Alg 2 Honors Unit 4 notes

Notes 4.1 Operations with Polynomials - Complete these before you do the practice problems for 4.1.

Examples: **Simplify the following expressions.**

Key

1) $-2 + 3(t^2 - 6t + 2) + 4(5t^2 - t - 8) - (6t + 1)$

$-2 + 3t^2 - 18t + 6 + 20t^2 - 4t - 32 - 6t - 1$

$\boxed{23t^2 - 28t - 29}$

You try:

2) $20 - \frac{1}{2}(-6t^2 + 8t - 3) - 3(t^2 - 6t + 5) + (t - 3)$

$20 \cancel{+3t^2} - 4t + \frac{3}{2} \cancel{-3t^2} + 18t - 15 + t - 3$

$20 - 15 - 3 + 3/2$

$\frac{4}{2} + \frac{3}{2} = \frac{7}{2}$

$15t + \frac{7}{2}$

Example 3) Subtract: $5z^2 - z + 3$ from $4z^2 + 9z - 12$

$(4z^2 + 9z - 12) - (5z^2 - z + 3)$

$4z^2 + 9z - 12 - 5z^2 + z - 3$

$\boxed{-z^2 + 10z - 15}$

You try:

4) Subtract: $-3k^2 + 6k$ from $5k^2 - k - 4$

$5k^2 - k - 4$
$-(-3k^2 + 6k)$

$\rightarrow$

$5k^2 - k - 4$
$3k^2 - 6k$

$8k^2 - 7k - 4$

Example 5) $-2(x + 3)(3x^2 - 2x + 4)$

$-2(3x^3 - 2x^2 + 4x + 9x^2 - 6x + 12)$

$-2(3x^3 + 7x^2 - 2x + 12)$

$\boxed{-6x^3 - 14x^2 + 4x - 24}$

You try:

6) $5(x^2+4)(-3x^2-8x-1)$

$(5x^2+20)(-3x^2-8x-1)$

$-15x^4-40x^3-5x^2$

$+\quad -60x^2-160x-20$

$\boxed{-15x^4-40x^3-65x^2-160x-20}$

Example 7) $(x-5)(x+2)(3x-1)$

$(x^2-3x-10)(3x-1)$

$3x^3-x^2-9x^2+3x-30x+10$

$\boxed{3x^3-10x^2-27x+10}$

You try:

8) $(2x-1)(x+4)(x-1)$

$(2x-1)(x^2+3x-4)$

$2x^3+6x^2-8x$

$+\quad -x^2-3x+4$

$\boxed{2x^3+5x^2-11x+4}$

Example 9) $2x(x-7)^2(x+3)$

$2x(x-7)(x-7)(x+3)$

$2x(x^2-14x+49)(x+3)$

$2x(x^3+3x^2-14x^2-42x+49x+147)$

$2x(x^3-11x^2+7x+147)$

$\boxed{2x^4-22x^3+14x^2+294x}$

You try:

10) $5x^2(x+3)^2(2x-3)$

$(x^2+6x+9)(10x^3-15x^2)$

$10x^5 - 15x^4$

$+60x^4 - \cancel{90x^3}$

$+ \cancel{90x^3} - 135x^2$

$\boxed{10x^5 + 45x^4 - 135x^2}$

Example 11) Write a polynomial to represent the volume of a cylinder with a radius of $(3x-2)$ inches and a height of $(x+5)$ inches. Leave your answer in terms of pi.

$(3x-2)(3x-2)$

$9x^2-12x+4$

$V=\pi r^2 h$

$V=\pi(3x-2)^2(x+5)$

$V=\pi(9x^2-12x+4)(x+5)$

$V=\pi(9x^3+45x^2-12x^2-60x+4x+20)$

$\boxed{V=\pi(9x^3+33x^2-56x+20)}$

or

$\boxed{V=9\pi x^3+33\pi x^2-56\pi x+20\pi}$

You try:

12) Write a polynomial to represent the volume of a cone with a radius of $(x+6)$ inches and a height of $h=(2x-1)$ inches. Leave your answer in terms of pi. *(hint: volume of a cone is one third the base times the height)*

$V=\pi r^2 \frac{h}{3}$

$V=\frac{\pi}{3}(x+6)^2(2x-1) = \frac{\pi}{3}(x^2+6x+36)(2x-1)$

$\frac{\pi}{3}\left(2x^3+12x^2+72x - x^2 - 6x - 36\right)$

$\frac{\pi}{3}(2x^3+11x^2+66x-36)$

$\boxed{\frac{2\pi}{3}x^3+\frac{11\pi}{3}x^2+22\pi-12\pi}$

2017

Key

4.2 Notes: Factoring and Solving Polynomials

Lesson Plan
*4.1 HW Questions
*4.1 Quiz
*4.2 Notes
*HW is wk 4.2

Sum of two cubes: $(a + b)(a^2 - ab + b^2)$

Difference of two cubes: $(a - b)(a^2 + ab + b^2)$

Examples: Factor each sum or difference of cubes.

1) $x^3 + 125$ ($125 = 5^3$)

$a^3 + b^3$ so $a = x$, $b = 5$

$(a+b)(a^2 - ab + b^2)$

$(x+5)(x^2 - 5x + 25)$

2) $27g^6 - 343$

$(3g^2)^3 - (7)^3$

$(3g^2 - 7)(9g^4 + 21g^2 + 49)$

3) $2w^3 - 54h^6$

$2(w^3 - 27h^6)$

$2(w^3 - (3h^2)^3)$

$2(w - 3h^2)(w^2 + 3h^2w + 9h^4)$

4) $1 + 8a^3 = 1^3 + (2a)^3$

$(1+2a)(1 - 2a + 4a^2)$

Summary of types of factoring:

***GCF**

$15x^5y - 27x^2y^3$

$3x^2y(5x^3 - 9y^2)$

***Difference of Perfect Squares**

$16y^4x^2 - 25z^8$

$(4y^2x + 5z^4)(4y^2x - 5z^4)$

***Trinomials**

$15x^2 - 19x - 10$

$(5x + 2)(3x - 5)$

***Sum/Difference of Cubes**

see prev. page

Examples: Factor each polynomial completely, use *i* if necessary.

4) $x^4 + x^2 - 20$

$(x^2 + 5)(x^2 - 4)$

$(x^2 - (-5))(x + 2)(x - 2)$

$\sqrt{-5} \cdot \sqrt{-5}$

$(x + i\sqrt{5})(x - i\sqrt{5})(x + 2)(x - 2)$

now we know the roots are $i\sqrt{5}, -i\sqrt{5}, -2, 2$, if set = to 0.

5) $c^4 - 81$

$9 \cdot 9$

$(c^2 + 9)(c^2 - 9)$

$(c^2 - -9)$ $\quad 3 \cdot 3$

$c \cdot c \quad 3i \; 3i$

$(c + 3i)(c - 3i)(c + 3)(c - 3)$

zeros are $\pm 3i, \pm 3$, if set = to 0.

6) $x^6 - 64$

$x^3 \cdot x^3 \quad 8 \cdot 8$

$(x^3 + 8)(x^3 - 8)$ ← now use difference/sum of cubes equation

$x \cdot x \cdot x \quad 2 \cdot 2 \cdot 2$

$\boxed{(x+2)(x^2-2x+4)(x-2)(x^2+2x+4)}$

7) $3x^8 + 18x^5 + 24x^2$

$3x^2(x^6 + 6x^3 + 8)$

$3x^2(x^3 + 2)(x^3 + 4)$

$\cancel{3x(x+\sqrt[3]{2})}$ leave it like that

8) $3(2x + 1) - 5x(2x + 1)^2$

like: $3y - 5xy^2$

$y(3 - 5xy)$

$(2x+1)(3 - 5x(2x+1))$

$(2x+1)(-10x^2 - 5x + 3)$

$-(2x+1)(10x^2 + 5x - 3)$

9) $4(3 - x)^3 - 9(3 - x)^2$

$(3-x)^2(4(3-x) - 9)$

$(3-x)^2(-4x + 3)$

$-(3-x)^2(4x - 3)$

Examples: Solve the following polynomial equations by factoring.

10) $6v^3 = 384$

$6v^3 - 384 = 0$

$6(v^3 - 64) = 0$

$v \cdot v \cdot v \quad 4 \cdot 4 \cdot 4$

$6(v-4)\underbrace{(v^2 + 4v + 16)}_{\text{quad formula}} = 0$

one root is 4

$\dfrac{-4 \pm \sqrt{16 - 4(1)(16)}}{2(1)}$

$= \dfrac{-4 \pm \sqrt{-48}}{2} = \dfrac{-4 \pm 4i\sqrt{3}}{2}$

$= -2 \pm 2i\sqrt{3}$

$\boxed{\text{roots are } 4, -2 \pm 2i\sqrt{3}}$

11) $6r^7 + 6r^5 = 9r^6 + 9r^4$

$6r^7 - 9r^6 + 6r^5 - 9r^4 = 0$

$3r^4(2r^3 - 3r^2 + 2r - 3) = 0$

$3r^4[r^2(2r-3) + (2r-3)] = 0$

$3r^4[(r^2+1)(2r-3)] = 0$

$3r^4(r+i)(r-i)(2r-3) = 0$

$\boxed{r = 0, -i, i, 3/2}$

12) $m^3 + 4m + 4 = 2m^2 + 12$

$m^3 - 2m^2 + 4m - 8 = 0$

$m^2(m-2) + 4(m-2) = 0$

$(m^2+4)(m-2) = 0$

$(m^2 - -4)(m-2)$

$2i \cdot 2i$

$(m+2i)(m-2i)(m-2) = 0$

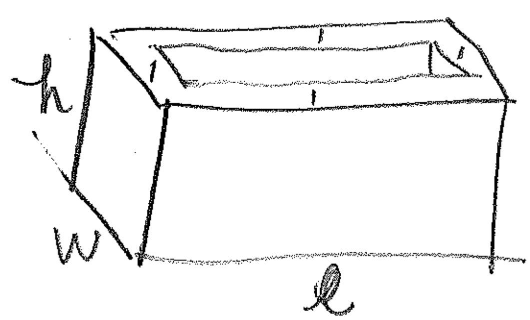
$\boxed{m = \pm 2i, 2}$

13) You are designing a marble planter for a city park. You want the length of the planter to be six times the height and the width to be three times the height. The sides should be one foot thick. Because the planter will be on the sidewalk, it does not need a bottom. What should the outer dimensions of the planter be if it is to hold 4 cubic feet of dirt?

$V = h(w-2)(l-2)$

$l = 6 \cdot h$

$w = 3 \cdot h$



$\overset{4}{\cancel{V}} = h(3h-2)(6h-2)$

$4 = h(18h^2 - 18h + 4)$

$0 = 18h^3 - 18h^2 + 4h - 4$

$0 = 9h^3 - 9h^2 + 2h - 2$

$0 = 9h^2(h-1) + 2(h-1)$

$0 = (9h^2+2)(h-1)$

$0 = (3h + i\sqrt{2})(3h - i\sqrt{2})(h-1)$

so $h = \cancel{\pm i\sqrt{2}}, 1$

$\boxed{h = 1}$

4.3: Dividing Polynomials: Long/Synthetic Division

Lesson Plan
4.2 HW ?s
4.2 Quiz
4.3 Notes
HW is wk 4.3

$$\begin{array}{r} 441 \text{ R}3 \\ 12\overline{)5295} \\ -48 \\ \hline 49 \\ -48 \\ \hline 15 \\ -12 \\ \hline 3 \end{array}$$

$$\frac{5295}{12} = \boxed{441\frac{3}{12}} = \boxed{441\frac{1}{4}}$$

Divide using long division

1) $f(x) = 3x^4 - 5x^3 + 4x - 6$ by $x^2 - 3x + 5$

$$\begin{array}{r} 3x^2 + 4x - 3 \\ x^2 - 3x + 5 \overline{)\, 3x^4 - 5x^3 + 0x^2 + 4x - 6} \\ -(3x^4 - 9x^3 + 15x^2) \\ \hline 4x^3 - 15x^2 + 4x \\ -(4x^3 - 12x^2 + 20x) \\ \hline -3x^2 - 16x - 6 \\ -(-3x^2 + 9x - 15) \\ \hline -25x + 9 \end{array}$$

$$\frac{3x^4 - 5x^3 + 4x - 6}{x^2 - 3x + 5} = 3x^2 + 4x - 3 + \frac{-25x + 31}{x^2 - 3x + 5}$$

2) $f(x) = (x^3 + 3x^2 - 7) \div (x^2 - x - 2)$ $= \boxed{x+4+\frac{6x+5}{x^2-x-2}}$

$$\begin{array}{r} x + 4 \\ x^2-x-2 \,\big)\, \cancel{x^3} + 3x^2 - 0x - 7 \\ -\cancel{x^3} - x^2 - 2x \\ \hline \cancel{4x^2} + 2x - 7 \\ -\cancel{4x^2} - 4x - 8 \\ \hline 6x + 15 \end{array}$$

$6x + 15$ ← remainder

3) $f(x) = 2x^3 + 10x^2 + 6x - 18$ by $2x + 6$

$$\begin{array}{r} x^2 \quad 2x \quad -3 \\ 2x+6 \,\big)\, \cancel{2x^3} + 10x^2 + 6x - 18 \\ \cancel{2x^3} + 6x^2 \\ \hline \cancel{4x^2} + 6x \\ \cancel{4x^2} + 12x \\ \hline -6x - 18 \\ -6x - 18 \\ \hline 0 \end{array}$$

$$\frac{2x^3+10x^2+6x-18}{2x+6} = \boxed{x^2+2x-3}$$

Divide using synthetic division

4) $f(x) = 2x^3 + x^2 - 8x + 5$ by $x + 3$

root is -3
So divide by that value

$$\begin{array}{r|rrrr} -3 & 2 & 1 & -8 & 5 \\ & \downarrow & -6 & 15 & -21 \\ \hline & 2 & -5 & 7 & \boxed{-16} \end{array}$$

$$\boxed{2x^2 - 5x + 7 - \frac{16}{x+3}}$$

5) $f(x) = 4x^3 - 3x + 7$ by $x - 1$

$4x^3 + 0x^2 - 3x + 7$

$$\begin{array}{r|rrrr} 1 & 4 & 0 & -3 & 7 \\ & & 4 & 4 & 1 \\ \hline & 4 & 4 & 1 & \boxed{8} \end{array}$$

$$\boxed{4x^2 + 4x + 1 + \frac{8}{x-1}}$$

6) The area of a rectangular parking lot can be represented by the polynomial $A = 3x^3 - 10x^2 - 26x + 5$. If the width of the parking lot is $(x - 5)$, what is the length?

$$A = w \cdot l$$

$$3x^3 - 10x^2 - 26x + 5 = (x-5) \cdot l$$

$$l = \frac{3x^3 - 10x^2 - 26x + 5}{x-5}$$

5	3	−10	−26	5
		15	25	−5
	3	5	−1	0

$$\boxed{l = 3x^2 + 5x - 1}$$

4.4 Notes: Remainder Theorem

Lesson Plan:
*4.3 HW ?s
*4.3 Quiz
*4.4 *Notes* Exploration (student do on their own)
*4.4 *Notes* Remainder Theorem and intro to Fundamental Thm of Alg
*Reading on Rational Root Theorem
*finish 4.4 *Notes*
*HW is wk 4.4

Exploration: Consider the polynomial $g(x) = x^3 + 2x^2 - 9x - 18$.

a) Write this polynomial in factored form.

$x^3 + 2x^2 - 9x - 18$

$x^2(x+2) - 9(x+2)$

$(x^2-9)(x+2)$

$(x+3)(x-3)(x+2)$

b) What are values of x that would make this function = 0? Divide the roots into the function using synthetic division.

$$\begin{array}{r|rrrr} -3 & 1 & 2 & -9 & -18 \\ & & -3 & 3 & 18 \\ \hline & 1 & -1 & -6 & \boxed{0} \end{array} \qquad \begin{array}{r|rrrr} 3 & 1 & 2 & -9 & -18 \\ & & 3 & 15 & 18 \\ \hline & 1 & 5 & 6 & \boxed{0} \end{array} \qquad \begin{array}{r|rrrr} -2 & 1 & 2 & -9 & -18 \\ & & -2 & 0 & 18 \\ \hline & 1 & 0 & -9 & \boxed{0} \end{array}$$

c) Find g(-2). $g(-2) = (-2)^3 + 2(-2)^2 - 9(-2) - 18$

$= -8 + 8 + 18 - 18 = 0$

$(-2, 0)$ means there's an x-intercept @ -2.

Remainder Theorem: A polynomial $f(x)$ has a factor $x - k$ if and only if $f(k)=0$ (REMAINDER = 0)

Examples:

1) Factor $f(x) = 3x^3 - 4x^2 - 28x - 16$ completely, given that $x + 2$ is a factor of $f(x)$.

-2	3	-4	-28	-16
		-6	20	16
	3	-10	-8	$\boxed{0}$

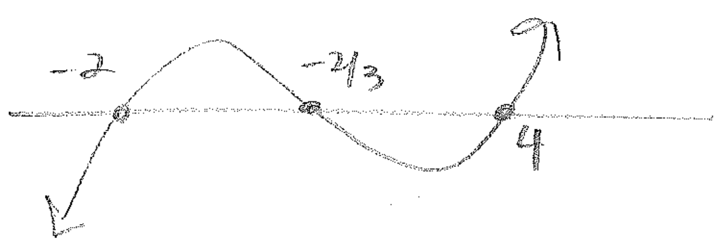
**Because you know x+2 is a factor, you know that f(-2) = 0. Use synthetic division to find the other factors.*

$3x^3 - 4x^2 - 28x - 16$

$(x+2)(3x^2 - 10x - 8)$

$(x+2)(3x+2)(x-4)$

$x = -2, -2/3, 4$



2) Find all the zeros of f given that $f(2) = 0$ and $f(x) = x^3 - x^2 - 22x + 40$

$$\begin{array}{r|rrrr} 2 & 1 & -1 & -22 & 40 \\ & & 2 & 2 & -40 \\ \hline & 1 & 1 & -20 & \boxed{0} \end{array}$$

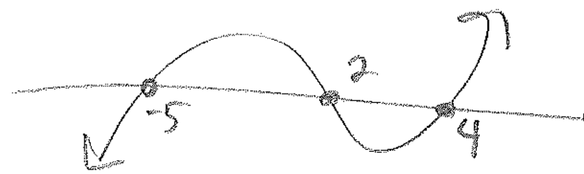
There is an x-int @ (2, 0) so (x − 2) must be a factor

$0 = x^3 - x^2 - 22x + 40$

$0 = (x-2)(x^2 + x - 20)$

$0 = (x-2)(x+5)(x-4)$

$\boxed{x = 2, -5, 4}$



The Fundamental Theorem of Algebra

The Fundamental Theorem of Algebra: Any polynomial of degree **n** has **n** roots, both real and complex.

Example: How many x-intercepts does the following function have?

$F(x) = 7x^5 - 4x^2 + 1$

5

Complete the reading on the Rational Root Theorem.

Find the zeros of a polynomial function:

possible factors: $\pm 1, \pm 3, \pm 9$

1) $f(x) = x^3 + 7x^2 + 15x + 9$ (graph to verify)

$$\begin{array}{r|rrrr} 1 & 1 & 7 & 15 & 9 \\ & & 1 & 8 & \\ \hline & 1 & 8 & & \end{array}$$

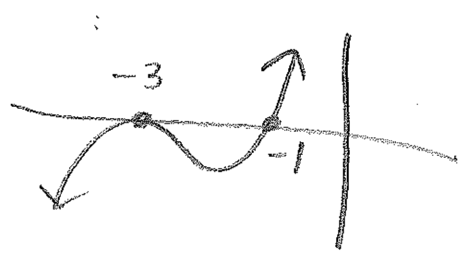
$$\begin{array}{r|rrrr} -1 & 1 & 7 & 15 & 9 \\ & & -1 & -6 & -9 \\ \hline & 1 & 6 & 9 & \boxed{0} \end{array}$$

$x^3 + 7x^2 + 15x + 9$

$(x+1)(x^2 + 6x + 9)$

$(x+1)(x+3)^2$

$x = -1, -3$



2) $f(x) = x^3 + x^2 - x + 2$

(find the complex zeros by using the quadratic formula. Graph to verify)

$$\begin{array}{r|rrrr} 1 & 1 & 1 & -1 & 2 \\ & & 1 & 2 & \\ \hline & 1 & 2 & 1 & \end{array}$$

$$\begin{array}{r|rrrr} -2 & 1 & 1 & -1 & 2 \\ & & -2 & 2 & -2 \\ \hline & 1 & -1 & 1 & \boxed{0} \end{array}$$

$$\begin{array}{r|rrrr} -1 & 1 & 1 & -1 & 2 \\ & & -1 & 0 & 1 \\ \hline & 1 & 0 & -1 & \end{array}$$

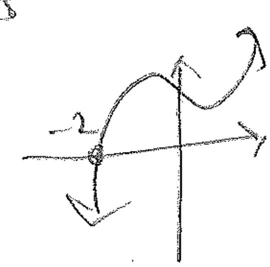
$$\begin{array}{r|rrrr} 2 & 1 & 1 & -1 & 2 \\ & & 2 & 6 & 10 \\ \hline & 1 & 3 & 5 & \end{array}$$

$x^3 + x^2 - x + 2$

$(x+2)(x^2 - x + 1)$

$\frac{1 \pm \sqrt{1 - 4(1)(1)}}{2(1)} = \frac{1 \pm i\sqrt{3}}{2}$

~~roots~~



roots @ $x = -2, \frac{1 + i\sqrt{3}}{2}, \frac{1 - i\sqrt{3}}{2}$

$\pm1, \pm3, \pm\frac{1}{2}, \pm\frac{3}{2}$

3) $f(x) = 2x^4 + 5x^3 - 5x^2 - 5x + 3$ $= (x-1)(2x^3+7x^2+2x-3)$

$\pm1, \pm3, \pm 3/2, \pm 1/2$

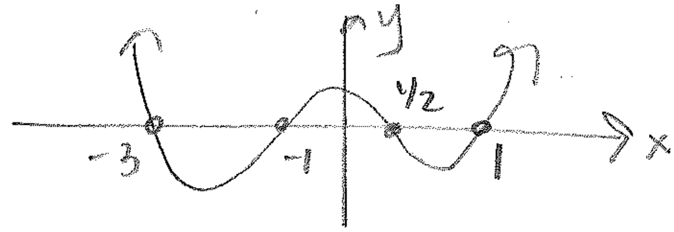
$$\begin{array}{r|rrrrr} 1 & 2 & 5 & -5 & -5 & 3 \\ & & 2 & 7 & 2 & -3 \\ \hline & 2 & 7 & 2 & -3 & \boxed{0} \end{array}$$

$(x-1)(x+3)(2x^2+x-1)$

$(x-1)(x+3)(2x-1)(x+1)$

$$\begin{array}{r|rrrr} -3 & 2 & 7 & 2 & -3 \\ & & -6 & -3 & 3 \\ \hline & 2 & 1 & -1 & \boxed{0} \end{array}$$

$x = 1, -3, 1/2, -1$



4) $f(x) = 2x^4 - 3x^3 + 7x^2 + 12x$ $= x(2x^3 - 3x^2 + 7x + 12)$

possible factors: $\cancel{\pm1}, \cancel{\pm3}, \cancel{\pm4}, \pm6, \cancel{\pm2}, \pm12,$
$\pm 1/2, \pm 3/2$

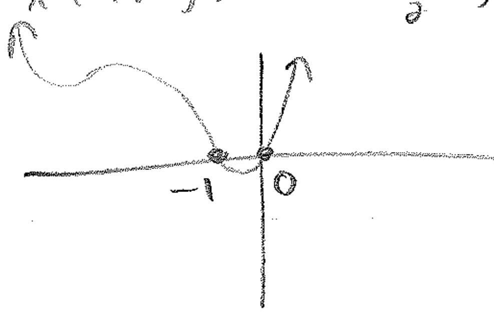
$$\begin{array}{r|rrrr} -1 & 2 & -3 & 7 & 12 \\ & & -2 & 5 & -12 \\ \hline & 2 & -5 & 12 & \boxed{0} \end{array}$$

$x(x+1)(2x^2-5x+12)$

$x(x+1)\cancel{(2x+3)(x-4)}$

$\frac{5 \pm \sqrt{25-4(2)(12)}}{2(1)} = \frac{5 \pm i\sqrt{71}}{2}$

$x(x+1)\left(x - \frac{5+i\sqrt{71}}{2}\right)\left(x - \frac{5-i\sqrt{71}}{2}\right)$



4.5 Notes: Exploring Key Features of Polynomials

Lesson Plan
*4.4 HW ?'s
*4.4 Quiz
*Foldable pages 1 - 7
*HW is wk4.5

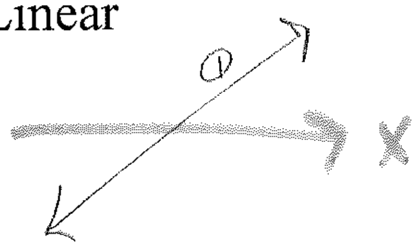
4.6 Notes: Exploring Key Features of Polynomials Part 2

Lesson Plan
*4.5 HW ?'s
*4.5 Quiz
*Foldable pages 8-11 (show them how to use graphing calculator to find key features)
*HW is wk4.6

(foldable following…)

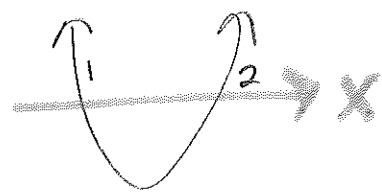
Polynomial
Function Families

Linear



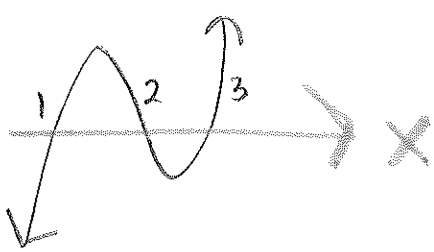
- no curves
one root

Quadratic



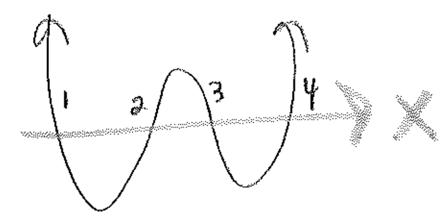
two roots

Cubic



three roots

Quartic



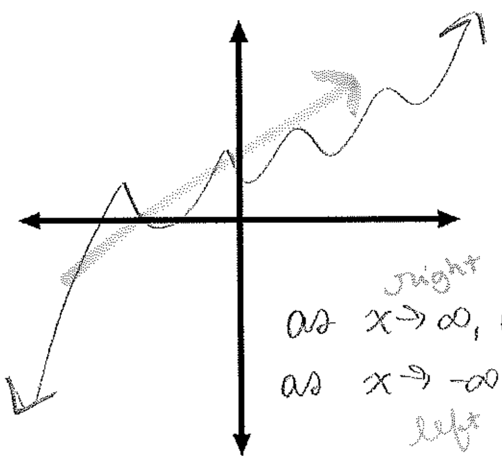
four roots

End Behavior

If we know the degree and leading coefficient of a polynomial, then we can get a general idea of what the graph would look like. End behavior describes what happens to the range as x goes to infinity and negative infinity.

Degree: Odd

Leading Coefficient: +



right, up

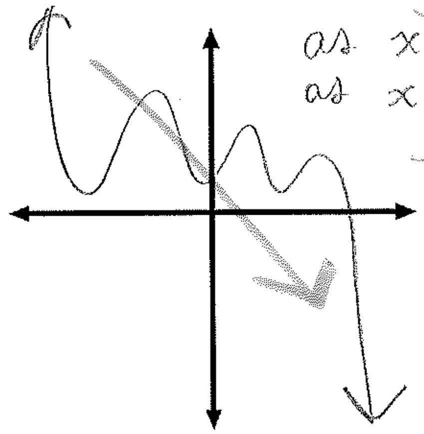
as $x \to \infty, y \to \infty$

as $x \to -\infty, y \to -\infty$

left, down

Degree: Odd

Leading Coefficient: −



right, down

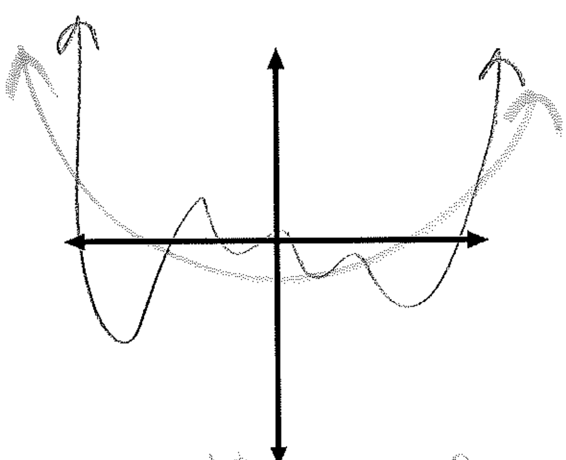
as $x \to \infty, y \to -\infty$

as $x \to -\infty, y \to \infty$

left, up

Degree: Even

Leading Coefficient: +



right, up

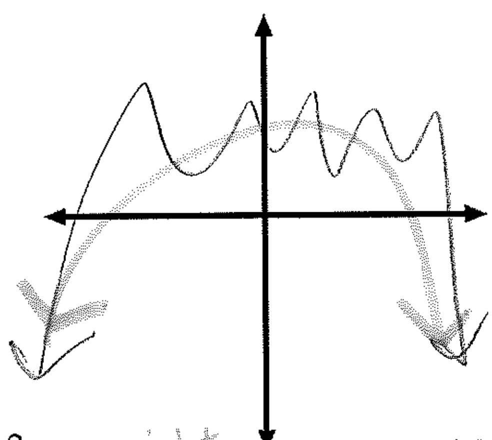
as $x \to \infty, y \to \infty$

as $x \to -\infty, y \to \infty$

left, up

Degree: Even

Leading Coefficient: −



right, down

as $x \to \infty, y \to -\infty$

as $x \to -\infty, y \to -\infty$

left, down

Examples:

Describe the end behavior of the graph of the polynomial functions:

1) $f(x) = -2x^2 + 5x^4 - 3x + 7$

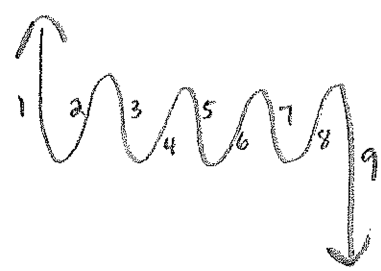
down

$\boxed{\text{as } x \to \pm\infty,\ y \to -\infty}$

means both left and right

2) $f(x) = (5x - 3 - 8x^5 + 12x^2)(3x^4 - 1)$

$(-8x^5 + \text{blah blah})(3x^4 - 1)$



$-24x^9 + \text{blah blah blah} \ldots$

9th degree, negative

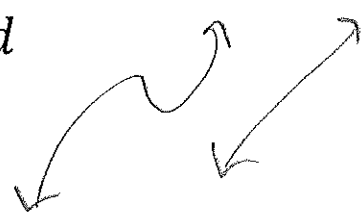
$\boxed{\begin{array}{l}\text{as } x \to \infty,\ y \to -\infty \\ \text{as } x \to -\infty,\ y \to \infty\end{array}}$

3) Create a function that has the following end behavior:

$as\ x \to -\infty, f(x) \to -\infty\ and$ (left) (down)

$as\ x \to \infty, f(x) \to \infty$

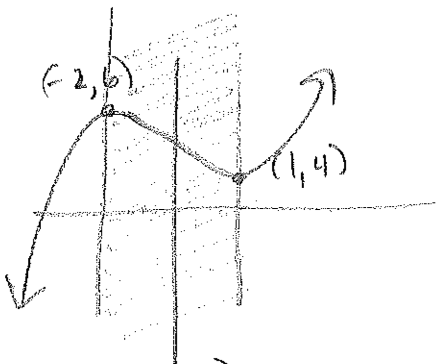
right up



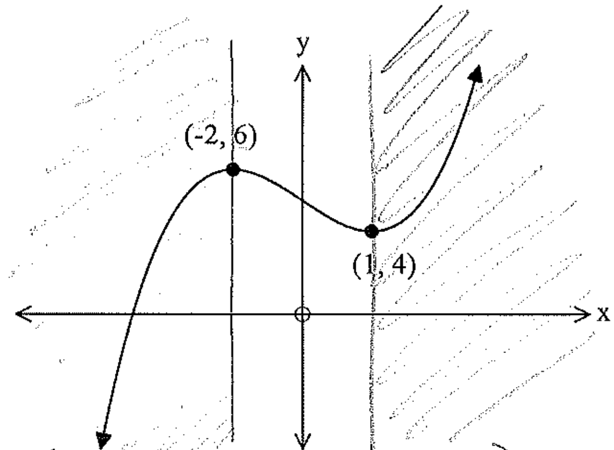
$\boxed{y = x}$

Increasing and Decreasing

Is the function graphed below increasing, decreasing or both? Explain.



(-2, 1)
decreasing



$(-\infty, -2) \cup (1, \infty)$ increasing

We describe the intervals where a function is either increasing or decreasing using the __x__-values in interval notation.

Highlight where the function is increasing. Describe the sections in interval notation.

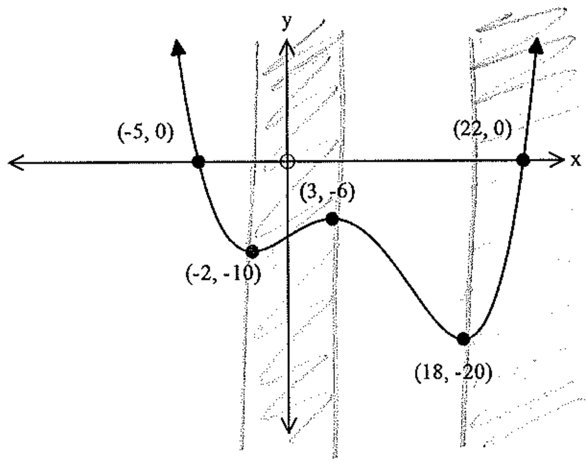
$(-\infty, -2) \cup (1, \infty)$

Highlight where the function is decreasing. Describe the section in interval notation.

$(-2, 1)$

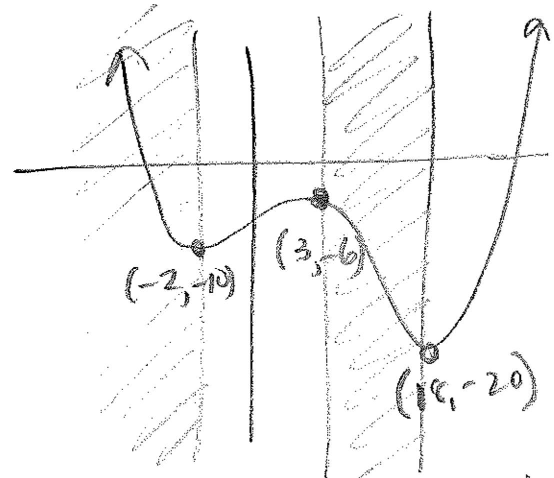
Example) Describe the end behavior of the following function as well as the intervals where the function is increasing and decreasing.

increasing



$(-2, 3) \cup (18, \infty)$

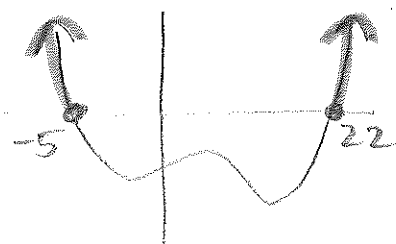
decreasing



$(-\infty, -2) \cup (3, 18)$

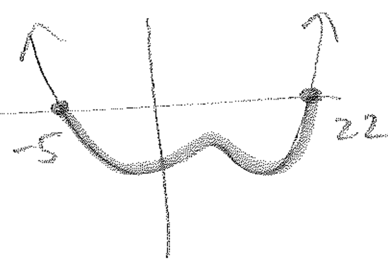
For which x-values is the function positive?

$(-\infty, -5) \cup (22, \infty)$



For which x-values if the function negative?

$(-5, 22)$



Key Features

X-intercepts: (-5, 0), (22, 0) or can say x = -5, 22

y-intercept: (0, -3) or can say y = -3

Local (relative) Maximum: -6 (or can say (3, -6))

Absolute Maximum: none

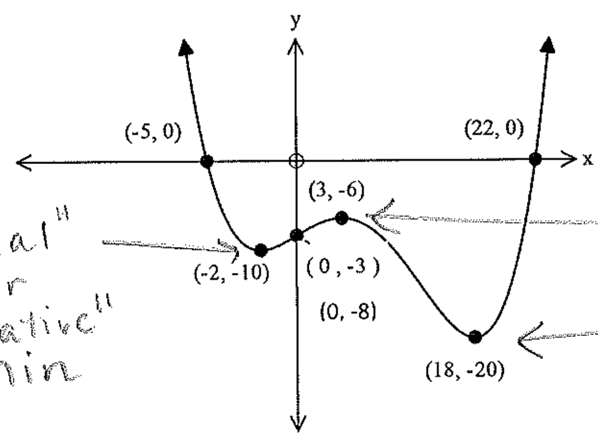
Local (relative) Minimum: -10 (or can say (-2, -10))

Absolute Mimimum: -20 (or can say (18, -20))

Example) Identify the key features of the function graphed below.

no "max"

"local" or "relative" min



"local" or "relative" max

THE ABSOLUTE min

Example) Identify all the key features of the function graphed below.

Odd or Even Degree?

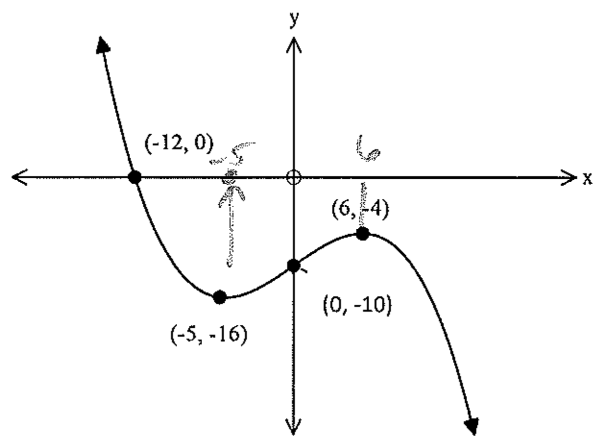
Positive or Negative Leading Coefficient?

x-intercepts: -12 (plus two complex :))

y-intercept: $(0, -10)$

Domain: $(-\infty, \infty)$ ⟷

Range: $(-\infty, \infty)$ ↕



Local Max: $(6, -4)$

Max: none

Local Min: $(-5, -16)$

Min: none

End Behavior:

as $x \to \infty$, $y \to +\infty$

as $x \to -\infty$, $y \to -\infty$

Increasing: $(-5, 6)$

Decreasing: $(-\infty, -5) \cup (6, \infty)$

Positive: $(-\infty, -12)$

Negative: $(-12, \infty)$

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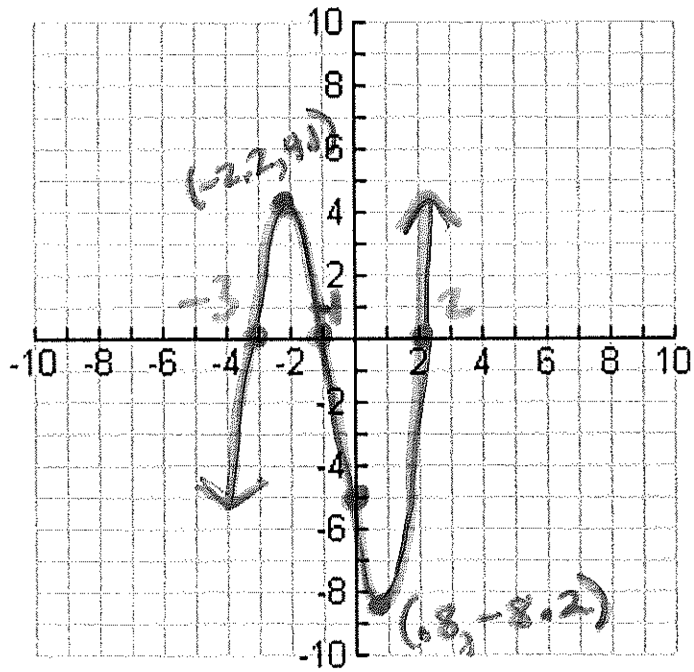
Cubic Functions: Positive Leading Coefficient

Example: $f(x) = x^3 + 2x^2 - 5x - 6$

Odd or Even Degree?



x-intercepts: $(-3,0), (-1,0), (2,0)$

y-intercept: $(0,-6)$

Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

Local Max: $(-2.2, 4.1)$

Max: none

Local Min: $(.8, -8.2)$

Min: none

Increasing: $(-\infty, -2.2) \cup (.8, \infty)$

Decreasing: $(-2.2, .8)$

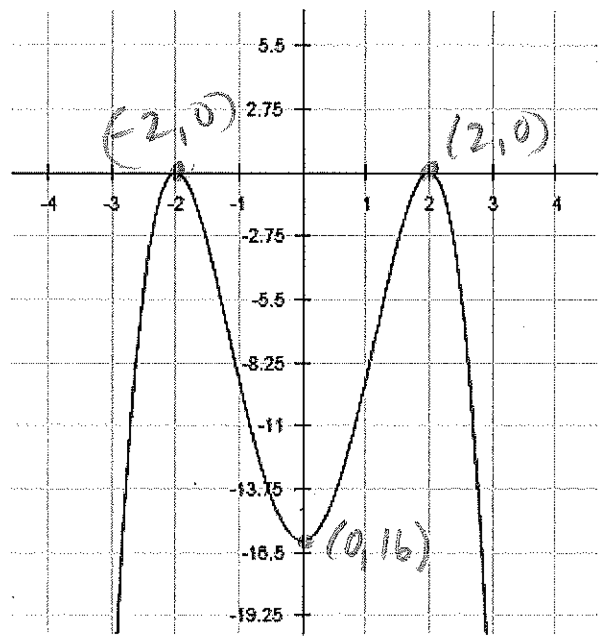
End Behavior:

as $x \to \infty$, $y \to \infty$

as $x \to -\infty$, $y \to -\infty$

Quartic Functions: Negative Leading Coefficient

Example: $f(x) = -x^4 + 8x^2 - 16$



Odd or Even Degree?

x-intercepts: $(-2,0), (2,0)$

y-intercept: $(0, 16)$

Domain: $(-\infty, \infty)$

Range: $(-\infty, 0]$

Local Max: $\emptyset$

Max: 0

Local Min: $(0, 16)$

Min: none

Increasing: $(-\infty, -2) \cup (0, 2)$

Decreasing: $(-2, 0) \cup (2, \infty)$

End Behavior:

as $x \to \pm\infty$, $y \to -\infty$

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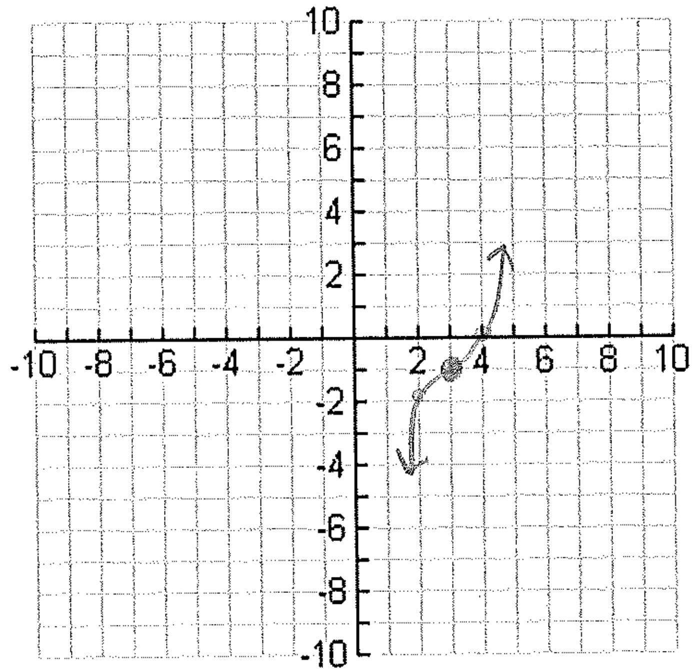
Graphing cubic functions in *h*, *k* form:

$$y = a(\mathrm{x} - h)^2 + k$$

Example #1:

$y = (x - 3)^3 - 1$

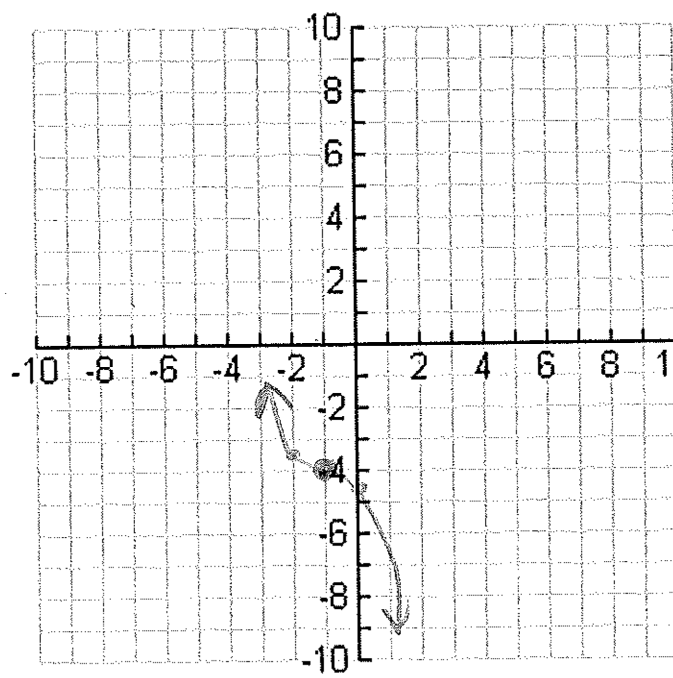
$\rightarrow 3 \quad \downarrow 1$



Example #2

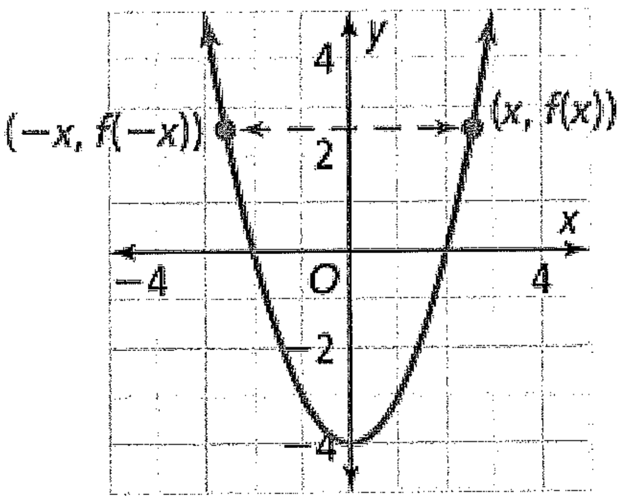
$y = -\frac{1}{2}(x + 1)^3 - 4$

$\leftarrow 1 \quad \downarrow 4$



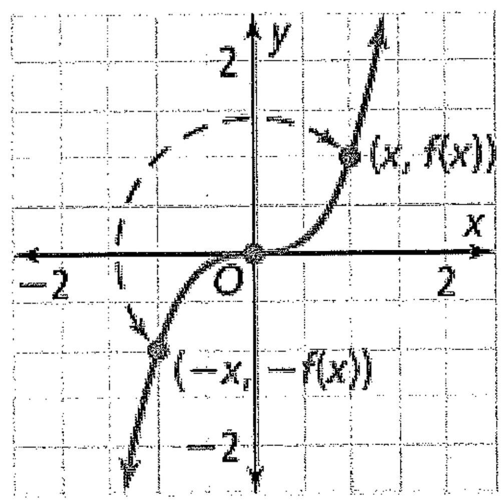
Even Functions & Odd Functions

Even Function



For all x in the domain,
$f(x) = f(-x)$

Odd Function



For all x in the domain,
$f(-x) = -f(x)$

Example: Is $f(x)$ odd, even, or neither?

(a) $f(x) = 4x^4 + 5$

$f(-x) = 4(-x)^4 + 5$

$\downarrow$

$4x+5$

$\boxed{\text{even}}$

(b) $f(x) = 2x^3 + 3x$

$f(-x) = 2(-x)^3 + 3(-x)$

$-2x^3 - 3x \neq f(x)$

so not even.

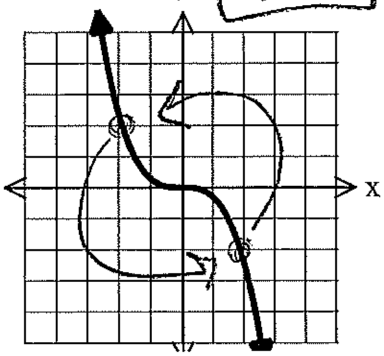
$-f(x) = -(2x^3 + 3x) = -2x^3 - 3x$

$f(-x) = -2x^3 - 3x$

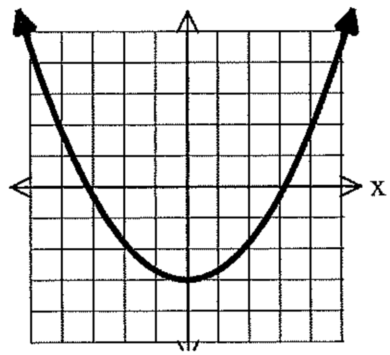
$\boxed{\text{odd, since}}$
$-f(x) = f(-x)$

Example: Is $f(x)$ odd, even, or neither?

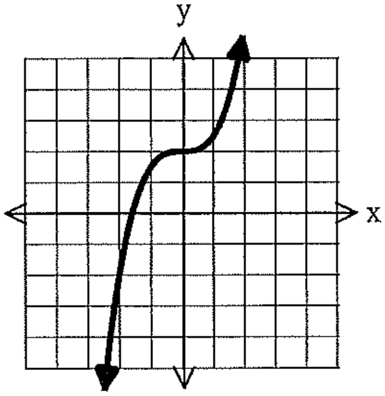
(a) ODD



(b) even

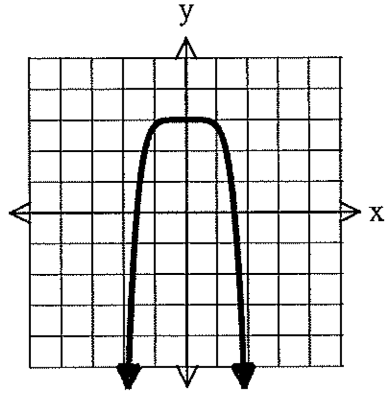


(c)



neither

(d)



even

4.7 Notes: Writing Polynomial Functions/Models/Systems

Lesson Plan
*4.6 HW ?
*4.6 Quiz
*4.7 Notes
*HW is wk4.7

Examples: Write a polynomial function f of least degree that has rational coefficients, a leading coefficient of 1, and the given zeros:

1) 3 and $2+\sqrt{5}$

$y = (x-3)(x-(2+\sqrt{5}))(x-(2-\sqrt{5}))$

$(x-3)(x-2-\sqrt{5})(x-2+\sqrt{5})$

$(x-3)\underbrace{((x-2)-\sqrt{5})((x-2)+\sqrt{5})}$

$(x-3)((x-2)^2 \overset{\text{foil}}{-} 5)$

$(x-3)(x^2-4x+4-5)$

~~2) 2, 2i, 4 − √6~~

$(x-3)(x^2-4x-1)$

x^3-4x^2-x
$-3x^2+12x+3$

$\boxed{x^3-7x^2+11x+3}$

1

(write a function of least degree w/ rational coefficients, $a=1$, given zeroes:)

2) $2, 2i, 4-\sqrt{6}$

$$y = (x-2)\underbrace{(x+2i)(x-2i)}(x-(4-\sqrt{6}))(x-(4+\sqrt{6}))$$

$$(x-2)(x^2+4) \cdot (x-4+\sqrt{6})(x-4-\sqrt{6})$$

$$\underbrace{(x-2)(x^2+4)}((x-4)+\sqrt{6})((x-4)-\sqrt{6})$$

$$(x^3-2x^2+4x-8)((x-4)^2-(\sqrt{6})^2)$$

$$(x^3-2x^2+4x-8)(x^2-8x+16-6)$$

$$(x^3-2x^2+4x-8)(x^2-8x+10)$$

$$x^5-8x^4+10x^3$$
$$-2x^4+16x^3-20x^2$$
$$4x^3-32x^2+40x$$
$$-8x^2+16x-80$$

$$\boxed{y = x^5-10x^4+30x^3-60x^2+56x-80}$$

3) Write the equation for a cubic function whose zeros are -4, 1, and 3, and whose y-intercept is at -6.

$$y = a(x+4)(x-1)(x-3)$$

$$f(0) = a(0+4)(0-1)(0-3) = 6$$

$$a(4)(-1)(-3) = 6$$

$$-12a = 6$$

$$a = -\frac{1}{2}$$

$$y = -\frac{1}{2}(x+4)(x-1)(x-3)$$

$$-\frac{1}{2}(x+4)(x^2-4x+3)$$

$$-\frac{1}{2}(x^3 \cancel{-4x^2} + 3x \cancel{+4x^2} - 16x + 12)$$

$$-\frac{1}{2}(x^3 - 13x + 12)$$

$$\boxed{y = -\frac{1}{2}x^3 + \frac{13}{2}x - 6}$$

4) Write a cubic function whose graph passes through the points (-2, 0), (-1, 0), (0, -8), (2, 0). y-int

$$f(x) = a(x+2)(x+1)(x-2)$$

$$f(0) = a(0+2)(0+1)(0-2) = -8$$

$$-4a = -8$$

$$a = 2$$

$$f(x) = 2(x+2)(x+1)(x-2)$$

$$(2x+4)(x^2-x-2)$$

$$2x^3 - 2x^2 - 4x$$
$$+4x^2 - 4x - 8$$

$$\boxed{y = 2x^3 + 2x^2 - 8x - 8}$$

3

5) According to data from the U.S. Census Bureau for the period 2000-2007, the number of male students enrolled in high school in the United States can be approximated by the function $M(x) = -0.004x^3 + 0.037x^2 + .049x + 8.11$ where x is the number of years since 2000 and $M(x)$ is the number of male students in the millions. The number of female students enrolled in high school in the United States can be approximated by the function $F(x) = -0.006x^3 + 0.029x^2 + 0.165x + 7.67$ where x is the number of years since 2000 and $F(x)$ is the number of female students in millions. Estimate the total number of students enrolled in high school in the United States in 2007. add male, femal

$M(x) = -.004x^3 + .037x^2 + .049x + 8.11$

$F(x) = -.006x^3 + .029x^2 + .165x + 7.67$

$T(x) = -.01x^3 \; .066x^2 + .214x + 15.78$

$T(7) = -.01(7)^3 + .066(7)^2 + .214(7) + 15.78 = 17.082$ million

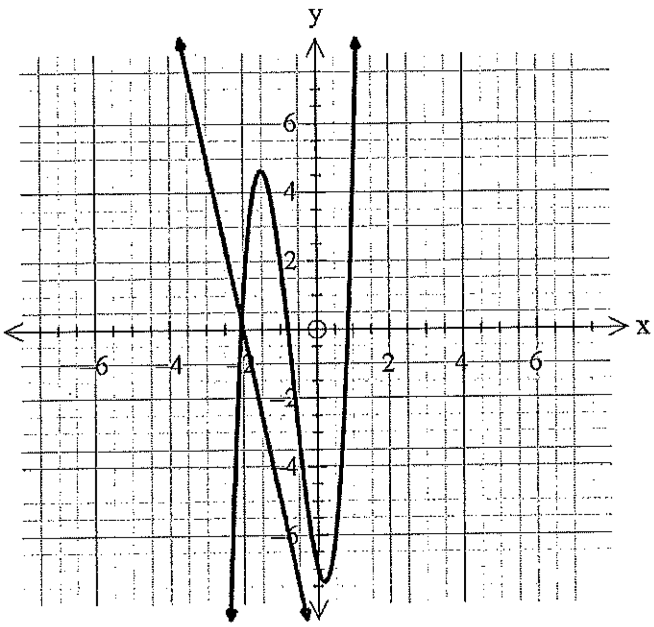
Reflect: Explain how you can use the given information to estimate how many more male high school students than female high school students there were in the United States in 2007.

You can find

$M(7) - F(7)$

Systems

What are the solution(s) to the following system?

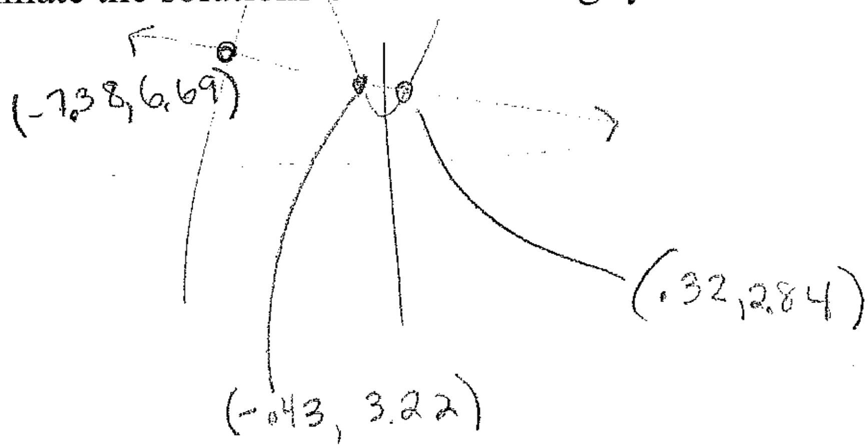


(-2,0)

Why are they the solutions?

Use a Calculator to approximate the solutions of the following system:

$$\begin{cases} y = 2x^3 + 15x^2 + x + 1 \\ y = -\frac{1}{2}x + 3 \end{cases}$$



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