} = 2^{2x-5}$.

$8^{x+3} = 2^{2x-5}$

$(2^3)^{x+3} = 2^{2x-5}$

$2^{3x+3} = 2^{2x-5}$

$\vdots$

X

STep 2 to step 3 must distribute 3

$2^{3x+9} = 2^{2x-5}$

For Examples 4 – 6: Solve each equation for the variable.

4) $\mathbf{5^x + 2 = 27}$

$-2 \quad -2$

$5^x = 25$

$x = 2$

5) $\mathbf{3^{x-7} + 1 = 4}$

$-1 \quad -1$

$3^{x-7} = 3^1$

$x - 7 = 1$

$x = 8$

6) $\left(\frac{1}{4}\right)^{5x} = \left(\frac{1}{4}\right)^{3x+8}$

$5x = 3x + 8$

$2x = 8$

$x = 4$

You try!

7) $\mathbf{2^x - 3 = 5}$

8) $\left(\frac{2}{3}\right)^{6x-1} = \left(\frac{2}{3}\right)^{4x+11}$

9) $\mathbf{4^{5x+1} + 3 = 19}$

REVIEW:

For examples 10 and 11: Solve each the systems using substitution.

10) $-3x - 3y = 3$ $y = -5x - 17$	11) $x + 3y = 1$ $-3x - 3y = -15$

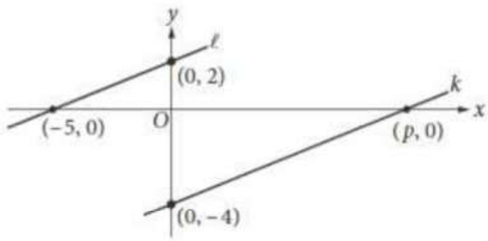
6.2 Day 1 Notes: Graphing Exponential Functions

Learning Objectives:

1) **Graph basic exponential functions and identify as exponential growth or decay.**
2) **Identify the domain/range and asymptotes of an exponential function.**
3) **Distinguish between linear functions and exponential functions.**

Warm up:

1)



In the *xy*-plane above, line ℓ is parallel to line k. What is the value of p?

A) 4
B) 5
C) 8
D) 10

2) $\ell = 24 + 3.5m$

One end of a spring is attached to a ceiling. When an object of mass m kilograms is attached to the other end of the spring, the spring stretches to a length of ℓ centimeters as shown in the equation above. What is m when ℓ is 73?

A) 14
B) 27.7
C) 73
D) 279.5

Key Vocabulary

Exponential Function:

A function of the form $f(x) = a \cdot b^x$ where $a \neq 0$, $b > 0$, and $b \neq 1$.

LINEAR functions change by a constant amount (difference). A table of values will make clear you are adding/subtracting a fixed amount every change in x.

Bonus: Exponential functions are **continuous** functions (when their domain is all real numbers).

Exponential Function : $1, 2, 4, 8, 16, 32, \ldots$ $\times 2$ $\times 2$ $\times 2$

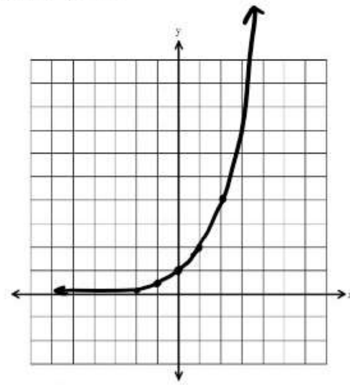
Recall: $a^{-n} = \dfrac{1}{a^n}$

Exponential Growth: When $f(x) = a \cdot b^x$ and $b > 1$

base

Example 1: Use a table of values to graph the function $y = 2^x$

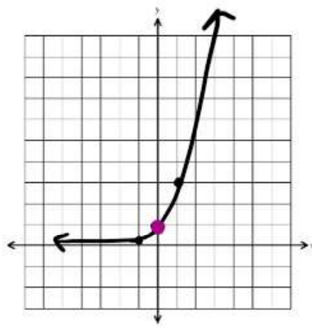
x	$y = 2^x$
−2	$2^{-2} = \frac{1}{2^2} = \frac{1}{4}$
−1	$2^{-1} = \frac{1}{2^1} = \frac{1}{2}$
0	$2^0 = 1$
1	$2^1 = 2$
2	$2^2 = 4$



Example 2: Graph each exponential growth function.

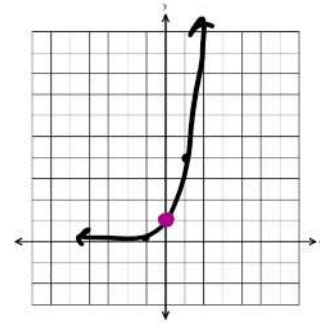
a) $f(x) = 3^x$

x	y
−1	$3^{-1} = 1/3$
0	1
1	3



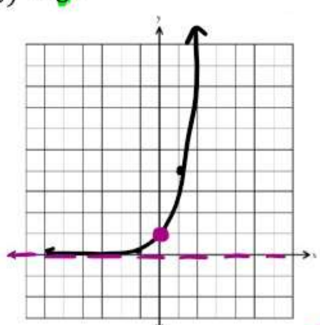
b) $f(x) = 4^x$

x	y
−1	1/4
0	1
1	4



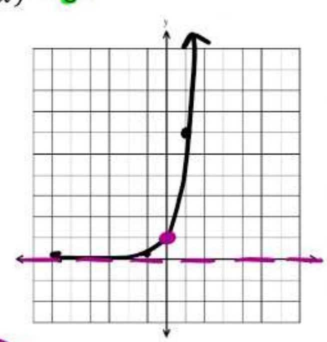
You Try! c) $f(x) = 5^x$

x	y
−1	1/5
0	1
1	5



You Try! d) $f(x) = 6^x$

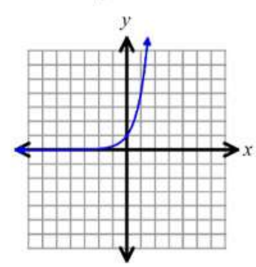
x	y
−1	1/6
0	1
1	6



Characteristics of the graphs of Exponential Growth

When $b > 1$, $f(x)$ has exponential growth

- There is a **horizontal asymptote** along the x-axis (at $y = 0$.)
- For **exponential growth, $f(x)$** moves **away** from the horizontal asymptote as the function moves to the right.
- There is an **anchor point** at (0, 1).
- **Domain:** all real numbers
- **Range:** $y > 0$
- The larger the base number, the faster the function grows in height as the function moves to the right.



$f(x) = 4^x$

Exponential Growth

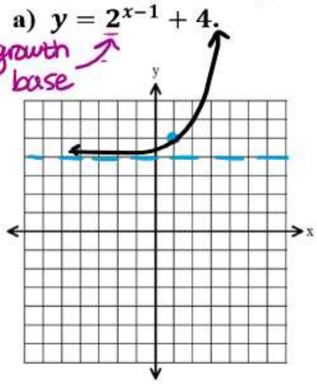
(h, k) Form of an Exponential Function: $f(x) = b^{x-h} + k$

- h will shift the graph left or right
- k will shift the graph up and down
- horizontal asymptote is the equation $y = k$

Example 3: Graph each function below. Identify each as growth or decay, find the domain/range, transformations, and give the equation of the asymptote.

a) $y = 2^{x-1} + 4.$

growth base



Domain: All real #'s

Range: $y > 4$

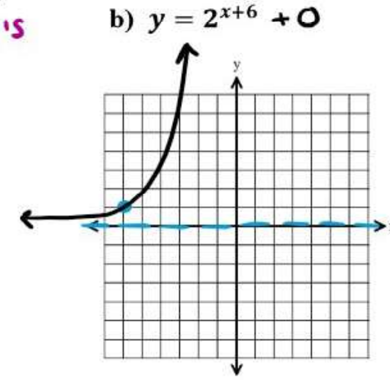
Transformations: right 1, up 4

Growth/Decay?

Asymptote: $y = 4$

x	y

b) $y = 2^{x+6} + 0$



Domain: All real #'s

Range:

Transformations: left 6

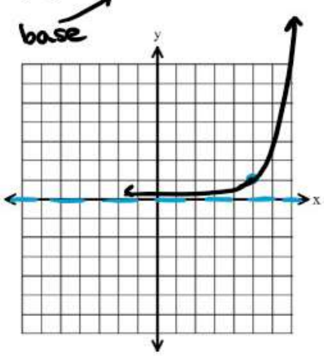
Growth/Decay?

Asymptote: $y = 0$

x	y

c) $y = 7^{x-5} + 0$

base



Domain: All real #'s

Range: $y > 0$

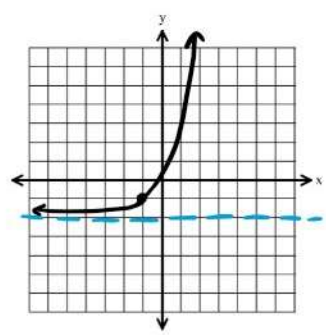
Transformations: right 5

Growth/Decay?

Asymptote: $y = 0$

x	y

You Try! d) $y = 5^{x+1} - 2$



Domain: All real #'s

Range: $y > -2$

Transformations: left 1, down 2

Growth/Decay?

Asymptote: $y = -2$

x	y

You Try! e) $y = 4^{x-3} + 1$

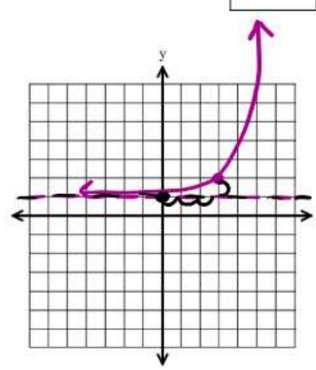
Domain: All real #'s **Growth**/Decay?

Range: $y > 1$ **Asymptote:** $y = 1$

Transformations:

right 3, up 1

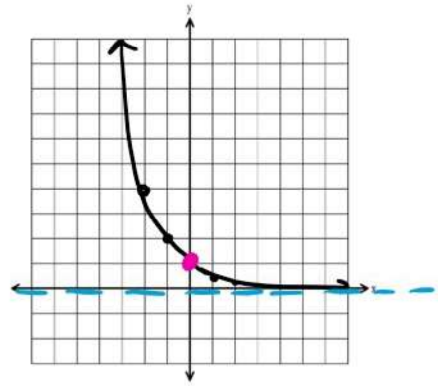
x	y



<u>Exponential Decay:</u> When $f(x) = a \cdot b^x$ and $0 < b < 1$

Example 4: Use a table of values to graph the function $y = \left(\frac{1}{2}\right)^x$

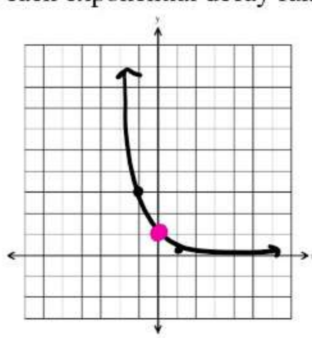
x	$y = \left(\frac{1}{2}\right)^x$
-2	$\left(\frac{1}{2}\right)^{-2} = \frac{1^{-2}}{2^{-2}} = \frac{2^2}{1^2} = 4$
-1	$\left(\frac{1}{2}\right)^{-1} = \frac{1^{-1}}{2^{-1}} = \frac{2^1}{1^1} = 2$
0	$\left(\frac{1}{2}\right)^0 = 1$
1	$\left(\frac{1}{2}\right)^1 = \frac{1}{2}$
2	$\left(\frac{1}{2}\right)^2 = \frac{1^2}{2^2} = \frac{1}{4}$



Example 5: Graph each exponential decay function.

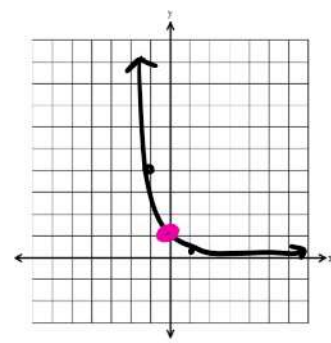
a) $f(x) = \left(\frac{1}{3}\right)^x$

x	y
-1	3
0	1
1	$1/3$



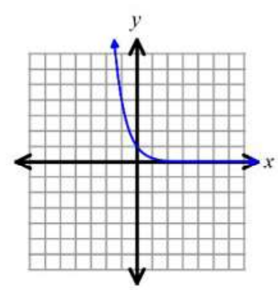
b) $f(x) = \left(\frac{1}{4}\right)^x$

x	y
-1	4
0	1
1	$1/4$



<u>Characteristics of the graphs of Exponential Decay</u>

- There is a **horizontal asymptote** along the x-axis (at $y = 0$.)
- For **exponential decay,** $f(x)$ moves **toward** the horizontal asymptote as the function moves to the right.
- There is an **anchor point** at (0, 1).
- **Domain:** all real numbers
- **Range:** $y > 0$
- The SMALLER the base number, the faster the function grows in height as the function moves to the left. $g(x) = \left(\frac{1}{4}\right)^x$ will grow faster to the left than $f(x) = \left(\frac{1}{2}\right)^x$.

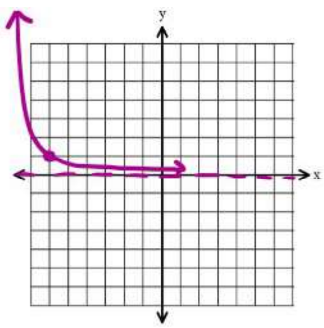


$f(x) = \left(\frac{1}{4}\right)^x$

Exponential Decay

Example 6: Graph each function below. Identify each as growth or decay, find the domain/range and give the equation of the asymptote.

a) $y = \left(\frac{1}{2}\right)^{x+6} + 0$



Domain: All real #'s

Range: y>0

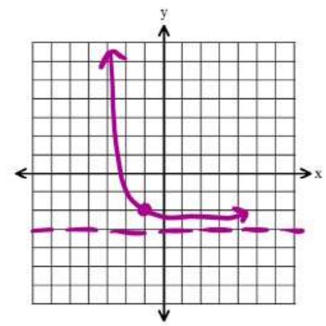
Transformations: left 6

Growth/Decay?

Asymptote: y=0

x	*y*

b) $y = \left(\frac{1}{5}\right)^{x+1} - 3$



Domain: All real #'s

Range: y > -3

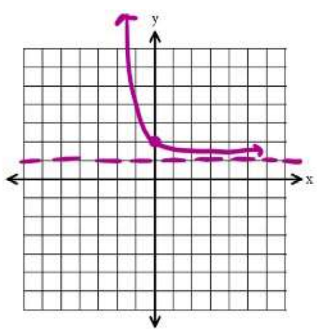
Transformations: left 1, down 3

Growth/Decay?

Asymptote: y = -3

x	*y*

You Try! c) $y = \left(\frac{1}{4}\right)^{x} + 1$



Domain: All real #'s

Range: y>1

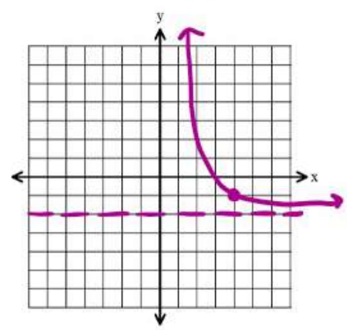
Transformations: up 1

Growth/Decay?

Asymptote: y=1

x	*y*

You Try! d) $y = \left(\frac{1}{3}\right)^{x-4} - 2$



Domain: All real #'s

Range: y > -2

Transformations: right 4, down 2

Growth/Decay?

Asymptote: y = -2

x	*y*

Rate of Change

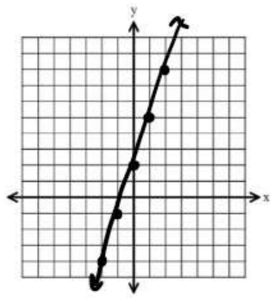
- For a line, the rate of change (or SLOPE) stays the same for the entire line.
- For an exponential graph, the rate of change is not the same for the entire curve.
- For exponential growth, the rate of change INCREASES as the function moves to the right. We can see this by looking at how the graph gets steeper as the function moves to the right.
- For exponential decay, the rate of change DECREASES as the function moves to the right. We can see this by looking at how the graph gets less steep as the function moves to the right.

Example 7: Graph the following functions, then classify as linear or exponential.

a)

x	***y***
-2	-4
-1	-1
0	2
1	5
2	8

+3 +3 +3 +3

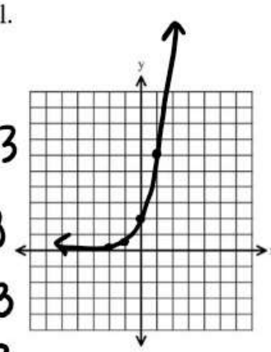


Linear

b)

x	***y***
-2	$\frac{2}{9}$
-1	$\frac{2}{3}$
0	2
1	6
2	18

×3 ×3 ×3 ×3



Exponential

Can you explain how to identify types of functions?

a) a linear function?

b) an exponential function?

What if you have a table, but no graph?

To recognize if a function is linear or exponential without an equation or graph, look at the differences of the *y*-values.

- If the first differences are constant (the same), then the table represents a linear function.

Example: 5, 7, 9, 11, …

+2 +2 +2 → Linear

- If the ratio of consecutive *y*-values is constant, then the table represents an exponential function.

Example: 3, 9, 27, 81, …

×3 ×3 ×3 → Exponential

Example 8: Based on each table, tell whether the function is linear, exponential, or neither.

a)

x	-3	-2	-1	0	1	2
y	-7	-5	-3	-1	1	3

+2

Linear

b)

x	-3	-2	-1	0	1	2
y	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4

×2 ×2

Exponential

c)

x	-3	-2	-1	0	1	2
y	9	4	1	0	1	4

neither

d)

x	-3	-2	-1	0	1
y	4	12	36	108	324

×3 ×3

Exponential

e) $y = 5^x$

exponential

f) $y = -4x + 5$

$y = mx + b$

Linear

You try! Linear, exponential, or neither?

g) $y = (x - 2)^2 - 3$

neither

h) $y = \left(\frac{1}{3}\right)^x$

exponential

i) $2x + 3y = 9$

linear

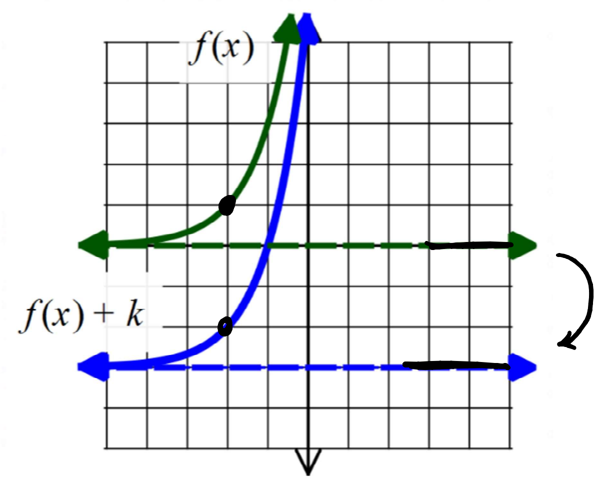
j)

x	-2	-1	0	1	2
y	12	5	0	-3	-4

neither

Example 9: Given $f(x)$ and $f(x) + k$ as graphed below… find the value of k.

$K = -3$



6.2 Day 2: Transformations of Exponential Functions $f(x) = a \cdot b^{x-h} + k$

Learning Objectives:

1) **Graph transformations of exponential functions.**
2) **Identify the anchor point and transformations.**
3) **Identify the domain/range and equation of the asymptote.**

Warm up:

1) If $16 + 4x$ is 10 more than 14, what is the value of $8x$?

A) 2
B) 6
C) 16
D) 80

$16+4x=10+14$

$16+4x=24$

$-16 \quad -16$

$4x=8$

$x=2$

2) For what value of n is $|n-1|+1$ equal to 0 ?

A) 0
B) 1
C) 2
D) There is no such value of n.

$|n-1|+1=0$

$-1 \quad -1$

$|n-1|=-1$

$\left(\frac{1}{4}\right)^{-x}=\left(\frac{4}{1}\right)^{x}$

Are the following exponential functions examples of growth or decay?

a) $y = \frac{1}{4}(2)^x$

$2>1$

$+x$

growth

b) $f(x) = 5\left(\frac{1}{3}\right)^x$

$\frac{1}{3}<1$

$+x$

decay

c) $y = -7\left(\frac{1}{4}\right)^{-x}$

$\frac{1}{4}<1$

$-x$

growth

d) $y = 2(6)^{-x}$

$6>1$

$-x$

decay

<u>Transformations of Exponential Functions</u>: $y = ab^{x-h} + k,\ a \neq 0$

To <u>graph</u> an exponential function in (h, k) form: $y = ab^{x-h} + k,\ a \neq 0$	
Method #1	**Method #2**
1) Graph the horizontal asymptote, ______________, as a dotted line. 2) Plot the anchor point $(h, a+k)$. $(b^0 = 1)$ 3) Check for reflections and translations using a, b, h, and k.	creating T-chart multiply y-values by a translated $\pm h, \pm k$

Example 1: Graph the following exponential functions. Graph the HA. Identify the D, R, and transformations. Tell whether the graph models growth or decay.

4 time R1 D3

a) $y = 4 \cdot 2^{x-1} - 3$

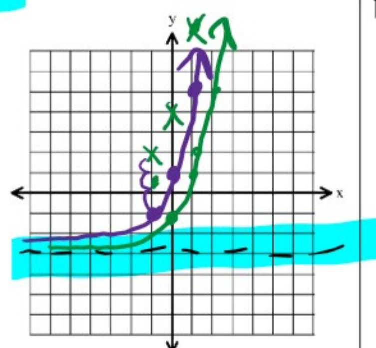
Domain:

Range:

Transformations:

Growth/Decay?

Asymptote:



x	y = 2^x	
−1	$2^{-1} = \frac{1}{2^1} = \frac{1}{2} \times 4 = 2$	
0	$2^0 = 1$	$1 \times 4 = 4$
1	$2^1 = 2$	$2 \times 4 = 8$

L3 D1

b) $y = 2 \cdot \left(\frac{1}{3}\right)^{x+3} - 1$

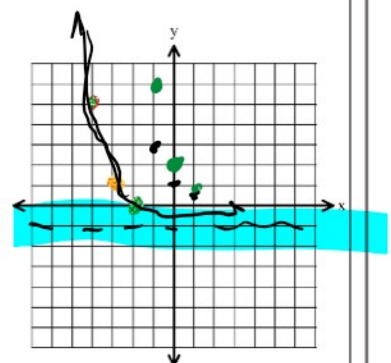
Domain:

Range:

Transformations

Growth/Decay?

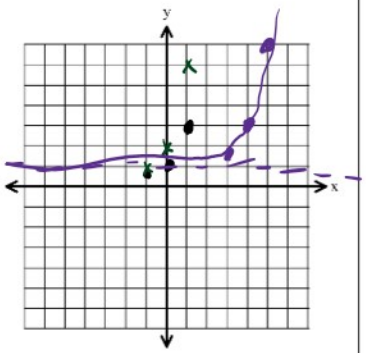
Asymptote:



x	y = $\left(\frac{1}{3}\right)^x$		
−1	$\left(\frac{1}{3}\right)^{-1} = \left(\frac{3}{1}\right)^1 = 3 \times 2 = 6$		
0	$\left(\frac{1}{3}\right)^0 = 1$	$\times 2$	2
1	$\left(\frac{1}{3}\right)^1 = \frac{1}{3}$	$\times 2$	$\frac{2}{3}$

c) $y = 2 \cdot 3^{x-4} + 1$ right 4 up 1

Domain:
Range:
Transformations:
Growth/Decay?
Asymptote:



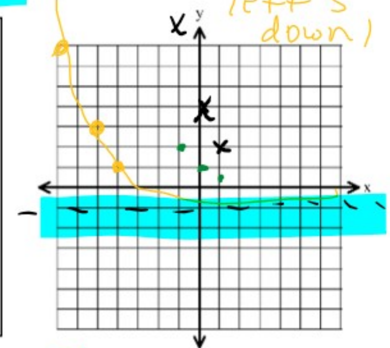
x	y = 3^x
-1	1/3
0	1
1	3

x2 = 2/3
x2 = 2
x2 = 0

d) $y = 4 \cdot \left(\frac{1}{2}\right)^{x+5} - 1$ left 5 down 1

Domain:
Range:
Transformations:
Growth/Decay?
Asymptote:

left 5 down 1

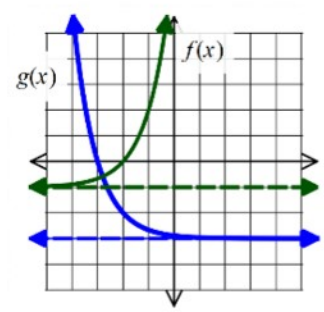


x	y = $\left(\frac{1}{2}\right)^x$
-1	2
0	1
1	1/2

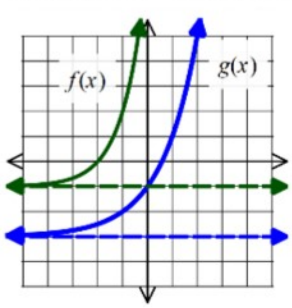
x 4 = 8
x 4 = 4
x 4 = 2

Example 2: Describe the transformation on $f(x)$ that created $g(x)$.
Choose all that apply.

a) $f(x)$ is shifted down 2.
b) $f(x)$ is shifted left 1.
c) $f(x)$ was vertically reflected.
d) the base of $f(x)$ was changed to its reciprocal.



Example 3: Which statement below is true?



A. $f(x)$ grows faster than $g(x)$.
B. $g(x)$ grows faster than $f(x)$.
C. $f(x)$ and $g(x)$ grow at the same rate of change.
D. It is impossible to determine which function grows faster.

Example 4: Graph the following exponential functions. Identify D, R, and Transformations.

a) $y = \left(\frac{1}{3}\right)^x$

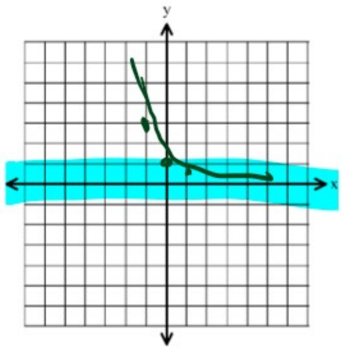
Domain: R

Range: $y > 0$

Transformations: none

Growth/Decay?

Asymptote: $y = 0$



x	y $= \left(\frac{1}{3}\right)^x$
-1	3
0	1
1	1/3

b) $y = -\left(\frac{1}{3}\right)^x$

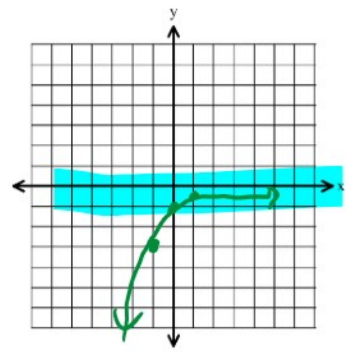
Domain: R

Range: $y < 0$

Transformations: reflected

Growth/Decay? neg

Asymptote: $y = 0$



x	y
-1	3
0	1
1	1/3

x -1 = -3
x -1 = -1
x -1 = -1/3

You try parts c and d!

c) $f(x) = 3^{x+1}$ left 1

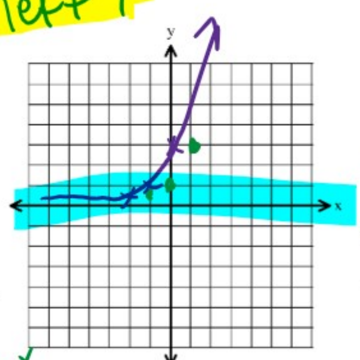
Domain: R

Range: $y > 0$

Transformations: left 1

Growth/Decay?

Asymptote: $y = 0$



x	y $= 3^x$
-1	1/3
0	1
1	3

d) $h(x) = -3^{x+1}$ left 1

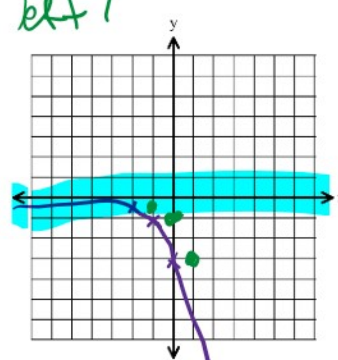
Domain: R

Range: $y < 0$

Transformations: reflect left 1

Growth/Decay?

Asymptote: $y = 0$



x	y
-1	1/3
0	1
1	3

x -1 → $-\frac{1}{3}$
x -1 → -1
x -1 → -3

Example 5: Graph the following exponential functions. Identify D, R, and Transformations.

a) $y = -2 \cdot 3^{x+1} - 4$ left 1 D 4

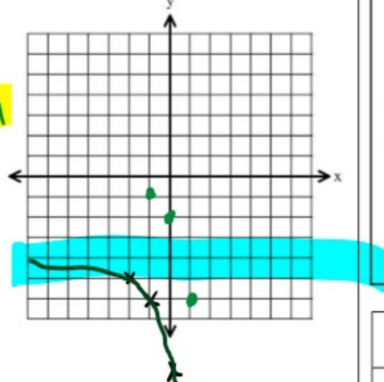
Domain: R

Range: y<4

Transformations:
reflect
stretch 2
left 1
down 4

Growth/Decay?

Asymptote:
y = -4



x	y
-1	1/3
0	1
1	3

x -2 = -2/3
x -2 = -2
x -2 = -6

You Try! b) $y = -3\left(\frac{1}{4}\right)^x + 2$

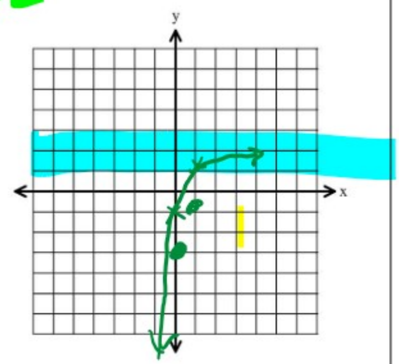
Domain: R

Range: y < 2

Transformations:
reflect
stretch 3
up 2

Growth/Decay?

Asymptote:
y = 2

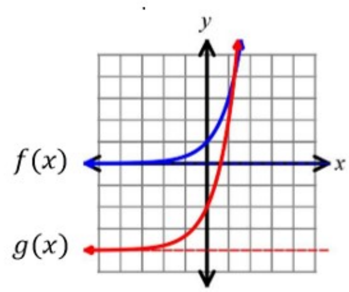


x	y
-1	4
0	1
1	1/4

x -3 = -12
x -3 = -3
x -3 = -3/4

Example 6: Consider $f(x)$ and $g(x)$, where $g(x)$ is a result of transformations of $f(x)$, such that $g(x) = k \cdot f(x) + h$. Which options below could be the values of k and h?

A) $k = -2, h = -4$
B) $k = 2, h = -4$
C) $k = 4, h = -2$
D) $k = -4, h = -2$



6.3: Writing an Equation for a Geometric Sequence/Exponential Function

Learning Objectives:

1) Write an equation for a geometric sequence in exponential function notation and recursive notation.
2) Use geometric sequences to model real life problems.

Warm-Up:

1) Salim wants to purchase tickets from a vendor to watch a tennis match. The vendor charges a one-time service fee for processing the purchase of the tickets. The equation $T = 15n + 12$ represents the total amount T, in dollars, Salim will pay for n tickets. What does 12 represent in the equation?

A) The price of one ticket, in dollars
B) The amount of the service fee, in dollars
C) The total amount, in dollars, Salim will pay for one ticket
D) The total amount, in dollars, Salim will pay for any number of tickets

2)

x	$w(x)$	$t(x)$
1	−1	−3
2	3	−1
3	4	1
4	3	3
5	−1	5

The table above shows some values of the functions w and t. For which value of x is $w(x) + t(x) = x$?

A) 1
B) 2 $3 + -1 = 2$ ✓
C) 3
D) 4

Key Vocabulary:

Geometric Sequence

A sequence with a common ratio like 2, 6, 18, 54… or 4/3, 4/3, 16/3, 64/3…

Exponential Function A continuous function with a multiplier

Writing an equation of the exponential functions:

1. Determine if the function is exponential.
2. Identify the value of "b"
3. Create an equation using the initial value, a, and the value of b.

$f(x) = a \cdot b^x$
$f(x)$ = the output
a = the initial value (when b = 0)
b = base (multiplier)
x = the input

For Examples 1 – 7: Write the function for each exponential function or geometric sequence.

1:

x	0	1	2	3
y	6	12	24	48

$a=6$
$b=2$
$\boxed{f(x)=6\cdot 2^x}$

2:

x	0	1	2	3
y	351	117	39	13

$a=351$
$b=\frac{1}{3}$
$\boxed{f(x)=351\cdot\left(\frac{1}{3}\right)^x}$

3: **3, 12, 48, 192, …**
~~**27, 9, 3, 1, …**~~

$b=4$
$a=3/4$
$\boxed{f(x)=\frac{3}{4}\cdot 4^x}$

4:

#4 27, 9, 3, 1

$b=\frac{1}{3}$ $a=81$
$\boxed{f(x)=81\left(\frac{1}{3}\right)^x}$

You try!

5:

x	0	1	2	3
y	3	9	27	81

$a=3$
$b=3$
$\boxed{y=3\cdot 3^x \text{ or } y=3^{x+1}}$

6:

x	0	1	2	3
y	224	112	56	28

$a=224$
$b=\frac{1}{2}$
$\boxed{f(x)=224\left(\frac{1}{2}\right)^x}$

7: **64, 16, 4, 1, …**

$b=\frac{1}{4}$
$a=256$
$\boxed{f(x)=256\left(\frac{1}{4}\right)^x}$

8: 10, 50, 250, 1250, …

$b=5$
$a=10/5=2$
$\boxed{f(x)=2(5)^x}$

For Examples 9 – 10: a. Write an equation for each function or sequence. b. Find $f(7)$.

9.

x	0	1	2	3
$f(x)$	64	16	4	1

$a = 64$
$b = 1/4$
$f(x) = 64\left(\frac{1}{4}\right)^x$
$f(7) = 64\left(\frac{1}{4}\right)^7 = \frac{64}{4^7} = \frac{4^3}{4^7} = \frac{1}{4^4} = \frac{1}{256}$

10.

x	0	1	2	3
$f(x)$	2	6	18	54

$a = 2$
$b = 3$
$f(x) = 2 \cdot 3^x$
$f(7) = 2 \cdot 3^7$
$= 4374$

For Examples 11 – 12: a. Write an equation for each function or sequence. b. Find $f(8)$.

11. $32, 16, 8, 4, \ldots$

$b = \frac{1}{2}$
$a = 64$
$f(x) = 64\left(\frac{1}{2}\right)^x$
$f(7) = 64 \cdot \left(\frac{1}{2}\right)^7 = \frac{64}{128} = \frac{1}{2}$

12. $\frac{1}{25}, \frac{1}{5}, 1, 5, \ldots$

$b = 5$
$a = \frac{1}{125}$ $\left(\frac{1}{25} \div 5 = \frac{1}{25} \cdot \frac{1}{5} = \frac{1}{125}\right)$
$f(x) = \frac{1}{125}(5)^x$
$f(7) = \frac{1}{125}(5)^7 = \frac{1}{5^3} \cdot 5^7 = 5^4 = 625$

Example 13: Suppose a ball is dropped from a height of 16 meters and it bounces up to 75% of its previous height after each bounce. Write an equation that will represent the height of the ball after "x" bounces.

$a =$ 16 $b =$ $\frac{3}{4}$ or $.75$ $f(x) =$ $16\left(\frac{3}{4}\right)^x$ or $f(x) = 16(.75)^x$

How high will the ball be after 10 bounces? (use a calculator) $16\left(\frac{3}{4}\right)^{10} = .901$ meters high

Example 14: A pendulum swings 80 cm on its first swing, 76 cm on its second swing, 72.2 cm on its third swing, and 68.59 cm on its fourth swing. Find the length of the 12th swing. (Hint: write an equation to represent this pattern.)

Swing #	1	2	3	4
dist	80	76	72.2	68.59

$b = \frac{76}{80}$ or $b \doteq .95$ $a = \frac{80}{76/80} = 80 \cdot \frac{80}{76} = \frac{6400}{76}$

$f(x) = \frac{6400}{76}\left(\frac{76}{80}\right)^x$ or $f(x) = \frac{6400}{76}(.95)^x$

$f(12) = \frac{6400}{76}(.95)^{12} = \boxed{45.504}$ (by calc)

Recursive Formula for a Geometric Sequence:

$r =$ multiplier $a_1 =$ first term

$a_n =$ next term (n^{th} term) $a_{n-1} =$ previous term ($n-1^{st}$ term)

$a_n = r(a_{n-1})$

$a_1 =$ ____

Example 15, write the recursive formula for each geometric sequence.

a) 9, 18, 36, 72, …

$a_1 = 9 \quad r = 2$

$a_n = 2(a_{n-1})$

b) $3, \frac{3}{2}, \frac{3}{4}, \frac{3}{8}, \frac{3}{16}, \ldots$

$a_1 = 3 \quad r = \frac{1}{2}$

$a_n = \frac{1}{2}(a_{n-1})$

You try!

c) $9, 3, 1, \frac{1}{3}, \frac{1}{9}, \ldots$

$a_1 = 9 \quad r = \frac{1}{3}$

$a_n = \frac{1}{3}(a_{n-1})$

d) $\frac{1}{2}, 2, 8, 32, 128, \ldots$

$a_1 = \frac{1}{2} \quad r = 2$

$a_n = 2(a_{n-1})$

Example 16: Which of the following represent the sequence $2, 12, 72, 432, \ldots$? Choose all that apply.

A) $y = 6 \cdot 2^x$ → no multiplier not 2

B) $y = 2 \cdot 6^x$ → No when $x = 0, y = 2$ and $x = 1, y = 12$

C) $a_n = 6(a_{n-1}); a_1 = 2$

D) $a_n = 2(a_{n-1});\ a_1 = 6$ → no, $a_1 \neq 6$

E) $y = \frac{1}{3} \cdot 6^x$ $\quad y = \frac{1}{3} \cdot 6^x$ when $x = 1$ $y = 2$ ✓

REVIEW:

Example 17: Solve the following systems using elimination.

$-2x - 9y = -25$) -1

$-4x - 9y = -23$

$$\begin{aligned} 2x + 9y &= 25 \\ -4x + 9y &= -23 \\ \hline -2x &= 2 \end{aligned}$$

$x = -1$

$y = 3$

$-4(-1) - 9y = -23$

$4 - 9y = -23$

$-9y = -27$

$y = 3$

$\boxed{(-1, 3)}$

$5x + 4y = -14$) -3

$3x + 6y = 6$) 5

$$\begin{aligned} -15x - 12y &= 42 \\ 15x + 30y &= 30 \\ \hline 18y &= 72 \end{aligned}$$

$y = 4$

$x = -6$

$3x + 6(4) = 6$

$3x + 24 = 6$

$3x = -18$

$x = -6$

$(-6, 4)$

6.4: Exponential Growth & Decay Functions

Learning Objectives:
1) Model exponential growth and decay from real life problems using a growth/decay rate and initial value.

Note: No warm-up to make time for Topic 6 3Acts today.

<u>Exponential Growth Model</u>: $y = a\underbrace{(1 + r)}^{t}$

a **is the** initial value r **is the** growth rate in decimal
~~1+~~$1+r$ **is the** growth factor in decimal t **is the** time in years

Example 1: Identify the initial amount, growth rate, and the growth factor for the following equations.

a.) $y = 20(1.25)^x$

inital value = 20

growth factor = 1.25

b.) $y = 1.2^x$

inital value = 1.

growth factor = 1.2

You try!

c.) $y = 150(1.4)^x$

inital value = 150

growth factor = 1.4

d.) $y = 3 \cdot 1.05^x$

inital value = 3

growth factor = 1.05

Modeling Exponential Growth Functions: $y = a \cdot b^x$ **where** $b = (1 + r)$

Example 2: Alex buys a rare baseball card for $150. The value of the card increases by 30% each year.

a.) Write an exponential growth function that could be used to find the value of the card t-years after he bought it.

$y = 150(1+.3)^t$

$y = 150(1.3)^t$

b.) Find the value of the card after 3 years (use a calculator).

$y = 150(1.3)^3 = 329.55$

Example 3: You put $250 into a savings account that earns 4% annual interest compounded yearly. You do not make any deposits or withdrawals.

a.) Write an exponential growth function that could be used to find the value of your savings account after t-years

$y = 250(1.04)^x$

b.) How much money is in the account after 5 years?

$y = 250(1.04)^5 = \$304.16$

Exponential Decay Model: $y = a(1 - r)^t$

a is the initial value

r is the decay rate in decimal

$1+r$ is the decay factor in decimal

$1-r$

t is the time in years

Example 4: Identify the initial amount, decay rate, and the decay factor for each equation.

a) $y = 20(.75)^x$

inital value = 20

decay factor = .75 or $\frac{3}{4}$

b) $y = 100\left(\frac{1}{4}\right)^x$

inital value = 100

decay factor = $\frac{1}{4}$ or .25

You try!

c) $y = 150(.70)^x$

inital value = 150

decay factor = .70

d) $y = (0.85)^x$

inital value = 1

decay factor = .85

Modeling Exponential decay functions: $y = a \cdot b^x$ where $b = (1 - r)$

Example 5: Henry buys a car for $25,000. The value then depreciates at a rate of 15% per year.

a.) Write an exponential decay function that could be used to find the value of the car after t-years.

$y = 25000(1 - .15)^x$ $y = 25000(.85)^t$

b.) What is the value of the car 3 years after Henry purchases the car? (use a calculator)

$y = 25000(.85)^3$ $y = \$15353.13$

Example 6: You buy a computer for \$1,500. It depreciates at the rate of 20% per year.

a.) Write an exponential decay function that could be used to find the value of the car after t-years.

$y = 1500(1-.2)^t$ $y = 1500(.8)^t$

b.) What is the value of the computer after 3 years?

$y = 1500(.8)^3$ $y = \$768$

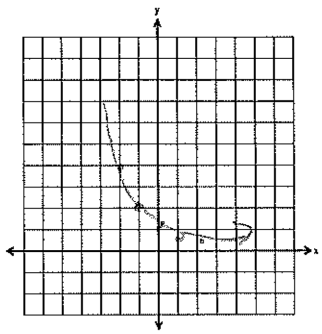
BEFORE WE LEAVE YOU: Graphing with negative exponents

Example 7: Complete the table and graph each function.

a) $y = 2^{-x}$

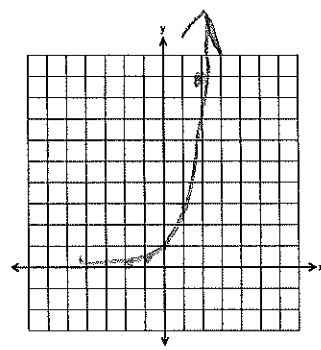
$y = \left(\frac{1}{2}\right)^x$

x	$y = 2^{-x}$
-2	4
-1	2
0	1
1	1/2
2	1/4



b) $y = \left(\frac{1}{3}\right)^{-x}$ $y = 3^x$

x	$y = \left(\frac{1}{3}\right)^{-x}$
-2	1/9
-1	1/3
0	1
1	3
2	9



Moral of the story…

Since $2^{-x} = \left(\frac{2}{1}\right)^{-x} = \left(\frac{1}{2}\right)^x$, exponential decay can also be modeled with negative exponents and b > 1

Since $\left(\frac{1}{3}\right)^{-x} = \left(\frac{3}{1}\right)^x = 3^x$, exponential growth can also be modeled with positive exponents and 0 < b < 1

Math is a beautiful thing ☺