Rules of Exponents: 7.1

1) $a^m \cdot a^n = a^{m+n}$
2) $\frac{a^m}{a^n} = a^{m-n}$
3) $(a^m)^n = a^{mn}$
4) $(ab)^n = a^n b^n$
5) $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
6) $\frac{1}{a} = a^{-1}$
7) $\frac{1}{a^n} = a^{-n}$
8) $a^0 = 1$
9) $\sqrt{a} = a^{\frac{1}{2}}$
10) $\sqrt[n]{a} = a^{\frac{1}{n}}$
11) $\sqrt{ab} = \sqrt{a}\sqrt{b}$
12) $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$
13) $\left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m} = a^{\frac{m}{n}}$

Source: http://www.algebralab.org/lessons/lesson.aspx?file=Algebra_ExponentsRules.xml

$a^m \cdot a^n = a^{m+n}$

If the bases of the exponential expressions that are multiplied are the same, then you can combine them into one expression by adding the exponents.

$$2^3 \bullet 2^4 = (2 \bullet 2 \bullet 2) \bullet (2 \bullet 2 \bullet 2 \bullet 2) = 2^7$$

$\frac{a^m}{a^n} = a^{m-n}$

If the bases of the exponential expressions that are divided are the same, then you can combine them into one expression by subtracting the exponents.

$$\frac{x^7}{x^3} = \frac{x \bullet x \bullet x \bullet x \bullet \cancel{x} \bullet \cancel{x} \bullet \cancel{x}}{\cancel{x} \bullet \cancel{x} \bullet \cancel{x}} = x \bullet x \bullet x \bullet x = x^4$$

$(a^m)^n = a^{mn}$

When you have an exponential expression raised to a power, you have to multiply the two exponents.

$$(3^2)^3 = 3^2 \bullet 3^2 \bullet 3^2 = 3^{2+2+2} = 3^6$$

$a^0 = 1$

Any number or variable raised to the zero power is always equal to 1.

$a^{-m} = \dfrac{1}{a^m}$

This is the rule used earlier dealing with negative exponents. It is important to note that if a negative exponents already appears in the denominator of a fraction, then it will move to the numerator as a positive exponent. In short, a negative exponent changes the location (numerator or denominator) of an expression and changes the sign of the exponent. This is seen in Example 2 below.

Examples:

1. $(2a^{12}b^3)(3a^2b^4) = \ (2a^{12}b^3)(3a^2b^4) = 6a^{12+2}b^{3+4} = 6a^{14}b^7$

Everything in this problem is multiplied. The base of *a* is common so we add the exponents of 12 and 2 to get 14. The base of *b* is common so we add the exponents of 3 and 4 to get 7. The coefficients of 2 and 3 do not have any exponents to worry about and we just multiply them as they are.

2. $\left(\dfrac{3x^4y^{-3}z^2}{4x^{-3}y^{10}z}\right)^2 =$

Using order of operations tells us that we should do what is inside the parentheses first and then deal with the exponent. To simplify within the parentheses involves working with several rules including the rule for negative exponents.

$$\left(\frac{3x^4y^{-3}z^2}{4x^{-3}y^{10}z}\right)^2 = \left(\frac{3x^4z^2\ \ x^3}{4y^{10}z\ \ y^3}\right)^2$$

This step shows that the negative exponents were moved and exponents became positive.

$$\left(\frac{3x^4z^2\ \ x^3}{4y^{10}z\ \ y^3}\right)^2 = \left(\frac{3x^{4+3}z^{2-1}}{4y^{10+3}}\right)^2$$

This step shows combining exponents for terms that have the same base. Two different rules were used in this step: both the multiplication rule and the division rule.

$$\left(\frac{3x^{4+3}z^{2-1}}{4y^{10+3}}\right)^2 = \left(\frac{3x^7z}{4y^{13}}\right)^2$$

This step is the final simplification of what is inside the parentheses. Now we have to raise each term in the parentheses to the power of 2.

$$\left(\frac{3x^7z}{4y^{13}}\right)^2 = \frac{(3)^2(x^7)^2(z)^2}{(4)^2(y^{13})^2}$$

It is not absolutely necessary to use this many parentheses, but it is useful in keeping track of each term that needs to be raised to the power of 2.

$$\frac{(3)^2(x^7)^2(z)^2}{(4)^2(y^{13})^2} = \frac{9x^{14}z^2}{16y^{26}}$$

The final step is to simplify each term that has been raised to the 2nd power. It requires using the power rule for exponents.

Rational Exponents: 7.1 Notes
If x is positive, p and q are integers and q is positive,

$$x^{\frac{p}{q}} = \sqrt[q]{x^p} = \left(\sqrt[q]{x}\right)^p$$

Source: http://www.wtamu.edu/academic/anns/mps/math/mathlab/col_algebra/col_alg_tut5_ratexp.htm

Example 1: Evaluate $49^{\frac{1}{2}}$.

$49^{\frac{1}{2}} =$

***Rewrite exponent 1/2 as a square root**

$\left(\sqrt{49}\right)^1 = 7$

Example 2: Evaluate $(-125)^{\frac{2}{3}}$.

***Rewrite exponent 2/3 as a cube root being squared**

***Cube root of -125 = -5**

$(-125)^{\frac{2}{3}} =$

$\left(\sqrt[3]{-125}\right)^2 =$

$(-5)^2 =$

25

Example 3: Evaluate . $\left(\frac{36}{49}\right)^{-\frac{3}{2}}$

$\left(\frac{36}{49}\right)^{-\frac{3}{2}} =$ ***Rewrite as recip. of base raised to pos. exp.**
***DO NOT take the reciprocal of the exponent, only the base**

$\left(\frac{49}{36}\right)^{\frac{3}{2}} =$ ***Rewrite exponent 3/2 as a square root being cubed**

$\left(\sqrt{\frac{49}{36}}\right)^3 =$ ***Square root of 49/36 = 7/6**

$\left(\frac{7}{6}\right)^3 = \frac{343}{216}$

Operations With Rational Exponents: 7.2

Source: http://www.wtamu.edu/academic/anns/mps/math/mathlab/col_algebra/col_alg_tut5_ratexp.htm

Example 1: Simplify. $\left(3a^{\frac{3}{2}}\right)\left(-7a^{\frac{1}{5}}\right)$

$\left(3a^{\frac{3}{2}}\right)\left(-7a^{\frac{1}{5}}\right) =$ Multiply the coefficients.

$-21a^{\frac{3}{2}+\frac{1}{5}} =$ Multiply like bases, and so add the exponents.

$-21a^{\frac{15}{10}+\frac{2}{10}} =$

$-21a^{\frac{17}{10}}$

Example 2: Simplify: $\dfrac{25x^{\frac{1}{3}}}{5x^{\frac{1}{4}}}$

$\dfrac{25x^{\frac{1}{3}}}{5x^{\frac{1}{4}}} =$ Reduce the coefficients.

$5x^{\frac{1}{3}-\frac{1}{4}} =$ Divide like bases, and so subtract exponents.

$5x^{\frac{4}{12}-\frac{3}{12}} =$

$5x^{\frac{1}{12}}$

Example 3: Simplify $\sqrt[10]{x^2}$ by reducing the index of the radical. *x* represents positive real numbers.

$\sqrt[10]{x^2} =$ *Rewrite tenth root of *x* squared as *x* to the 2/10 power

$x^{\frac{2}{10}} =$

$x^{\frac{1}{5}} =$ *Simplify exponents by reducing.

$\sqrt[5]{x}$ *Rewrite exponent 1/5 as a fifth root