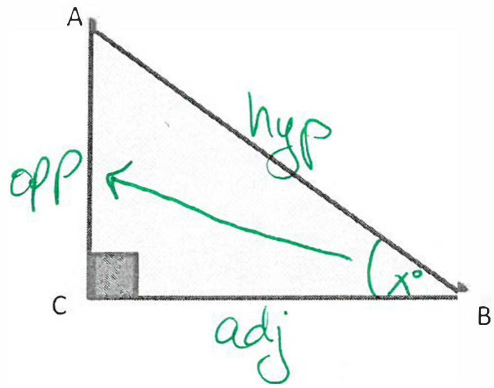
9.1 Notes: Right Triangle Trigonometry

Reminder of Trig Functions:

$\text{Sin}\angle = \dfrac{opp}{hyp}$

$\text{Cos}\angle = \dfrac{adj}{hyp}$

$\text{Tan}\angle = \dfrac{opp}{adj}$



To find a segment of a right triangle, write an equation with the appropriate trig function, and then isolate the variable.

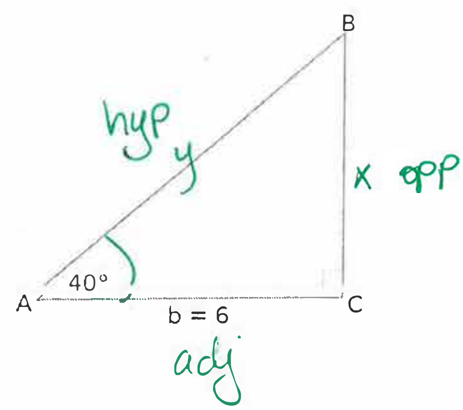
Trig Function Angle = Ratio

Examples: Find the requested values. Round to four decimal places.

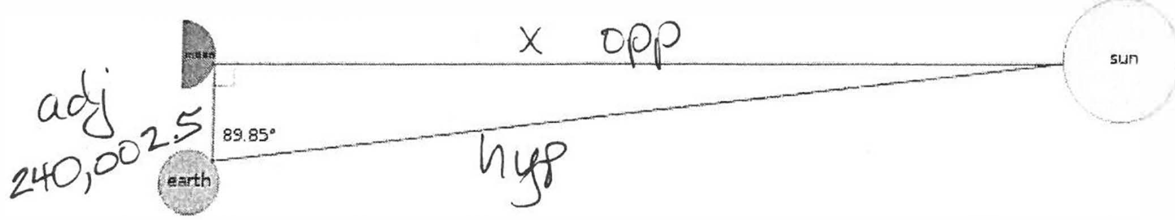
1) Find BC and AB.

$\tan 40 = \dfrac{x}{6}$

$\overline{BC} = 5.0346$



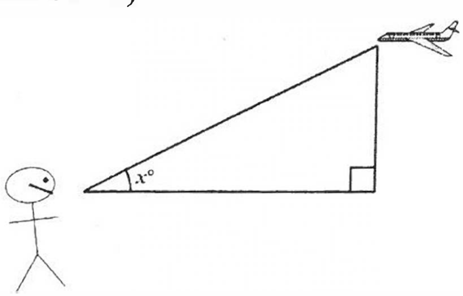
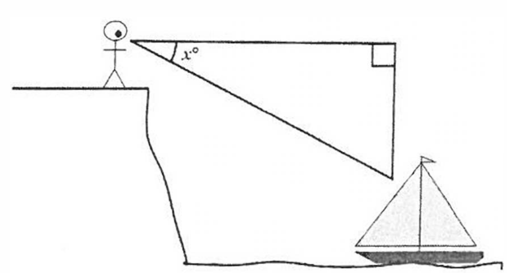
2) The earth, moon, and sun create a right triangle during the first quarter moon. The distance from the earth to the moon is approximately 240,002.5 miles. What is the distance from the sun to the moon?



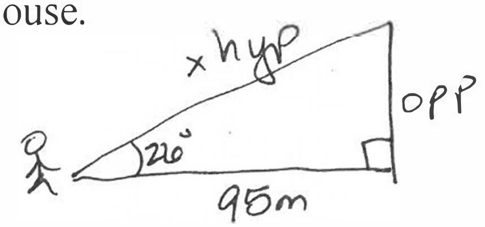
$$\tan 89.85 = \frac{x}{240{,}002.5}$$

$$\boxed{91{,}673{,}992.71 \text{ miles}}$$

Angle of Elevation and Angle of Depression: * made w/ horizontal line

Angle of Elevation (looking up from the horizontal)	Angle of Depression (looking down from the horizontal)
	

3) A man is standing 95 meters away from the base of a lighthouse, with an angle of elevation of 26 degrees. Find distance from the man's feet to the top of the lighthouse.



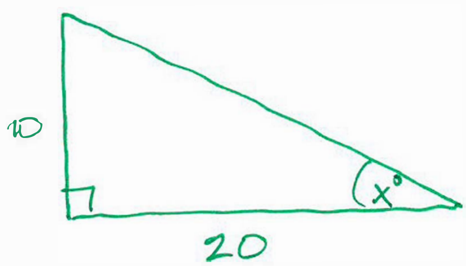
$$\cos 26 = \frac{95}{x}$$

$$x = 105.6972$$

To find an angle in a right triangle, use the appropriate inverse trig function.

$$\sin^{-1} x \qquad \cos^{-1} x \qquad \tan^{-1} x$$

4) A 10-foot tall lamppost casts a shadow of 20 feet. Find the angle of elevation from the tip of the shadow to the top of the lamppost.

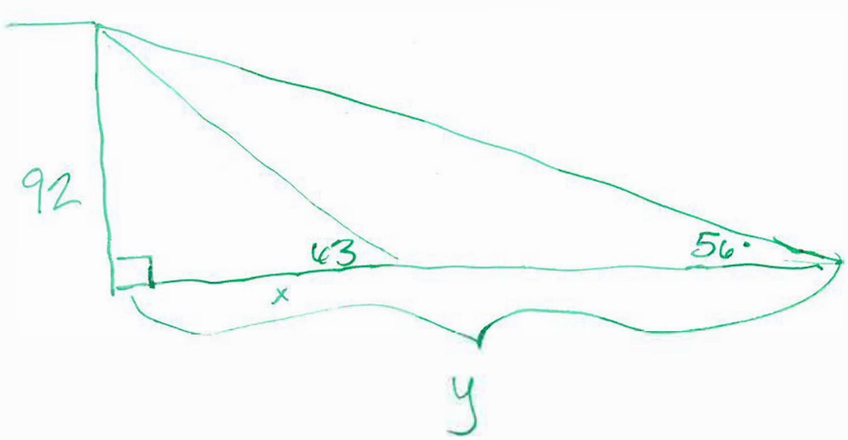


★ inverse

$\tan^{-1}\left(\frac{10}{20}\right)$

$= 26.5651$

5) A man on a cliff is looking down at two ships on the ocean below. The angles of elevation from the two ships are 56 degrees and 63 degrees. If the man is 6 feet tall and the top of the cliff is 86 feet above the ocean, find the distance between the two ships.



find $x - y$

$\tan 63 = \frac{92}{x}$

$x = 46.8763$

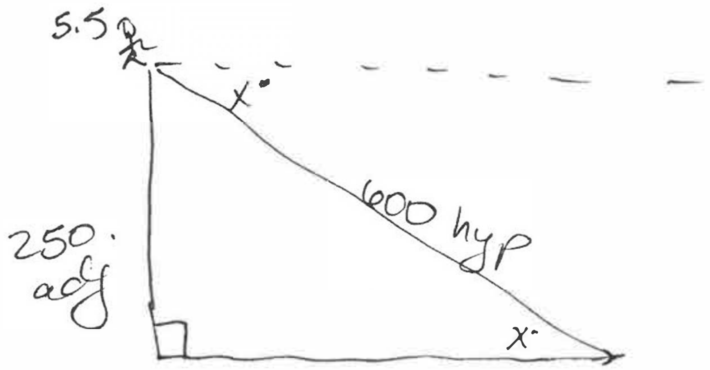
$\tan 56 = \frac{92}{y}$

$y = 62.0548$

$\boxed{15.1784 \text{ ft}}$

6) Miguel is 5.5 feet tall and is standing on an observation platform at the Statue of Liberty, and he is 250 feet above the ground. Find his angle of depression to a ship that is 600 feet away from his position.

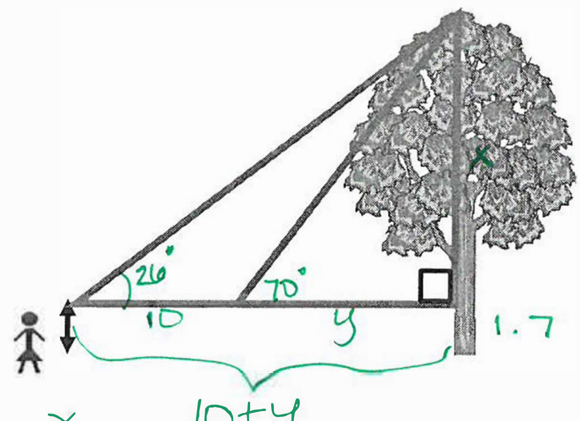
* miguel is 5.5 ft tall



$\sin\left(\frac{255.5}{600}\right)$

$\boxed{x = 25.2034°}$

7) To estimate the height of a tree she wants removed, Christina sights the tree's top at a 70° angle of elevation. She then steps back 10 meters and sights the tops at a 26° angle. If Christina's line of sight is 1.7 meters above the ground, how tall is the tree, to the nearest meter?



$\tan 70 = \frac{x}{y}$ $\quad\quad$ $\tan 26 = \frac{x}{y+10}$

$x = y \tan 70$ $\quad\quad$ $x = (y+10)\tan 26$

$y \tan 70 = (y+10)\tan 26$

$y \tan 70 - y \tan 26 = 10 \tan 26$

$y(\tan 70 - \tan 26) = 10 \tan 26$

$$y = \frac{10 \tan 26}{\tan 70 - \tan 26}$$

so $x = y \tan 70$

$$x = \left(\frac{10 \tan 26}{\tan 70 - \tan 26}\right)\tan 70$$

$x \approx 5.930$
$+ 1.7$

tree = 7.6300 meters

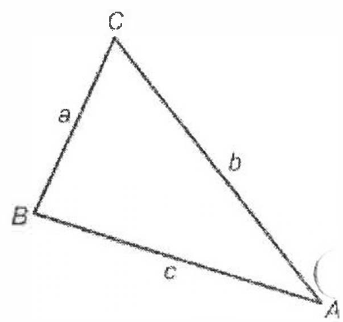
9.2 Notes: Oblique Triangle Trigonometry

In a non-right (oblique) triangle, the Law of Sines or the Law of Cosines may be used to find missing parts.

Proof of the Law of Sines: guided worksheet

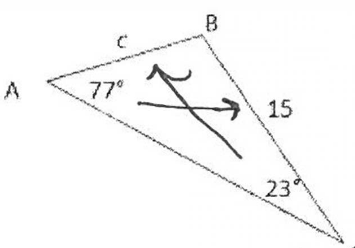
With ASA or AAS

The Law of Sines: $\dfrac{\sin\angle A}{a} = \dfrac{\sin\angle B}{b} = \dfrac{\sin\angle C}{c}$



Examples: Round to four decimal places.

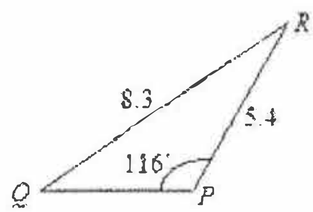
1) Find *c*.



$$\frac{\sin 23}{c} = \frac{\sin 77}{15}$$

$\boxed{16.0151}$

2) Find the measure of <Q.



*inverse

$$\frac{\sin 116}{8.3} = \frac{\sin Q}{5.4}$$

$\boxed{35.7859^\circ}$

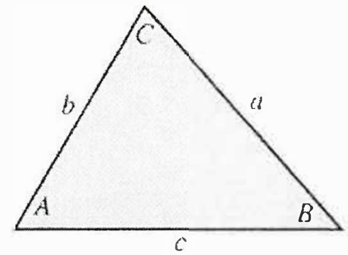
Proof of the Law of Cosines: Guided worksheet

With SAS or SSS

The Law of Cosines: $c^2 = a^2 + b^2 - 2ab\cos\angle C$

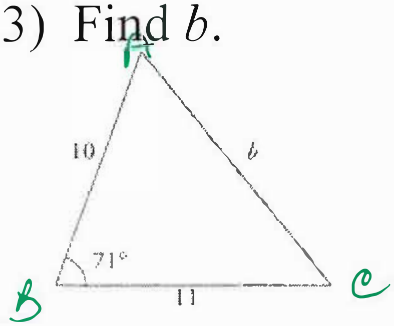
or $a^2 = c^2 + b^2 - 2cb\cos\angle A$

or $b^2 = a^2 + c^2 - 2ac\cos\angle B$



Examples: Round to four decimal places.

3) Find *b*.



$b^2 = 10^2 + 11^2 - 2(110)\cos 71$

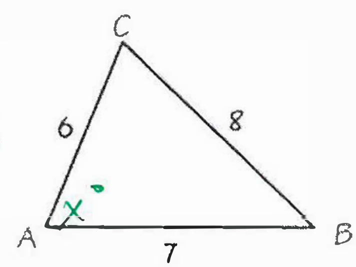
$b^2 = 221 - 220\cos 71$

$\sqrt{b^2} = \sqrt{149.375006}$

$b = 12.2219$

4) Find $m\angle A$.

* inverse



$8^2 = 6^2 + 7^2 - 2(42)\cos x$

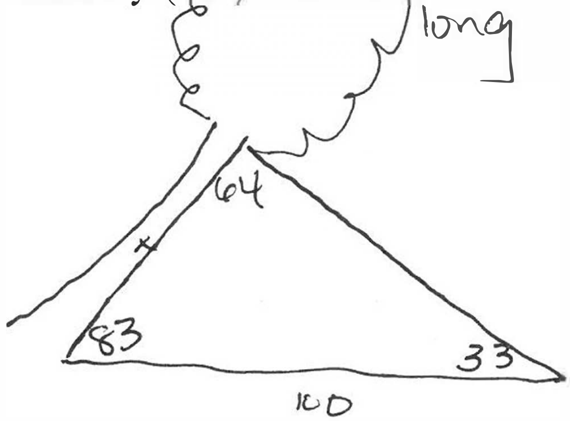
$64 = 85 - 84\cos x$

$\cos^{-1}\ \frac{-21}{-84} = \cos x\ \cos^{-1}$

$\cos^{-1}\frac{1}{4}$

$x = 75.5225°$

5) John wants to measure the height of a tree. He walks exactly 100 feet from the base of the tree and looks up. The angle from the ground to the top of the tree is 33°. This particular ~~tree~~ grows at an angle of 83° with respect to the ground rather than vertically (90°). How ~~tall~~ long is the tree?

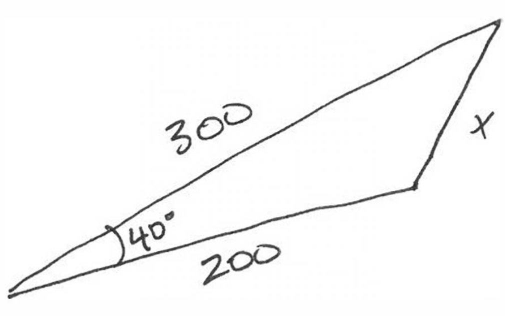


$$\frac{\sin 33}{x} = \frac{\sin 64}{100}$$

$$100 \sin 33 = x \sin 64$$

60.5966 ft

6) Two airplanes leave an airport, and the angle between their flight paths is 40°. An hour later, one plane has traveled 300 miles while the other has traveled 200 miles. How far apart are the planes at this time?



$$x^2 = 200^2 + 300^2 - 2(60{,}000)\cos 40$$

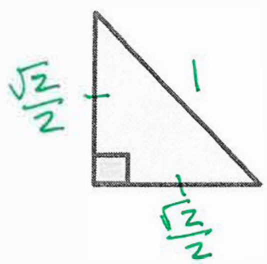
$$x^2 = 130{,}000 - 120{,}000 \cos 40$$

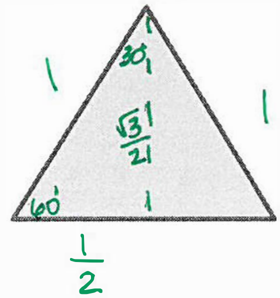
$$x^2 = 38{,}074.66683$$

$$x = 195.1273$$

9.3 Notes: Special Right Triangles and Triples

45-45-90 Triangles (hyp = 1) 30-60-90 Triangles (side = 1)





$\frac{\sqrt{2}}{2} : \frac{\sqrt{2}}{2} : 1$

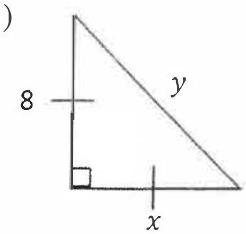
$\frac{1}{2} : \frac{\sqrt{3}}{2} : 1$

Triples: Right triangles with integer sides:

3, 4, 5	5, 12, 13	7, 24, 25	8, 15, 17
9, 40, 41	20, 21, 29	There are more…!	

Example 1: Find the following measures.

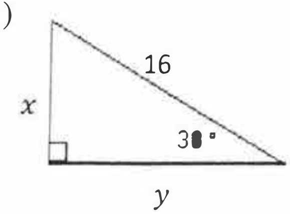
a.)



$x =$ 8

$y =$ $8\sqrt{2}$

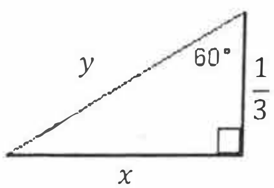
b.)



$x =$ 8

$y =$ $8\sqrt{3}$

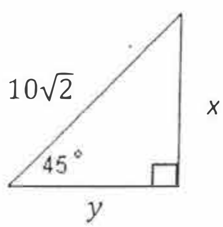
c.)



$x =$ $\frac{\sqrt{3}}{3}$

$y =$ $2/3$

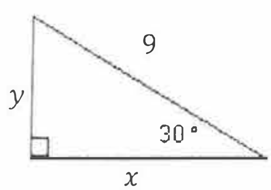
d.)



$x =$ 10

$y =$ 10

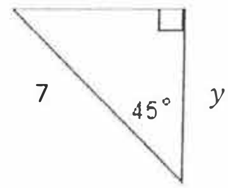
e.)



$x =$ $\frac{9\sqrt{3}}{2}$

$y =$ $9/2$

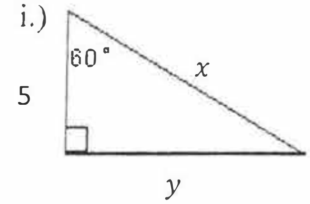
h.)



$x =$ $\frac{7\sqrt{2}}{2}$

$y =$ $\frac{7\sqrt{2}}{2}$

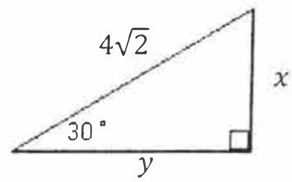
i.)



$x =$ 10

$y =$ $5\sqrt{3}$

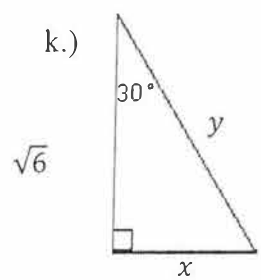
j.)



$x =$ $2\sqrt{2}$

$y =$ $2\sqrt{6}$

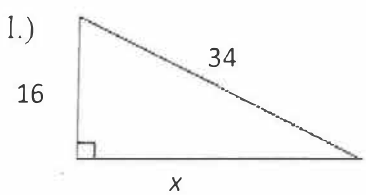
k.)



$x =$ $\sqrt{2}$

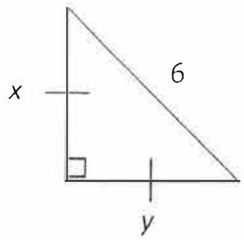
$y =$ $2\sqrt{2}$

l.)



$x =$ 30

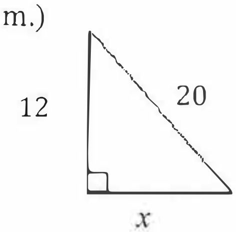
f.)



$x = 3\sqrt{2}$

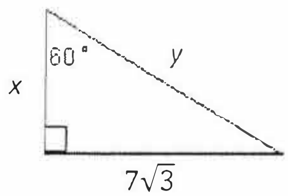
$y = 3\sqrt{2}$

m.)



$x = 16$

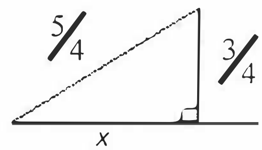
g.)



$x = 7$

$y = 14$

n.)



$x = 1$

Examples: Find the requested ratios without using a calculator. Hint: draw a special right triangle.

1) sin 45

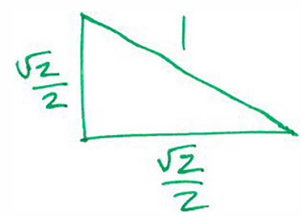
$\frac{\sqrt{2}}{2}$

2) tan 45

1

3) cos 45

$\frac{\sqrt{2}}{2}$



4) cos 60

$\frac{1}{2}$

5) tan 30

$\frac{1}{2} \cdot \frac{2}{\sqrt{3}}$

$\frac{\sqrt{3}}{3}$

6) cos 30

$\frac{\sqrt{3}}{2}$

7) sin 90

$\frac{1}{1}$ $\frac{opp}{hyp}$

8) sin 60

$\frac{\frac{\sqrt{3}}{2}}{1}$ $\frac{\sqrt{3}}{2}$

9) tan 60

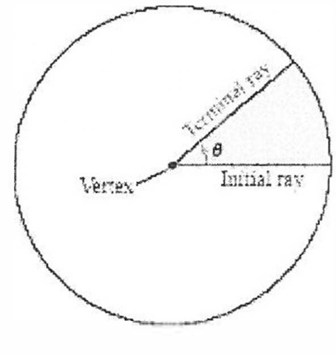
$\frac{\frac{\sqrt{3}}{\cancel{2}}}{\frac{1}{\cancel{2}}} = \sqrt{3}$

10) How do the measures in #4, 6, 8 compare to the lengths of the sides of a 30-60-90 triangle with hypotenuse = 1?

They are the same

9.4 Notes: The Unit Circle

Angles on a coordinate system:

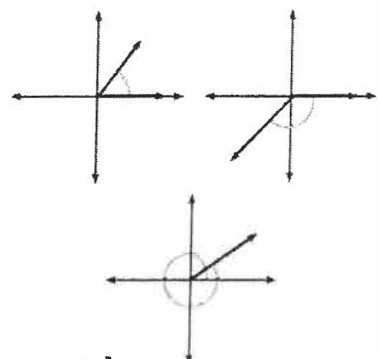


Standard position of an angle

Typical (lowercase) Greek letters used for angle measures:

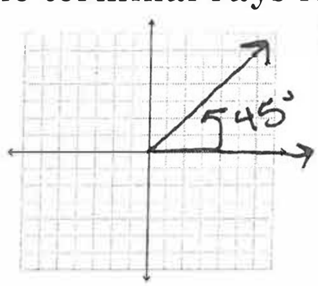
θ	α	β	γ
(theta	alpha	beta	gamma)

Samples of standard positions of angles:

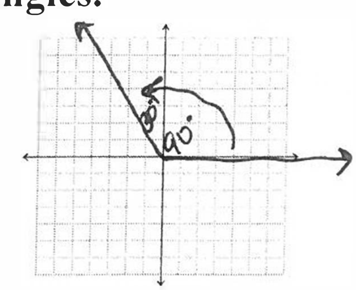


Examples: Draw the terminal rays for the following angles.

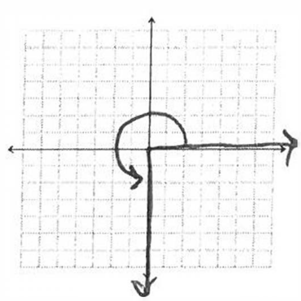
1) 45 degrees



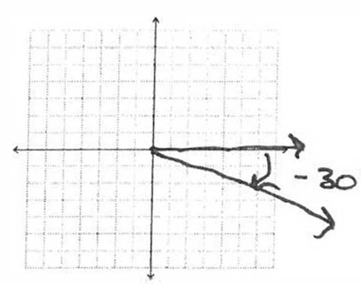
2) 120 degrees



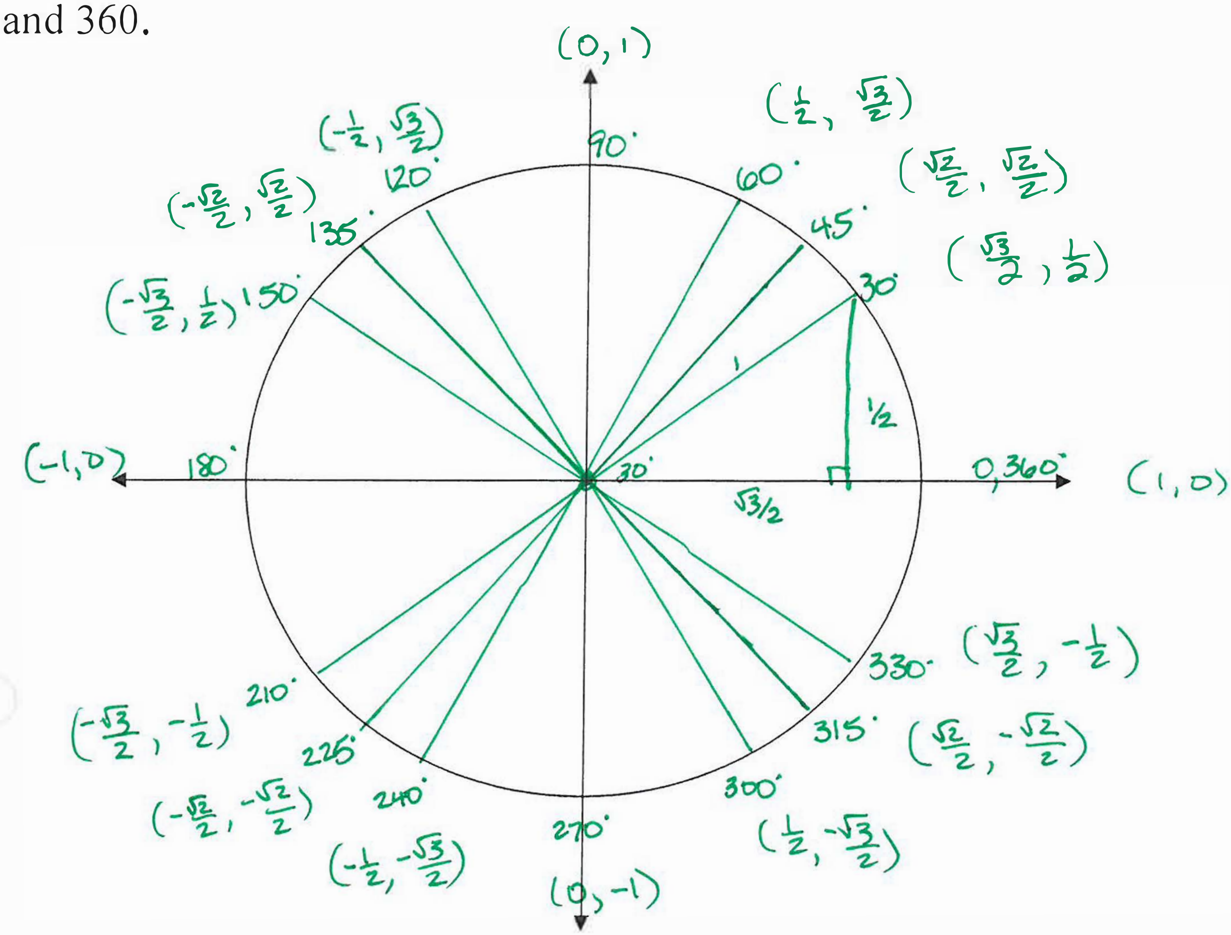
3) 270 degrees



4) -30 degrees



The Unit Circle: Draw a circle with a radius of one and a center of (0,0). Find the ordered pairs for the endpoint of the terminal ray at the following angle measures (in degrees): 30, 45, 60, 90, 120, 135, 150, 180, 210, 225, 240, 270, 300, 315, 330, and 360.



How do the coordinates that you found compare to the sin and cos of each angle?

$(\cos\theta, \sin\theta)$

x-coordinate y-coordinate

What does the tangent of each angle tell you? What happens at 0° and 180°? 90° and 270°?

$\tan = \frac{opp}{adj}$ $\frac{\uparrow}{\rightarrow}$

$\tan = \frac{\sin}{\cos}$

$\tan 0 = \frac{\sin}{\cos} = \frac{0}{1} = 0$

$\tan 90 = \frac{1}{0} =$ undefined

$\tan 180 = \frac{0}{-1} = 0$

$\tan 270 = \frac{-1}{0} =$ undefined

Good visual for unit circle: http://www.mathsisfun.com/geometry/unit-circle.html
The info below is also from the same website.

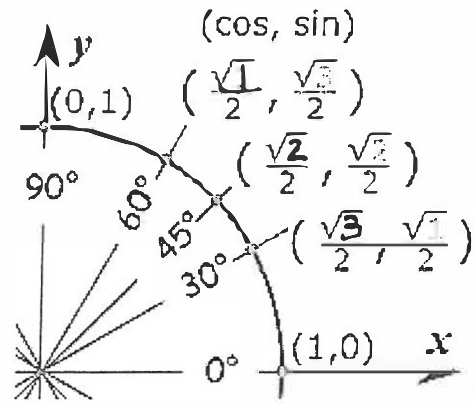
How To Remember?

To help you remember, think **"1,2,3"** :

- $\sin(30°) = \sqrt{1}/2 = 1/2$ (because $\sqrt{1} = 1$)
- $\sin(45°) = \sqrt{2}/2$
- $\sin(60°) = \sqrt{3}/2$

And cos goes **"3,2,1"**

- $\cos(30°) = \sqrt{3}/2$
- $\cos(45°) = \sqrt{2}/2$
- $\cos(60°) = \sqrt{1}/2 = 1/2$ (because $\sqrt{1} = 1$)



Quadrant	I	II	III	IV
	A	Smart	Trig	Class
Trig Function with Positive Values	All (sin, cos, tan)	sin	tan	cos

What about tan?

$\tan = \frac{opposite}{adjacent} = \frac{sin}{cos}$, so you can calculate this value for each angle:

30° 45° 60°

$\frac{\sin}{\cos} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}$

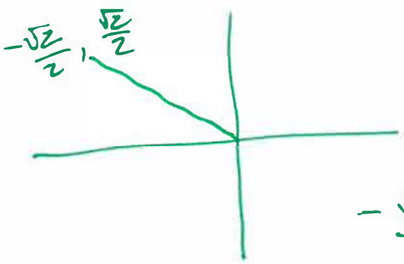
$\frac{1}{2} \cdot \frac{2}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

$\frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = 1$

$\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$

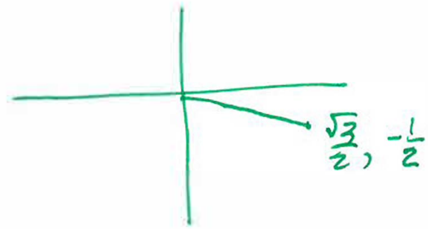
Examples: Find the following values without using a calculator. Try to re-create the portion of the unit circle that you need, rather than looking at the previous page.

1) Find cos 135°.



$-\frac{\sqrt{2}}{2}$

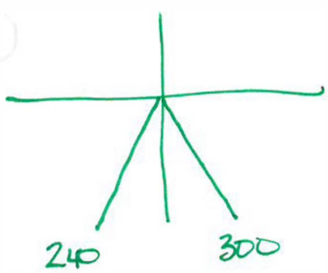
2) Find tan 330°.



$\frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}} = -\frac{\sqrt{3}}{3}$

3) Find all possible values for the missing angle in the equation, where $0° \leq \theta \leq 360°$: $\sin\theta = -\frac{\sqrt{3}}{2}$.

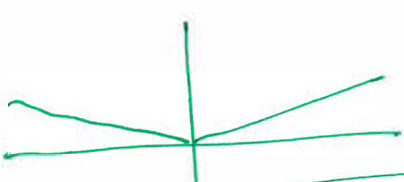
→ negative y value → Quad III or IV



$\boxed{240° \text{ \& } 300°}$

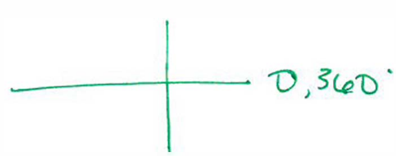
Examples: Solve each equation *without* a calculator. Do not use a completed unit circle, but recreate the portion you need only. Include all possible values when $0° \leq x \leq 360°$:

4) $\sin x = \frac{1}{2}$



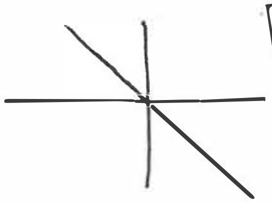
$\boxed{30° \text{ \& } 150°}$

5) $\cos x = 1$



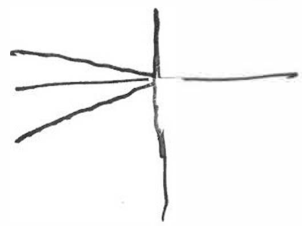
$\boxed{0° \text{ \& } 360°}$

6) $\tan x = -1$



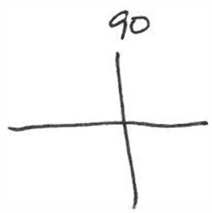
$\boxed{135^\circ \,\&\, 315^\circ}$

7) $\cos x = -\frac{\sqrt{3}}{2}$



$\boxed{150^\circ \,\&\, 210^\circ}$

8) $\sin x = 1$



$\boxed{90^\circ}$

9) $\tan x = 0$

$\boxed{0^\circ, 180^\circ, 360^\circ}$

9.5: Radians and Degrees

The unit circle has a radius = 1 unit. Find the circumference of the circle.

$C = 2\pi(1)$

$C = 2\pi$

$360^\circ = 2\pi$

What is the length of the semicircle with a radius of 1? Of an arc formed by a central angle of 90 degrees?

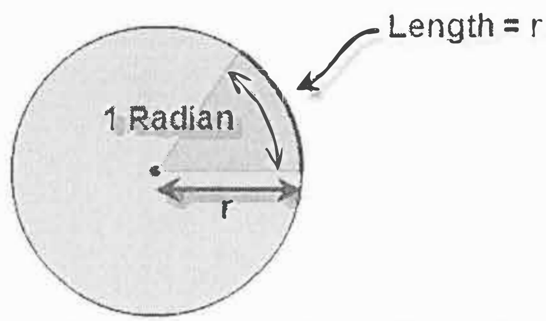
Semicircle : $\frac{2\pi}{2} = \pi$ (180°)

90° arc : $\frac{2\pi}{4} = \frac{\pi}{2}$

$C = 2\pi r$

A circle can be divided up in degrees (360 units), or can divided up in **radians.**

What is a **radian?**



- One radian is the angle of an arc created by wrapping the radius of a circle around its circumference.
- One Radian is $180/\pi$ degrees, or about 57.296°
- So, a **Radian** "cuts out" a length of a circle's circumference equal to the radius.
- There are 2π radians in a unit circle.
- Because the radian is based on the pure idea of "*the radius being laid along the circumference*", it gives simple and natural results to many angle-related mathematics.
- No unit of measurement is typically written.

http://lsquaredmath.us/radians/index.php

Examples:

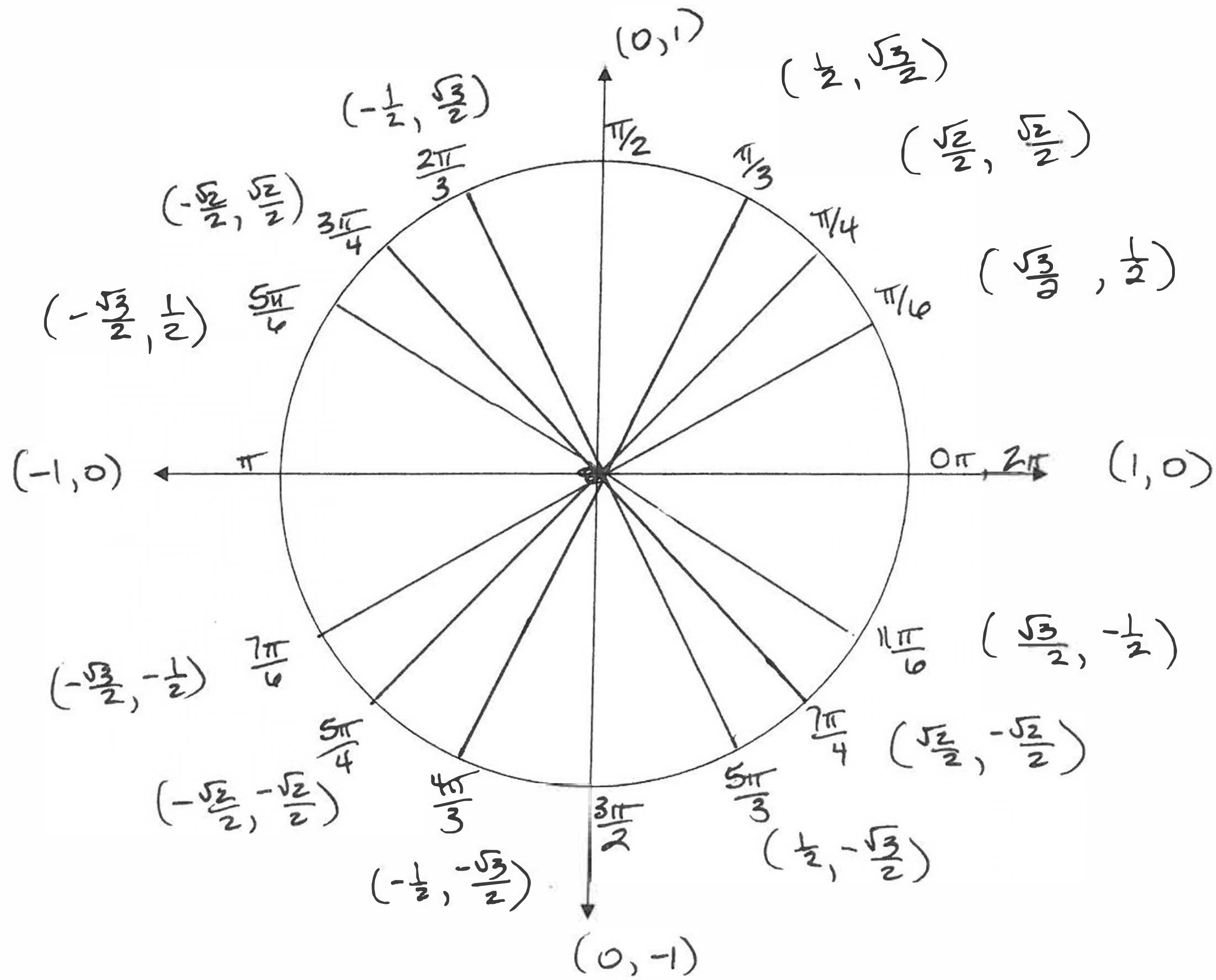
1) An angle has a measure of 60 degrees. Find the corresponding radian measurement for this angle.

$$60 \cdot \frac{\pi}{180} = \frac{6\pi}{18} = \frac{\pi}{3}$$

2) An angle has a measure of $\frac{3\pi}{2}$. Find the corresponding degree measurement for this angle.

$$\frac{3\pi}{2} \cdot \frac{180}{\pi} = 270^\circ$$

Label the unit circle below, for radian measures.



Examples: Find the requested values:

3) $\sin \frac{\pi}{6}$

$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$\frac{1}{2}$

4) $\cos \frac{5\pi}{4}$

$-\frac{\sqrt{2}}{2}$

5) $\tan \pi$

$\frac{0}{-1} = 0$

6) $\cos 2\pi$

1

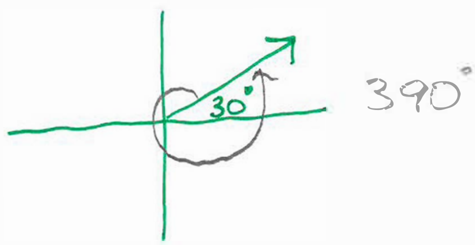
7) $\tan \frac{5\pi}{3}$

$\frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}}$ $\quad -\sqrt{3}$

8) $\sin\left(\frac{7\pi}{6}\right)$

$-\frac{1}{2}$

Coterminal Angles: Two angles with the same initial and terminal rays but possibly different rotations.



Examples: Find one positive co-terminal angle (that is less than 360 degrees or 2π) for each standard positional angle below.

9) 420 degrees

-360

$60°$

10) -120 degrees

$+360$

$240°$

11) $\frac{17\pi}{6} - \frac{12\pi}{6}$

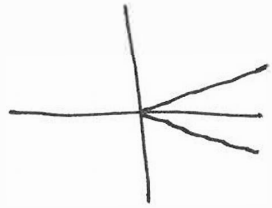
$\frac{5\pi}{6}$

12) $-\frac{2\pi}{3} + \frac{6\pi}{3}$

$\frac{4\pi}{3}$

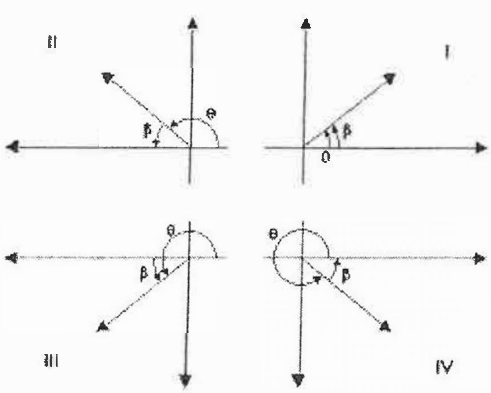
13) For what values of θ in the interval $[0, 2\pi)$ does the $\cos\theta$ have the same value as $\sin\frac{2\pi}{3}$?

$\to \frac{\sqrt{3}}{2}$



$\frac{\pi}{6}$ & $\frac{11\pi}{6}$

9.6: Reference Angles

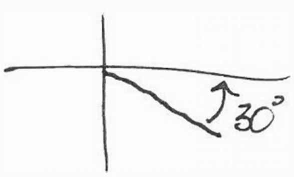


Reference angles: Any angle θ that is not acute in standard position can be referenced by a **reference angle β**, which is the positive acute angle formed by the terminal ray and the x-axis. In other words, the related angle in the **first quadrant.**

what does it take to get back to the x-axis

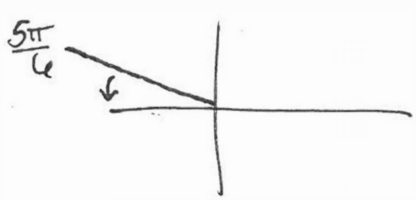
Examples: Find the reference angle β for each of the following angles.

1) 330°



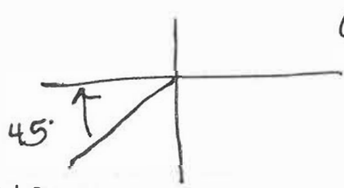
30°

2) $\frac{5\pi}{6}$



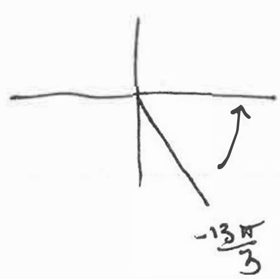
$\frac{\pi}{6}$

3) -135°



45°

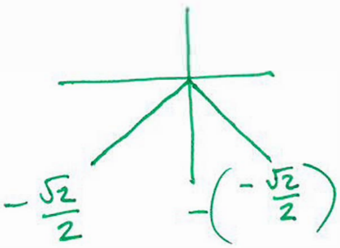
4) $-\frac{13\pi}{3}$



$\frac{\pi}{3}$

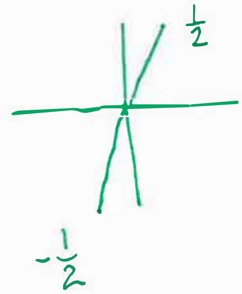
Examples: Are the following statements True or False? Justify your conclusion.

5) sin 225° = - sin (-45°)



False

6) $\cos\frac{4\pi}{3} = -\cos\frac{\pi}{3}$



$-\frac{1}{2} = -\left(\frac{1}{2}\right)$

True

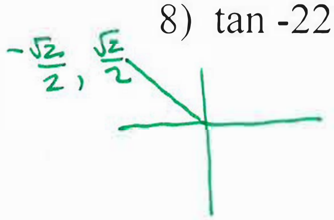
Examples: Use reference angles to find the exact value of each of the following trigonometric functions. Do not use a calculator.

7) $\cos\frac{17\pi}{6} - \frac{12\pi}{6}$

$\cos\frac{5\pi}{6}$

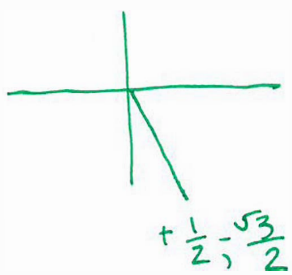
$-\frac{\sqrt{3}}{2}$

8) tan -225° $+360 = 135$



-1

9) $\sin\left(-\frac{19\pi}{3}\right) + \frac{18\pi}{3} = -\frac{\pi}{3}$



$-\frac{\sqrt{3}}{2}$

Examples: Let $f(x) = \sin x$ and $g(x) = \cos x$. Find the exact value of each expression. Do not use a calculator.

10) $f\left(\frac{4\pi}{3}\right) + f\left(\frac{\pi}{6}\right)$

$\sin\frac{4\pi}{3} + \sin\frac{\pi}{6}$

$-\frac{\sqrt{3}}{2} + \frac{1}{2}$

$\frac{1-\sqrt{3}}{2}$

11) $g\left(\frac{5\pi}{6}\right) - g\left(\frac{\pi}{6}\right)$

$\cos\frac{5\pi}{6} - \cos\frac{\pi}{6}$

$-\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}$

$-\frac{2\sqrt{3}}{2}$

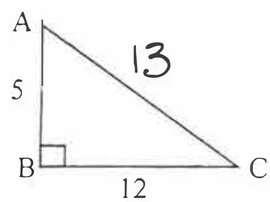
$-\sqrt{3}$

8. 9.7 Notes: Reciprocal Trig Functions

Reciprocal Functions	
$\sin\theta = \frac{1}{\csc\theta}$	$\csc\theta = \frac{1}{\sin\theta}$
$\cos\theta = \frac{1}{\sec\theta}$	$\sec\theta = \frac{1}{\cos\theta}$
$\tan\theta = \frac{1}{\cot\theta}$	$\cot\theta = \frac{1}{\tan\theta}$

Also Important
$\tan\theta = \frac{\sin\theta}{\cos\theta}$
$\cot\theta = \frac{\cos\theta}{\sin\theta}$

Examples: Find the requested ratios.



1) sin A

$\frac{12}{13}$

2) cot C $\frac{1}{\tan}$ or $\frac{a}{o}$

$\frac{12}{5}$

3) csc C $\frac{h}{o}$

$\frac{13}{5}$

4) tan A $\frac{o}{a}$

$\frac{12}{5}$

4) sec A $\frac{h}{a}$

$\frac{13}{5}$

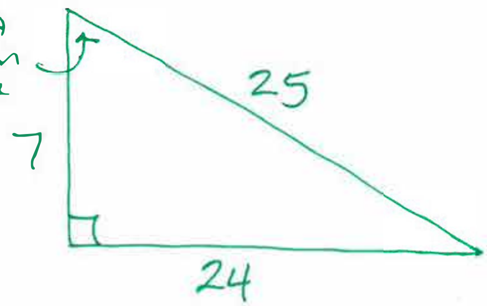
5) cot A $\frac{a}{o}$

$\frac{5}{12}$

For #6 – 7: A right triangle has legs of 7 and 24.

6) Find the sec of the larger acute angle.

Larger ∢ across from longer side



$\frac{1}{\cos} = \frac{h}{a} = \frac{25}{7}$

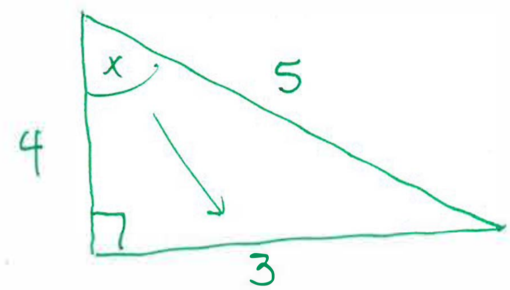
7) Find the csc of the larger actue angle.

$\frac{1}{\sin} = \frac{h}{o} = \frac{25}{24}$

8) Find the cot x, given that $\sin x = \frac{3}{5}$. $\frac{o}{h}$

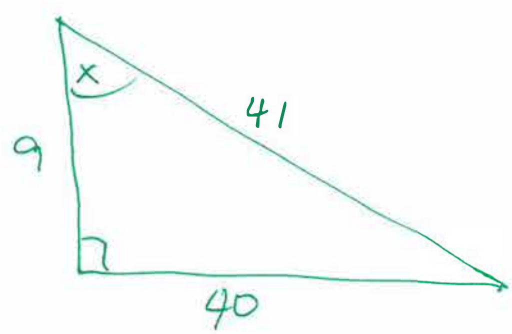
$\frac{\cos}{\sin}$ or $\frac{a}{o}$

$\frac{4}{3}$



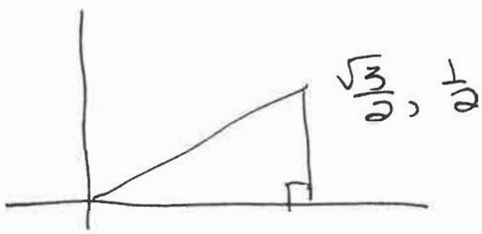
9) Find the csc x, given that $\sec x = \frac{41}{9}$. $\frac{h}{a}$

$\frac{41}{40}$



10) Find x, if the $\cot x = \sqrt{3}$. Note: x is in the first quadrant.

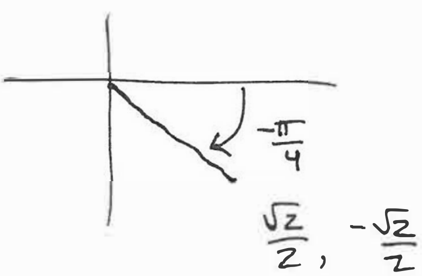
x is an $\angle$ $\rightarrow$ $\frac{1}{\tan} = \frac{\cos}{\sin} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}$



$\frac{\pi}{6}$

11) Find the $\sec x$, $\csc x$, and $\cot x$, given that $x = -\frac{\pi}{4}$.

$\frac{1}{\cos}$ $\frac{1}{\sin}$ $\frac{1}{\tan} = \frac{\cos}{\sin}$



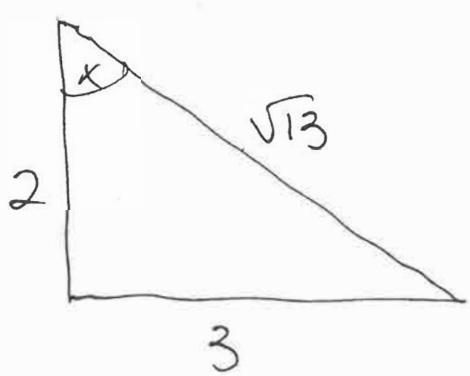
$\sec x = \frac{2}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$

$\csc x = -\frac{2}{\sqrt{2}} = \frac{-2\sqrt{2}}{2} = -\sqrt{2}$

$\cot x = \frac{1}{-1} = -1$

12) Find the $\cos x$, given that $\cot x = \frac{2}{3}$. $\frac{a}{o}$

$\frac{1}{\tan} = \frac{a}{o}$



$\cos x = \frac{2}{\sqrt{13}} = \frac{2\sqrt{13}}{13}$