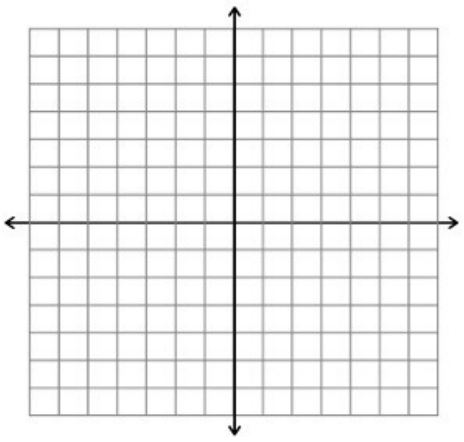
8.1 – Graph Exponential Growth and Decay Functions and Compound Interest

Exploration: Use a table of values to graph the function $y = 3^x$. Describe its domain and range. Does the graph show exponential growth or decay?



In groups, graph the following on the same coordinate plane (in different colors) and state the domain and range for each graph. Also, describe the transformation from the parent function.

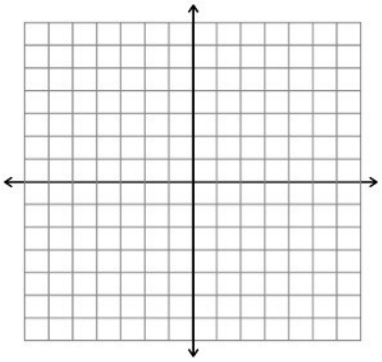
Group 1	Group 2	Group 3	Group 4
$f(x) = 3^x$	$f(x) = 3^x$	$f(x) = 3^x$	$f(x) = 3^x$
$g(x) = 3^x + 2$	$g(x) = 3^{x-5}$	$g(x) = -3^x$	$g(x) = 2(3^x)$
$h(x) = 3^x - 4$	$h(x) = 3^{x+1}$	$h(x) = 3^{-x}$	$h(x) = .5(3^x)$

Transformations for exponential functions:

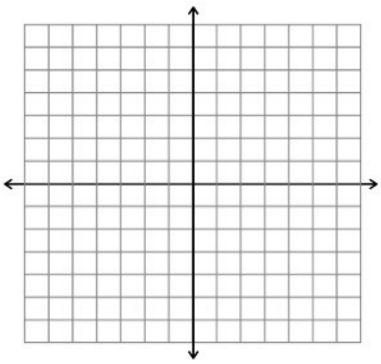
$$f(x) = ab^{x-h} + k \quad for\ b \neq 1$$

Examples: Graph the following functions. Describe the transformations from $y = 3^x$ and also the domain, range, and end behavior. Does the graph show exponential growth or decay?

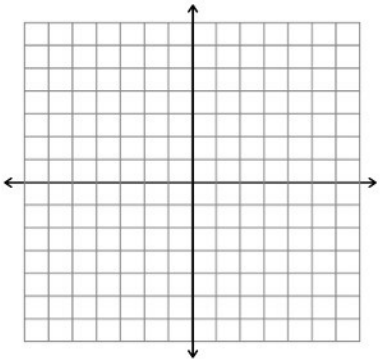
1) $y = 3^{x-2} + 4$



2) $y = -3^{x+4} - 2$



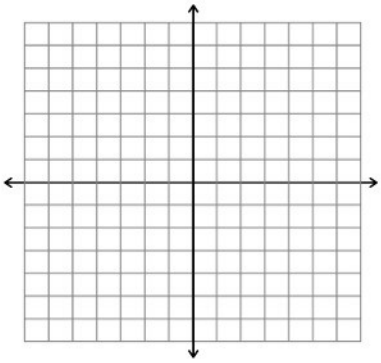
Explore: Use a table of values to graph the function $y = \left(\frac{1}{2}\right)^x$. Describe its domain and range. Does the graph show exponential growth or decay?



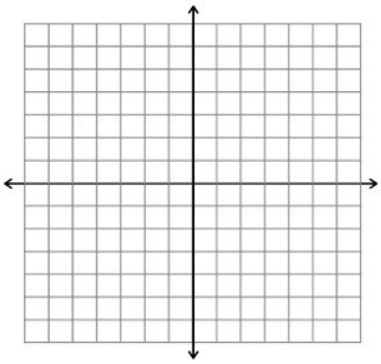
How does the graph change when 0 < b < 1?

Examples: Graph the following functions. Describe the transformations from $y = \left(\frac{1}{2}\right)^x$ and also the domain, range, and end behavior. Does the graph show exponential growth or decay?

3) $f(x) = -\left(\frac{1}{2}\right)^x + 2$



4) $g(x) = 3\left(\frac{1}{2}\right)^x - 5$



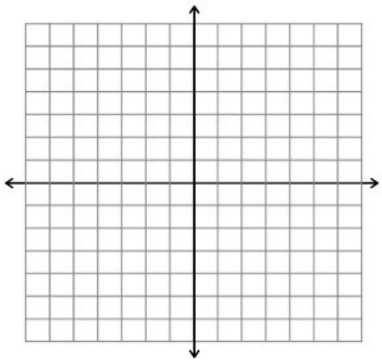
Example 7: Determine whether the following functions demonstrate exponential growth or decay.

a) $g(x) = \left(\frac{5}{6}\right)^x$ b) $h(x) = -2(7)^{x+5}$ c) $f(x) = 6(5)^{-3x}$

How would you change these to be growth functions?

8.2 Notes: Use Functions Involving *e*

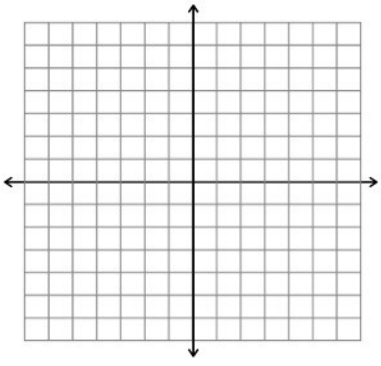
Exploration: Use a table of values to graph the function $y = e^x$. Describe its domain and range. Does the graph show exponential growth or decay?



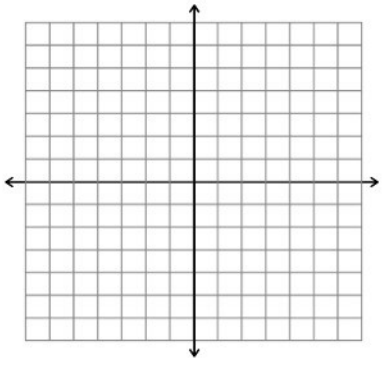
How does the graph of $y = e^x$ compare to the graphs of $y = 2^x$ and $y = 3^x$?

Examples: Graph the following functions, state their transformations from $y = e^x$, and tell whether the function is an example of exponential growth or exponential decay. Also state the domain, range, and end behavior.

4) $y = e^x - 3$



5) $y = -2e^x$



8) A person wants to invest $3,000 in a saving account for 2 years. The bank has 2 options. The first option compounds interest weekly at a rate of 5.4%. The second option compounds interest continuously at a rate of 5%. Which option should you choose? Explain your choice.

8.3– Evaluate Logarithms and Natural Logarithms

11) The wind speed s (in miles per hour) near the center of a tornado can be modeled by $s = 93 \log d + 65$ where d is the distance (in miles) that the tornado travels. In 1925, a tornado traveled 220 miles through three states. Estimate the wind speed near the tornado's center.

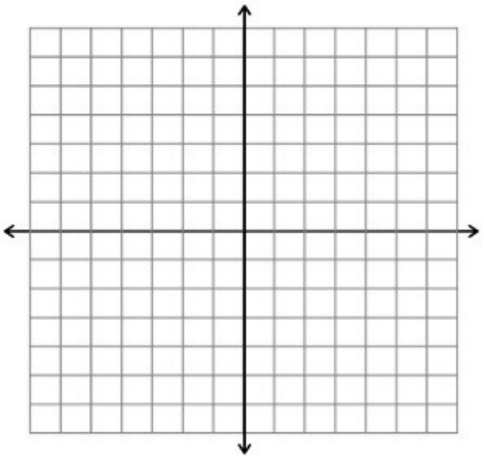
Inverse operations: Logarithms and exponentials are inverses if they have the **same base**.

Properties of Logs	$\log_b b^m =$	$\log_b 1 =$	$\log_b b =$

18) A negative integer is the **logarithm** (base 10) of the number A. What do you know about A?

8.4 Notes: Inverse and Graphs of Logarithmic Functions

Exploration: Use a table of values to graph $y = \log_3 x$. Determine the domain and range. Then sketch the graph of $y = 3^x$ on the same coordinate system. What do you notice?

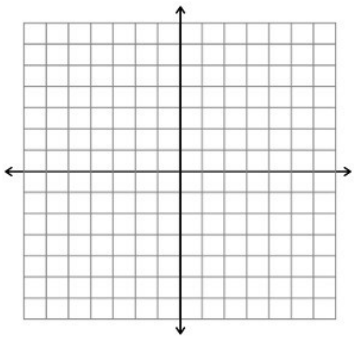


Transformations for logarithmic functions:

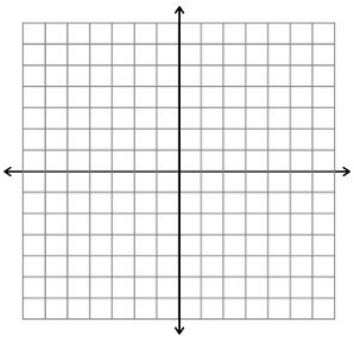
$$f(x) = a\log_b(x - h) + k \quad for\ b > 1$$

Examples: Graph the following functions, state the transformations from $y = \log_3 x$, and find the domain and range.

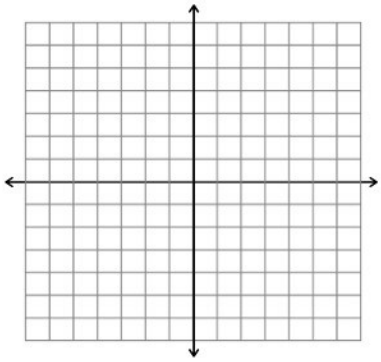
3) $y = \log_3 x - 1$



4) $y = -\log_3(x - 2)$

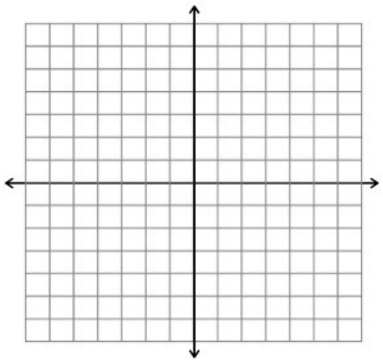


Exploration: Use a table of values to sketch a graph of $y = \ln x$. Determine the domain and range.

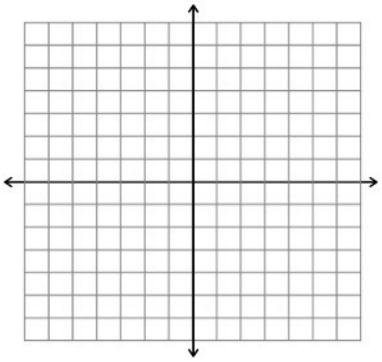


Examples: Graph the following functions, state the transformations from $y = \ln x$, and find the domain and range.

6) $f(x) = -2\ln x$



7) $f(x) = \ln(x + 3)$



9) Write the function $f(x) = \log_6 x$ after it has been compressed vertically by a factor of ¼, shifted up 3 units, and left 6 units.

8.5– Apply Properties of Logarithms

Product Property:

Quotient Property:

Power Property:

Examples: Use properties of logarithms to evaluate the following without a calculator:

Use $\log_6 5 \approx 0.898$ and $\log_6 8 \approx 1.161$ to evaluate the logarithm

1) $\log_6 \left(\frac{5}{8}\right)$ 2) $\log_6 40$ 3) $\log_6 64$

8.6 Notes: Solving Exponential Equations

13) How long would you have to invest $30,000 in an account earning 6% interest compounded continuously so that you have a total of $40,000?

14) Sally invests $350 in an account earning 5% interest, compounded annually. How long will it take her to earn $50 in interest?

15) You want to have $1000 in your savings account. Find the amount that you should deposit if the account pays 4% annual interest over a period of 5 years.

16) The graph of an exponential function in the form $y = ab^x$ passes through the points $(3, 12)$ and $(7, 192)$. What is the value of $f(-2)$?

8.7 Notes: Solving Logarithmic Equations

10) The population of deer in a forest preserve can be modeled by the equation

P = 50 + 200 ln (t + 1), where t is the time in years from the present. In how many years will be deer population reach 500?

11) One of the strongest earthquakes in recent history occurred in Mexico City in 1985 and measured 8.1 on the Richter scale. Find the amount of energy, E, released by this earthquake.

Use the formula: $M = \frac{2}{3}\log\frac{E}{10^{11.8}}$

12)

A student is trying to find an exponential function in the form $y = ab^x$ that models a set of data given by the teacher. The student does not write down the original problem and only partially completes the work before going to a tutor for help. The student's work is shown below. Complete the work to find the exponential function.

x	0	1	2	3	4	5
ln y	0.405	1.099	1.792	2.485	3.178	3.871

$m = \frac{3.178 - 1.099}{4 - 1} = 0.693$

$\ln y - 3.178 = 0.693(x - 4)$

?

A. $y = 1.5(0.4)^x$

C. $y = 1.5(2)^x$

B. $y = 0.4(1.5)^x$

D. $y = 2(1.5)^x$

8.8 Notes: Modeling Logarithmic Equations

All living things contain carbon-14. When a plant or animal dies, the carbon-14 in it begins to decay, or change to another substance. The process is very slow. It takes 5,730 years for just half of it to decay, then another 5,730 years for half the remaining amount to decay, and so on. By using a method similar to the one in this lesson, scientists can determine the amount of carbon-14 in a fossil and can use that amount to determine it age.

Radioactive substances decay to other substances over time. The half-life of a radioactive substance is the time it takes for one-half of the substance to decay. ***How can you determine the length of time it takes a given radioactive substance to decay to a specified percent?***

The isotope bismuth-210 has a half-life of 5 days. Complete the table showing the decay of a sample of bismuth-210.

Number of half-lives	Number of Days (t)	Percent of isotope remaining (p)
0	0	100
1	5	50
2	10	
3		
4		

A. Write an exponential decay function for bismuth-210.

a. What is the decay rate as a fraction:
b. Write an expression for the number of half-lives in t days:
c. Write the exponential decay function that models this situation. The function $p(t)$ should give the percent of the isotope remaining after t days.

*Check that your model is correct by plugging in a t value from the table above.

d. Every 5 days, the amount of bismuth-210 decreases by 50%. By what percent does the amount of bismuth-210 decrease *each day*? Explain.

B. Convert the exponential decay function to a logarithmic function.
   a. Write the inverse of the decay function by solving for t.

* Check that the logarithmic function is correct by substituting 50 for p. What is the resulting value of t? Compare it to the table above.

C. Which equation would you use to determine how many days it would take to have 10% of the bismuth-210 remaining? How many days would it take?

D. Which equation would you use it you wanted to know what percent of bismuth-210 is remaining after 40 days? What is the percent?

E. Will there ever be 0% of the bismuth-210 remaining? Explain your reasoning.