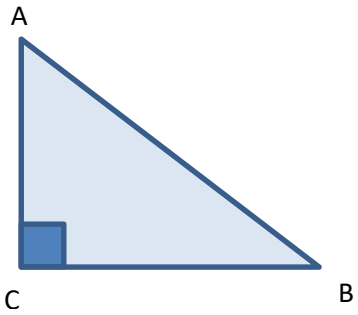
9.1 Notes: Right Triangle Trigonometry

Reminder of Trig Functions:

$$\text{Sin} = \frac{opp}{hyp}$$

$$\text{Cos} = \frac{adj}{hyp}$$

$$\text{Tan} = \frac{opp}{adj}$$

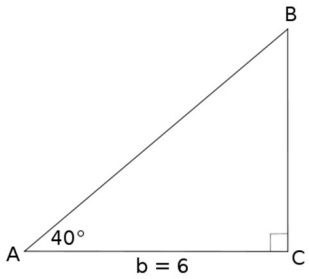


To find a segment of a right triangle, write an equation with the appropriate trig function, and then isolate the variable.

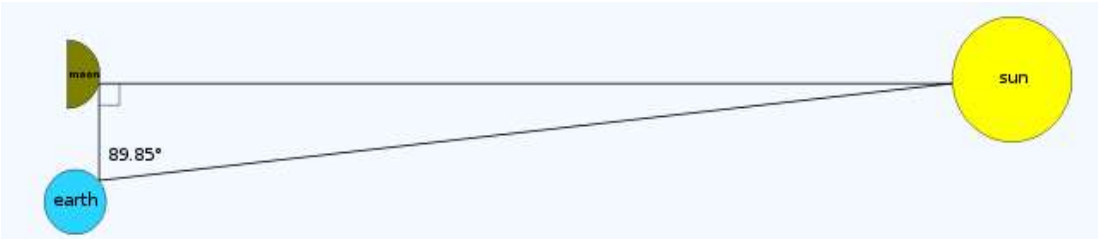
Trig Function Angle = Ratio

Examples: Find the requested values. Round to four decimal places.

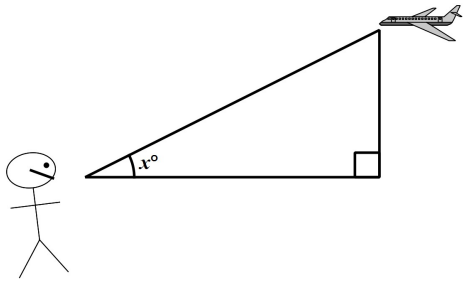
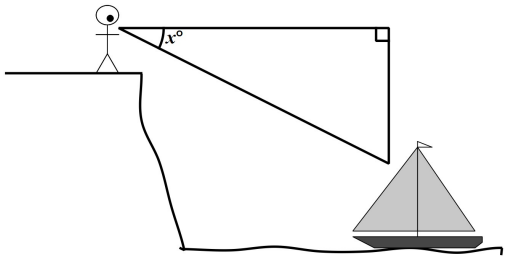
1) Find BC and AB.



2) The earth, moon, and sun create a right triangle during the first quarter moon. The distance from the earth to the moon is approximately 240,002.5 miles. What is the distance from the sun to the moon?



Angle of Elevation and Angle of Depression:

Angle of Elevation (looking up from the horizontal)	Angle of Depression (looking down from the horizontal)
	

3) A man is standing 95 meters away from the base of a lighthouse, with an angle of elevation of 26 degrees. Find distance from the man's feet to the top of the lighthouse.

To find an angle in a right triangle, use the appropriate inverse trig function.

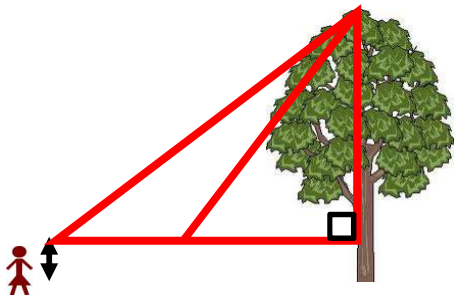
$$\sin^{-1} x \qquad \cos^{-1} x \qquad \tan^{-1} x$$

4) A 10-foot tall lamppost casts a shadow of 20 feet. Find the angle of elevation from the tip of the shadow to the top of the lamppost.

5) A man on a cliff is looking down at two ships on the ocean below. The angles of elevation from the two ships are 56 degrees and 63 degrees. If the man is 6 feet tall and the top of the cliff is 86 feet above the ocean, find the distance between the two ships.

6) Miguel is 5.5 feet tall and is standing on an observation platform at the Statue of Liberty, and he is 250 feet above the ground. Find his angle of depression to a ship that is 600 feet away from his position.

7) To estimate the height of a tree she wants removed, Christina sights the tree's top at a 70º angle of elevation. She then steps back 10 meters and sights the tops at a 26º angle. If Christina's line of sight is 1.7 meters above the ground, how tall is the tree, to the nearest meter?



9.2 Notes: Oblique Triangle Trigonometry

5) John wants to measure the height of a tree. He walks exactly 100 feet from the base of the tree and looks up. The angle from the ground to the top of the tree is 33º. This particular tree grows at an angle of 83º with respect to the ground rather than vertically (90º). How long is the tree?

6) Two airplanes leave an airport, and the angle between their flight paths is 40º. An hour later, one plane has traveled 300 miles while the other has traveled 200 miles. How far apart are the planes at this time?

9.3 Notes: Special Right Triangles and Triples

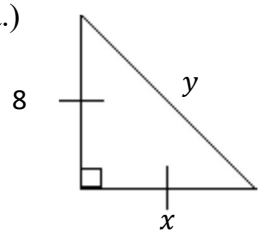
Triples: Right triangles with integer sides:

3, 4, 5 5, 12, 13 7, 24, 25 8, 15, 17

9, 40, 41 20, 21, 29 There are more…!

Example 1: Find the following measures.

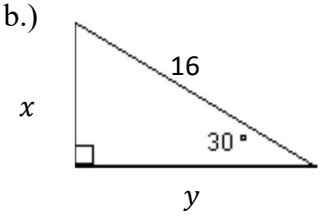
a.)



$x =$ _____

$y =$ _____

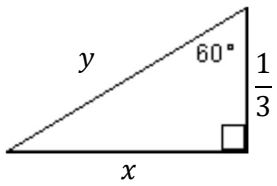
b.)



$x =$ _____

$y =$ _____

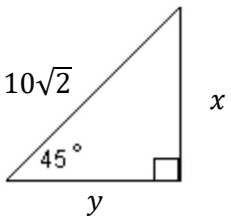
c.)



$x =$ _____

$y =$ _____

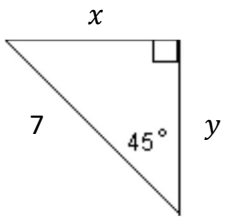
d.)



$x =$ _____

$y =$ _____

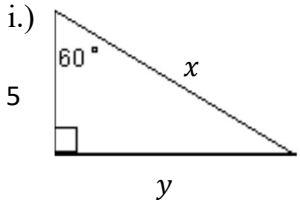
h.)



$x =$ _____

$y =$ _____

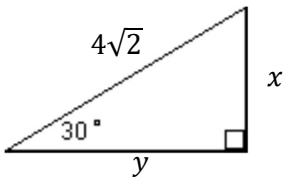
i.)



$x =$ _____

$y =$ _____

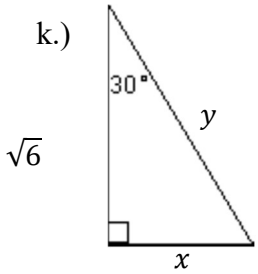
j.)



$x =$ _____

$y =$ _____

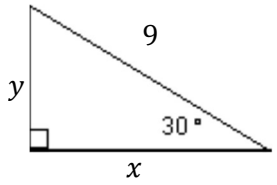
k.)



$x =$ _____

$y =$ _____

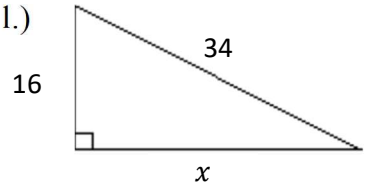
e.)



$x =$ _____

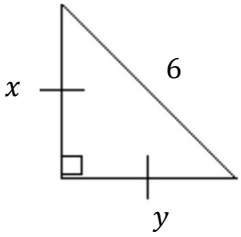
$y =$ _____

l.)



$x =$ _____

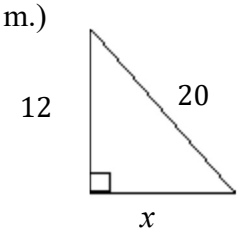
f.)



$x =$ _____

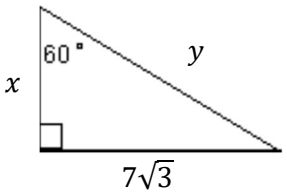
$y =$ _____

m.)



$x =$ _____

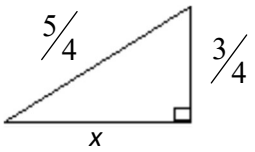
g.)



$x =$ _____

$y =$ _____

n.)



$x =$ _____

Examples: Find the requested ratios without using a calculator. Hint: draw a special right triangle.

1) sin 45°

2) tan 45°

3) cos 45°

4) cos 60°

5) tan 30°

6) cos 30°

7) sin 90°

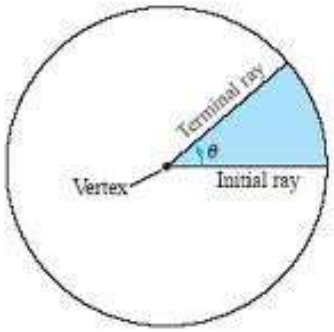
8) sin 60°

9) tan 60°

10) How do the measures in #4, 6, 8 compare to the lengths of the sides of a 30-60-90 triangle with hypotenuse = 1?

9.4 Notes: The Unit Circle

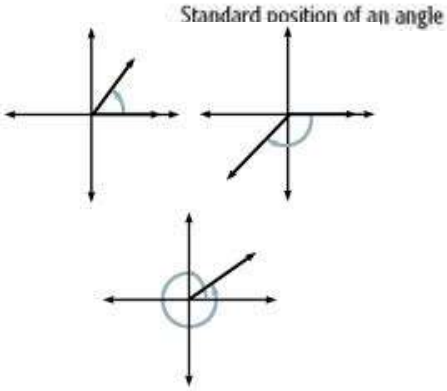
Angles on a coordinate system:



Typical (lowercase) Greek letters used for angle measures:

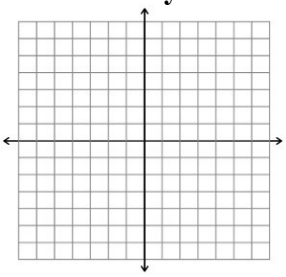
θ	α	β	γ
(theta	alpha	beta	gamma)

Samples of standard positions of angles:

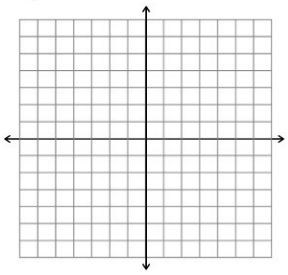


Examples: Draw the terminal rays for the following angles.

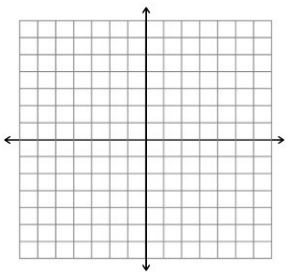
1) 45 degrees



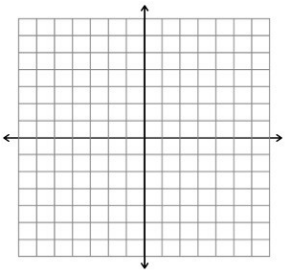
2) 120 degrees



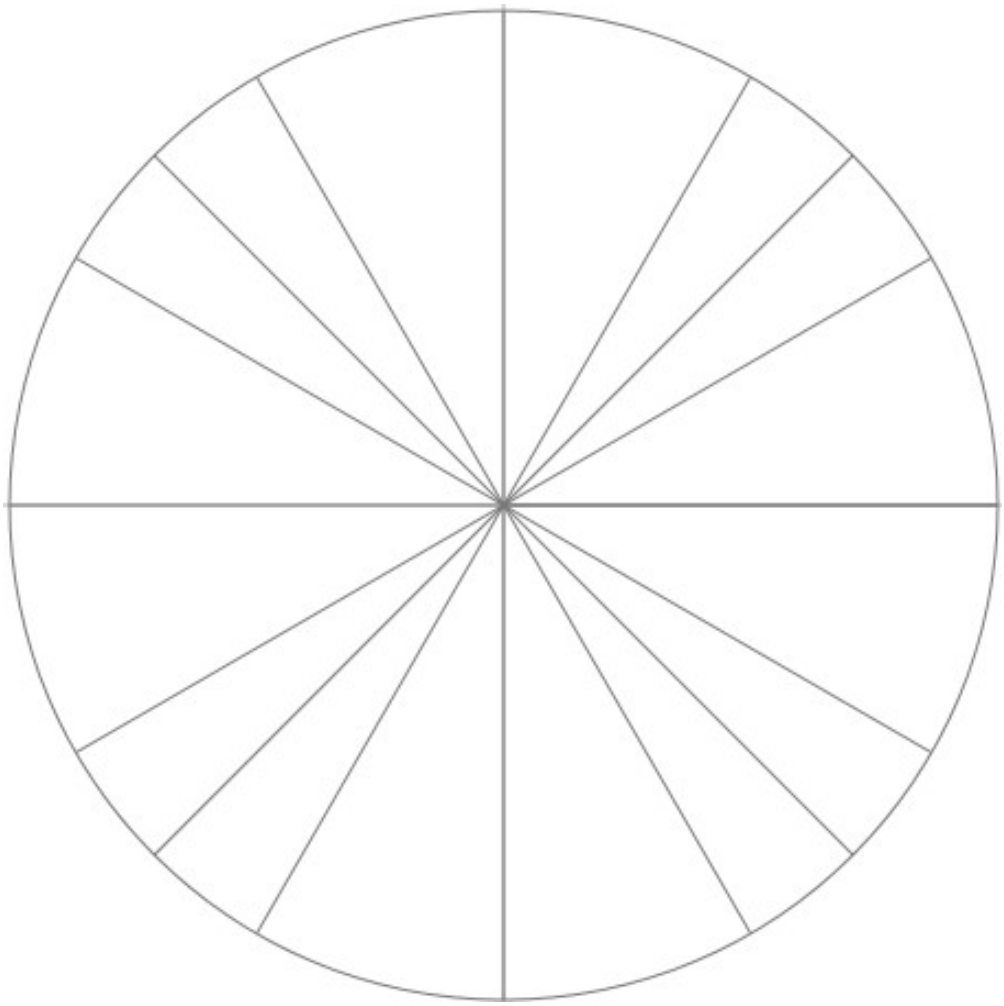
3) 270 degrees



4) -30 degrees



The Unit Circle: Draw a circle with a radius of one and a center of (0,0). Find the ordered pairs for the endpoint of the terminal ray at the following angle measures (in degrees): 30, 45, 60, 90, 120, 135, 150, 180, 210, 225, 240, 270, 300, 315, 330, and 360.



How do the coordinates that you found compare to the sin and cos of each angle?

What does the tangent of each angle tell you? What happens at 0^o and 180^o? 90^o and 270^o?

Good visual for unit circle: http://www.mathsisfun.com/geometry/unit-circle.html
The info below is also from the same website.

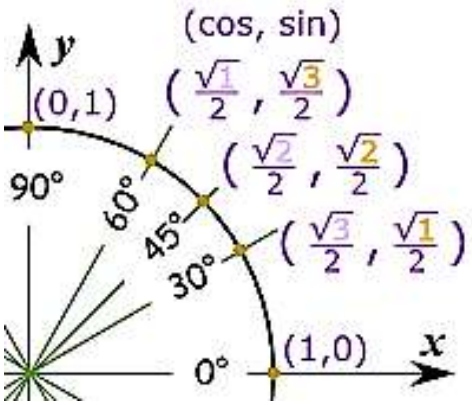
How To Remember?

To help you remember sin, think **"1,2,3"** :

- $\sin(30°) = \sqrt{1}/2 = 1/2$ (because $\sqrt{1} = 1$)
- $\sin(45°) = \sqrt{2}/2$
- $\sin(60°) = \sqrt{3}/2$

And cos goes **"3,2,1"**

- $\cos(30°) = \sqrt{3}/2$
- $\cos(45°) = \sqrt{2}/2$
- $\cos(60°) = \sqrt{1}/2 = 1/2$ (because $\sqrt{1} = 1$)



Quadrant	I	II	III	IV
	A	**S**mart	**T**rig	**C**lass
Trig Function with Positive Values	All (sin, cos, tan)	sin	tan	cos

What about tan?

$\tan = \frac{opposite}{adjacent} = \frac{sin}{cos}$, so you can calculate this value for each angle:

30° 45° 60°

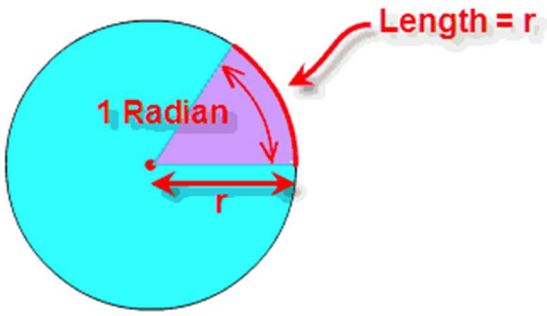
9.5: Radians and Degrees

The unit circle has a radius = 1 unit. Find the circumference of the circle.

What is the length of the semicircle with a radius of 1? Of an arc formed by a central angle of 90 degrees?

A circle can be divided up in degrees (360 units), or can divided up in **radians.**

What is a **radian?**



- One radian is the angle of an arc created by wrapping the radius of a circle around its circumference.
- One Radian is 180/π degrees, or about 57.296°
- So, a **Radian** "cuts out" a length of a circle's circumference equal to the radius.
- There are 2π radians in a unit circle.
- Because the radian is based on the pure idea of *"the radius being laid along the circumference*", it gives simple and natural results to many angle-related mathematics.
- No unit of measurement is typically written.

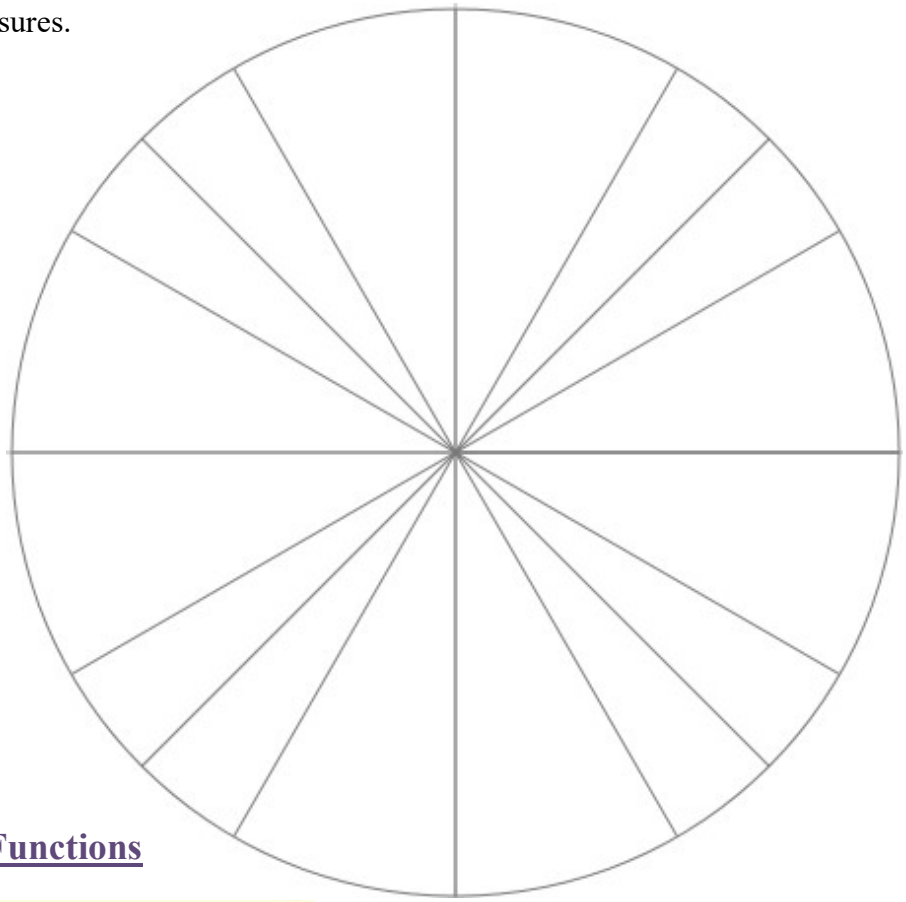
http://lsquaredmath.us/radians/index.php

Examples:

1) An angle has a measure of 60 degrees. Find the corresponding radian measurement for this angle.

2) An angle has a measure of $\frac{3\pi}{2}$. Find the corresponding degree measurement for this angle.

Label the unit circle for radian measures.



9.6 Notes: Reciprocal Trig Functions

Reciprocal Functions	
$\sin\theta = \frac{1}{\csc\theta}$	$\csc\theta = \frac{1}{\sin\theta}$
$\cos\theta = \frac{1}{\sec\theta}$	$\sec\theta = \frac{1}{\cos\theta}$
$\tan\theta = \frac{1}{\cot\theta}$	$\cot\theta = \frac{1}{\tan\theta}$

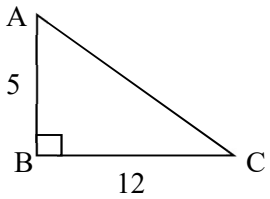
Also Important
$\tan\theta = \frac{\sin\theta}{\cos\theta}$
$\cot\theta = \frac{\cos\theta}{\sin\theta}$

Examples: Find the requested ratios.

1) sin A

2) cot C

3) csc C



4) tan A

5) sec A

6) cot A

For #6 – 7: A right triangle has legs of 7 and 24.

6) Find the sec of the larger acute angle.

7) Find the csc of the larger actue angle.

8) Find the cot x, given that $\sin x = \frac{3}{5}$.

9) Find the csc x, given that $\sec x = \frac{41}{9}$.

10) Find x, if the $\cot x = \sqrt{3}$. Note: x is in the first quadrant.

11) Find the sec x, csc x, and cot x, given that $x = -\frac{\pi}{4}$.

12) Find the cos x, given that $\cot x = \frac{2}{3}$.