# 8.1 –Graph Exponential Growth and Decay Functions, and Compound Interest

**Exploration:** Use a table of values to graph the function $y = 3^x$. Describe its domain and range. Does the graph show exponential growth or decay?

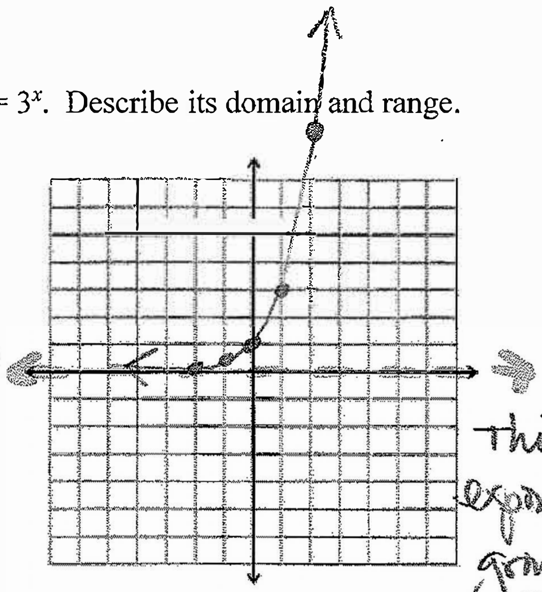
| $x$ | $y=3^x$ |
|---|---|
| $-2$ | $3^{-2} = \frac{1}{9}$ |
| $-1$ | $3^{-1} = 1/3$ |
| $0$ | $3^0 = 1$ |
| $1$ | $3^1 = 3$ |
| $2$ | $3^2 = 9$ |

asymptote at $y=0$

D: $(-\infty, \infty)$

R: $(0, \infty)$

exclusive



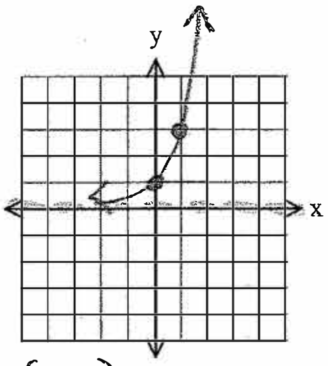
this is exponential growth

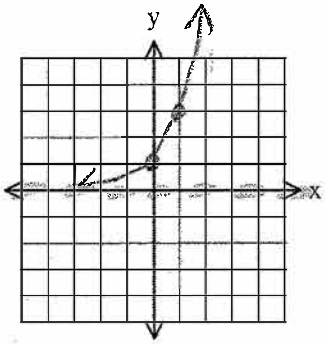
In groups, graph the following on the same coordinate plane (in different colors) and state the domain and range for each graph. Also, describe the transformation from the parent function.

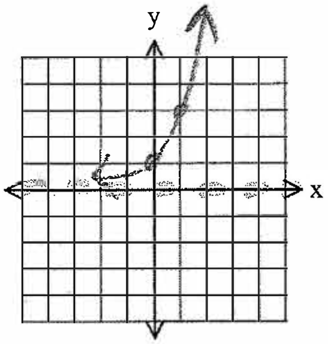
| Group 1 | Group 2 | Group 3 | Group 4 |
|---|---|---|---|
| $f(x) = 3^x$ | $f(x) = 3^x$ | $f(x) = 3^x$ | $f(x) = 3^x$ |
| $g(x) = 3^x + 2$ | $g(x) = 3^{x-5}$ | $g(x) = -3^x$ | $g(x) = 2(3^x)$ |
| $h(x) = 3^x - 4$ | $h(x) = 3^{x+1}$ | $h(x) = 3^{-x}$ | $h(x) = .5(3^x)$ |

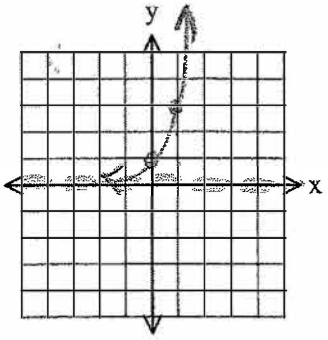
Group 1 Group 2 Group 3 Group 4

$f(x) = 3^x$ $f(x) = 3^x$ $f(x) = 3^x$ $f(x) = 3^x$





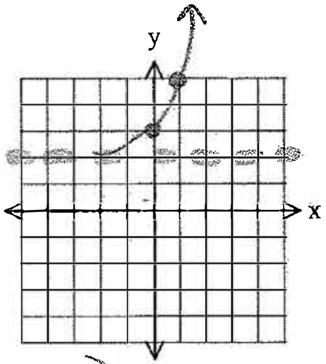


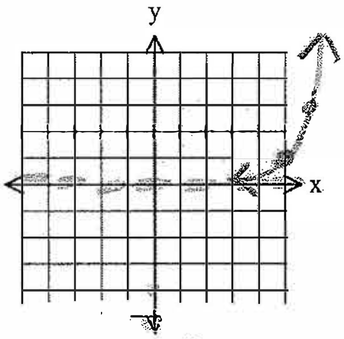


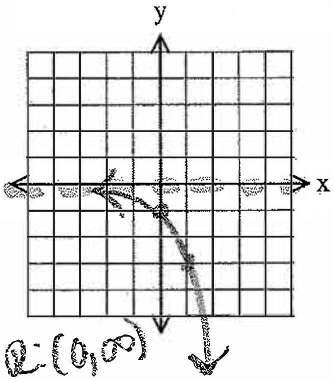
R: (0,∞)

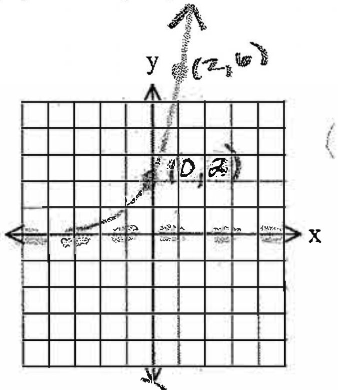
↑2 →5 vertically reflected stretched by 2

$g(x) = 3^x + 2$ $g(x) = 3^{x-5}$ $g(x) = -3^x$ $g(x) = 2(3^x)$





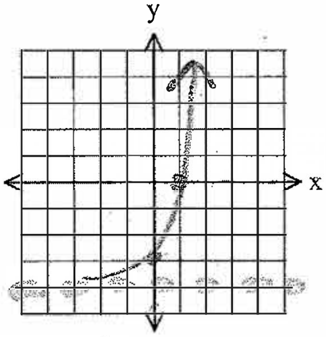


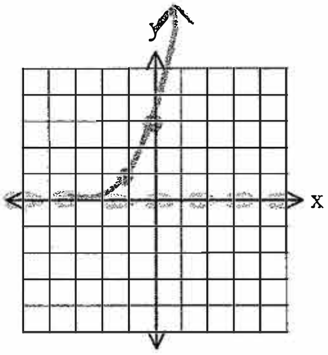


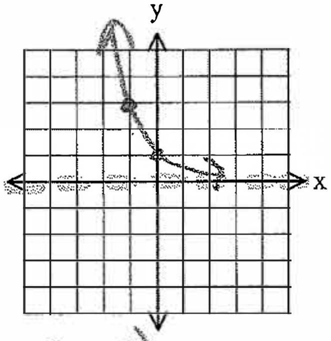
R: (2,∞) R: (0,∞) R: (0,∞) R: (0,∞)

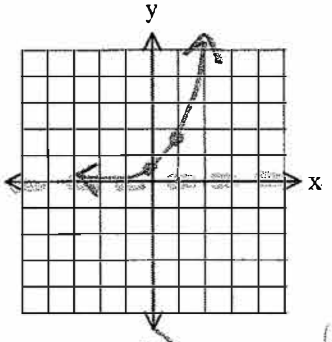
↓4 ←1 horizontally reflected x compressed by ½

$h(x) = 3^x - 4$ $h(x) = 3^{x+1}$ $h(x) = 3^{-x}$ $(\frac{1}{3})$ $h(x) = .5(3^x)$









R: (-4,∞) R: (0,∞) R: (0,∞) R: (0,∞)

## Transformations for exponential functions:

k is actually the horizontal asymptote

$$f(x) = a b^{x \overset{\leftrightarrow}{-h}} \underset{\updownarrow}{+ k} \text{ for } \cancel{b > 1}$$

$b \neq 1$

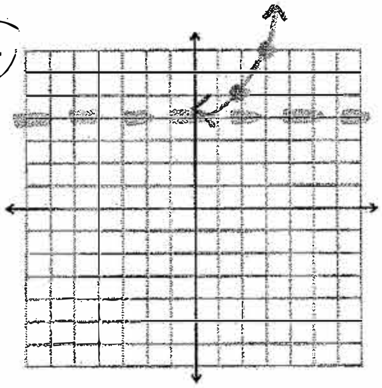
Vert. reflect if <0

stretch compress

**Examples:** Graph the following functions. Describe the transformations from $y = 3^x$ and also the domain, range, and end behavior. Does the graph show exponential growth or decay?

1) $y = \underset{\text{base}}{3}^{x \overset{\rightarrow 2}{-2}} + 4$

asymptote



D: $(-\infty, \infty)$

R: $(4, \infty)$

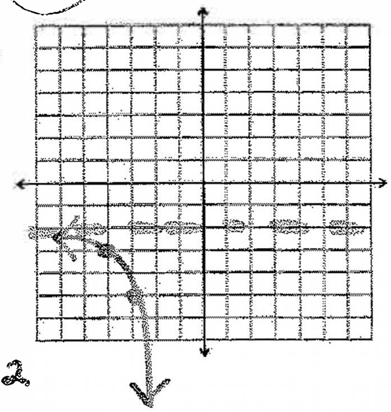
as $x \to \infty$, $y \to \infty$

as $x \to -\infty$, $y \to 4$

2) $y = -3^{x \overset{\leftarrow 4}{+4}} - 2$

asymptote

vert. reflected



D: $(-\infty, \infty)$

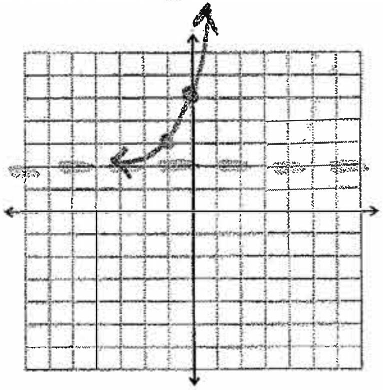
R: $(-\infty, -2)$

as $x \to \infty$, $y \to -\infty$

as $x \to -\infty$, $y \to -2$

3) $f(x) = 3^{x \overset{\leftarrow 1}{+1}} + 2$

asymptote



D: $(-\infty, \infty)$

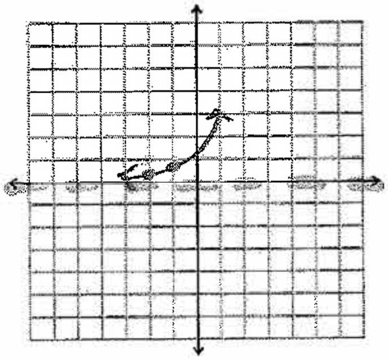
R: $(2, \infty)$

as $x \to -\infty$, $y \to 2$

as $x \to \infty$, $y \to \infty$

4) $f(x) = \underset{\text{compressed}}{\tfrac{1}{4}}(3)^{x \overset{\leftarrow 2}{+2}} + 0$

asymptote



D: $(-\infty, \infty)$

R: $(0, \infty)$

as $x \to -\infty$, $y \to 0$

as $x \to \infty$, $y \to \infty$

**Explore:** Use a table of values to graph the function $y = \left(\frac{1}{2}\right)^x$. Describe its domain, range, and end behavior. Does the graph show exponential growth or decay?

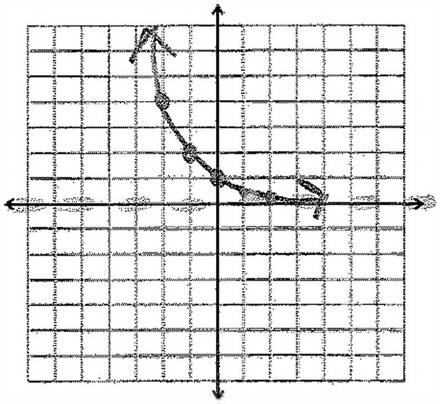
| $x$ | $y = \left(\frac{1}{2}\right)^x$ |
|---|---|
| $-2$ | $\left(\frac{1}{2}\right)^{-2} = 4$ |
| $-1$ | $\left(\frac{1}{2}\right)^{-1} = 2$ |
| $0$ | $(1/2)^0 = 1$ |
| $1$ | $(1/2)^1 = 1/2$ |
| $2$ | $(1/2)^2 = 1/4$ |

D: $(-\infty, \infty)$

R: $(0, \infty)$

as $x \to -\infty, y \to \infty$

as $x \to \infty, y \to 0$



this is exponential decay

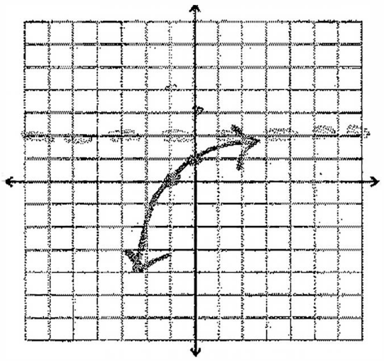
**How does the graph change when $0 < b < 1$?** DECAY APPROACHES H.A.

**Examples:** Graph the following functions. Describe the transformations from $y = \left(\frac{1}{2}\right)^x$ and also the domain, range, and end behavior. Does the graph show exponential growth or decay?

5) $f(x) = -\left(\frac{1}{2}\right)^x + 2$ asymptote

reflected

decay instead of growth. Also can think of it as a horizontal reflection of $2^x$



D: $(-\infty, \infty)$

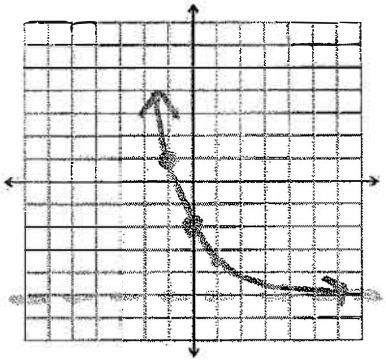
R: $(-\infty, 2)$

as $x \to -\infty, y \to -\infty$

as $x \to \infty, y \to 2$ (decay)

6) $g(x) = 3\left(\frac{1}{2}\right)^x - 5$ asymptote

means is decay.



D: $(-\infty, \infty)$

R: $(0, \infty)$

as $x \to \infty, y \to -5$ (decay)

as $x \to -\infty, y \to \infty$

**Example 7:** Determine whether the following functions demonstrate exponential growth or decay.

a) $g(x) = \left(\frac{5}{6}\right)^x$

$0 < \frac{5}{6} < 1$

so decay

b) $h(x) = -2(7)^{x+5}$

$7 > 0$

growth

c) $f(x) = 6(5)^{-3x}$

$= 6\left(\frac{1}{5^3}\right)^x$

$= 6\left(\frac{1}{125}\right)^x$

$0 < \frac{1}{125} < 1$

decay

**How would you change these to be growth functions?**

**Compound Interest:** $A = P\left(1+\frac{\boxed{r}}{n}\right)^{nt}$

rate, as a decimal

n is determined by how it's compounded

t is time

initial value

resulting total

(Do examples on foldable for compound interest and modeling growth and decay.) ✓

## *8.2 Notes: Use Functions Involving e*

The Natural Base *e*: 2.718281828...

**Examples:** Simplify the following expressions. For #1, 2: a) w/o a calculator b) with calculator

1) $2e^2 \cdot 5e^5$

2ee · 5eeeee

$\boxed{= 10e^7}$

2) $\frac{\overset{4}{\cancel{12}}e^4}{\underset{1}{\cancel{3}}e^3} = \boxed{4e}$

3) $(5e^{-3x})^2$

$25e^{-6x} = \boxed{\frac{25}{e^{6x}}}$

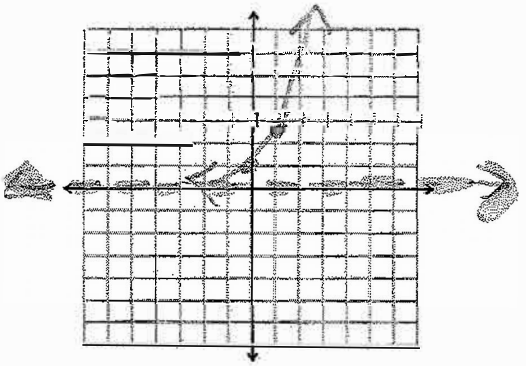
4) $4e^{4x} \cdot 3e^{x+1} \cdot e^{2x}$

$12e^{(4x+x+1+2x)}$

$\boxed{= 12e^{7x+1}}$

**Exploration:** Use a table of values to graph the function $y = e^x$. Describe its domain and range. Does the graph show exponential growth or decay?

$e \approx 2.718$

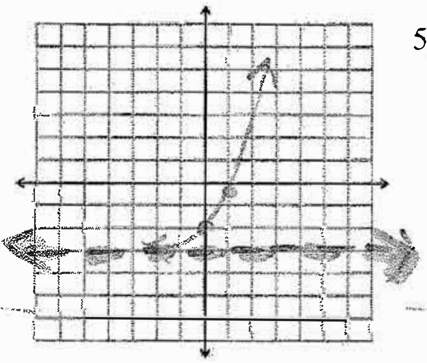


**How does the graph of $y = e^x$ compare to the graphs of $y = 2^x$ and $y = 3^x$?**

in between these 2

**Examples:** Graph the following functions, state their transformations from $y = e^x$, and tell whether the function is an example of exponential growth or exponential decay. Also state the domain, range and end behavior.

4) $y = e^x - 3$



down 3

growth

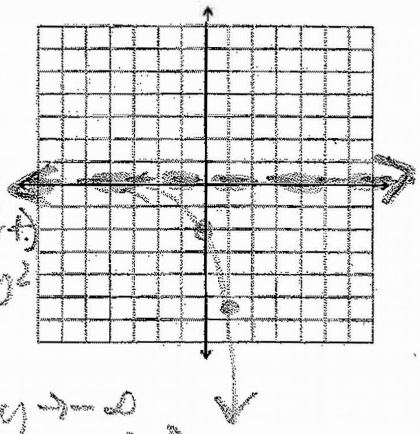
D: $(-\infty, \infty)$

R: $(-3, \infty)$

at $x \to \infty, y \to \infty$

at $x \to -\infty, y \to -3$

5) $y = -2e^x$



$-2(2.718)^1$

$= -5.437$

reflected (vert)

stretched by 2

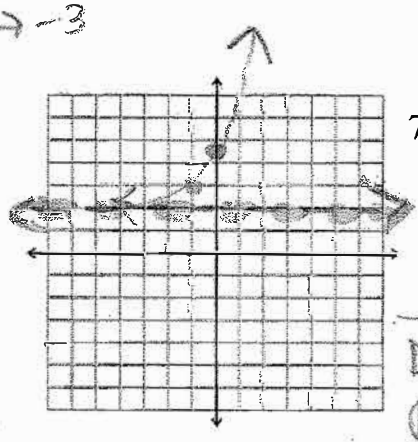
D: $(-\infty, \infty)$

R: $(-\infty, 0)$

as $x \to \infty, y \to -\infty$

as $x \to -\infty, y \to 0$

6) $f(x) = e^{x+1} + 2$ ←1



← 1, ↑ 2

growth

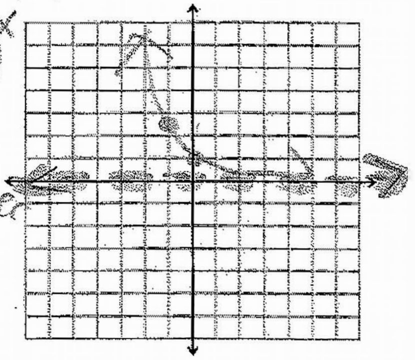
D: $(-\infty, \infty)$

R $(2, \infty)$

as $x \to -\infty, y \to 2$

as $x \to +\infty, y \to +\infty$

7) $g(x) = e^{-x}$ $= \left(\frac{1}{e}\right)^x$



decay

horiz. reflected

D: $(-\infty, \infty)$

R: $(0, \infty)$

as $x \to -\infty, y \to \infty$

as $x \to +\infty, y \to 0$

**Continuously Compounded Interest:** $A = Pe^{rt}$

Complete the foldable examples (compounded continuously section)

$n=52$

**8) A person wants to invest $3,000 in a saving account for 2 years. The bank has 2 options. The first option compounds interest weekly at a rate of 5.4%. The second option compounds interest continuously at a rate of 5%. Which option should you choose? Explain your choice.**

Option 1

$A(t) = P(1+\frac{r}{n})^{nt}$

$= 3000(1+\frac{.054}{52})^{52(2)}$

$= \$3341.96$

option 2

$A(t) = {}^{rt}$

$A(t) = 000e$

$= \$331$

would choose option 2. more

## 8.3 - *Evaluate Logarithms and Natural Logarithms*

Logarithm with Base b:

$\log_b x \to b^{?} = x$

Logarithm with base 10:

$\log_{10}$ og

don't need write 10)

Logarithm with base *e*: the Natural Log (*ln*)

$\log_e$

$\log_e$ is so common it has its own notation)

**Examples:** Rewrite the following equations in logarithm form or exponential form.

1) $\log_3 81 = 4$

$3^4 = 81$

2) $\log_{10} 1 = 0$

$10^0 = 1$

3) $2^3 = 8$

$\log_2 8 = 3$

4) $(1/4)^{-1} = 4$

$\log_{(1/4)} 4 = -1$

**Examples:** Evaluate the following (calculator only on #9 and 10):

5) $\log_3 81$

$3^? = 81$

$\boxed{4}$

6) $\log_{10} 0.001$

$10^? = .001$

$\boxed{-3}$

7) $\log_{1/4} 256 = x$

$\left(\frac{1}{4}\right)^x = 256$

$\left(\frac{1}{4}\right)^4 = \frac{1}{256}$

$\left(\frac{1}{4}\right)^{-4} = 256$

$\boxed{-4}$

8) $\log_{64} 2 = x$

$64^x = 2$

$2^6 = 64 \Rightarrow \sqrt[6]{64} = 2$

$\boxed{\frac{1}{6}}$

9) $\ln 6$

$\ln 6 = \log_e 6$

$e^? = 6$

$\boxed{1.7918}$

10) $\ln 54.2$

$\boxed{3.9927}$

11) The wind speed $s$ (in miles per hour) near the center of a tornado can be modeled by $s = 93 \log d + 65$ where d is the distance (in miles) that the tornado travels. In 1925, a tornado traveled 220 miles through three states. Estimate the wind speed near the tornado's center.

$s = 93 \log 220 + 65 = 282.8$ mph

**Inverse operations:** Logarithms and exponentials are inverses if they have the **same base.**

| | | | |
|---|---|---|---|
| Properties of Logs | $\log_b b^m = m$ | $\log_b 1 = 0$ | $\log_b b = 1$ |

$b^? = b^m$  $b^? = 1$  $b^? = b$

**Examples:** Simplify the following expressions

12) $10^{\log 4}$

$10^{\log_{10} 4} = \boxed{4}$

13) $e^{\ln 9} = \boxed{9}$

14) $\log_3 \overset{3^3}{27}$

$\log_3 3^3 = \boxed{3}$

15) $\log_5 \overset{5^2}{25}^{-2x} = \log_5 (5^2)^{-2x}$

$= \log_5 5^{-4x} = \boxed{-4x}$

16) $\ln e^{-2x}$

$\boxed{-2x}$

17) $\log_3 \overset{3^3}{27} + \ln(e^3) - \log_{10} 10^4 - 3\log_2 \overset{2^5}{32} + e^{\ln 3}$

$3 + 3 - 4 - 3 \cdot 5 + 3$

$2 - 15 + 3 = \boxed{-10}$

18) A negative integer is the **logarithm** (base 10) of the number A. What do you know about A?

$\log_{10} A = -\#$

$10^{-\#} = A$

A must be between 0 and 1.

## *8.4 Notes: Inverse and Graphs of Logarithmic Functions*

[redacted]

**Examples:** Find the inverse of the following functions

1) $y = 8^x$

$x = 8^y$

$\log_8 x = \log_8 8^y$

$\boxed{y = \log_8 x}$

2) $y = \ln(x - 4) + 5$

$x = \ln(y-4) + 5$

$x - 5 = \ln(y-4)$

$e^{x-5} = e^{\ln(y-4)}$

$e^{x-5} = y - 4$

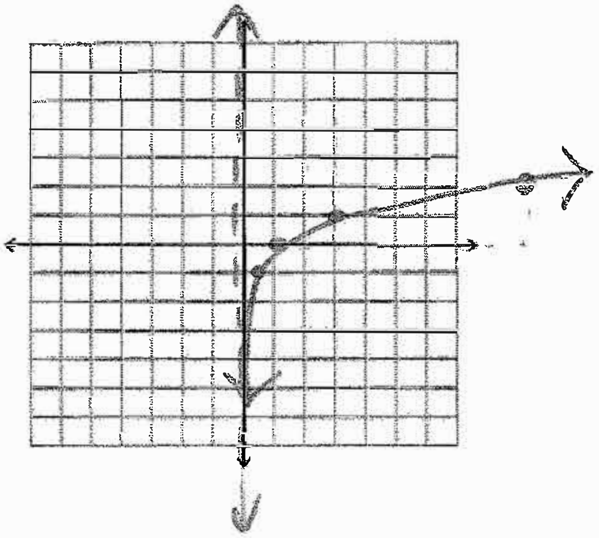
$\boxed{y = e^{x-5} + 4}$

3) Determine if $y = 4^x + 1$ and $y = \log_4 x - 1$ are inverses.

$4^{\log_4 x - 1} + 1$ ← can't simplify so <u>no</u>

**Exploration:** Use a table of values to graph $y = \log_3 x$. Determine the domain and range. Then sketch the graph of $y = 3^x$ on the same coordinate system. What do you notice?

| $x$ | $y = \log_3 x$ |
|---|---|
| 1 | $\log_3 1 = 0$ |
| 3 | $\log_3 3 = 1$ |
| 9 | $\log_3 9 = 3$ |
| $\frac{1}{3}$ | $\log_3(\frac{1}{3}) = -1$ |



**Transformations for logarithmic functions:**

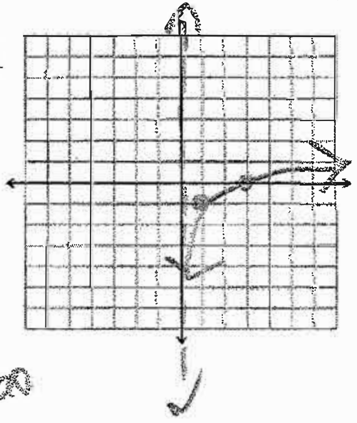
$$f(x) = a\log_{\overset{\text{base}}{b}}(x - h) + \overset{\updownarrow}{k} \quad for\ b > 1$$

stretch, reflect, compress

asymptote ↔

**Examples:** Graph the following functions, state the transformations from $y = \log_3 x$, and find the domain and range.

3) $y = \log_3 x - 1$



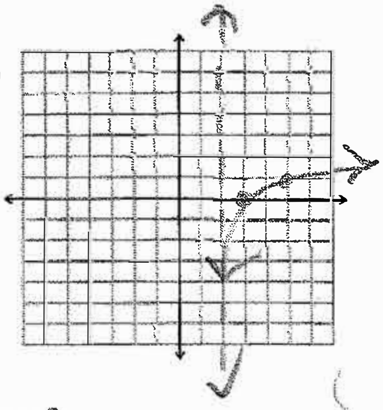
↓ 1

D: $(0, \infty)$

R: $(-\infty, \infty)$

as $x \to \infty$, $y \to \infty$

4) $y = -\log_3(x - 2)$



→2

D: $(2, \infty)$

R: $(-\infty, \infty)$

as $x \to \infty$, $y \to \infty$

5) $f(x) = 2\log_3(\overset{2}{x} + 1) + 3$

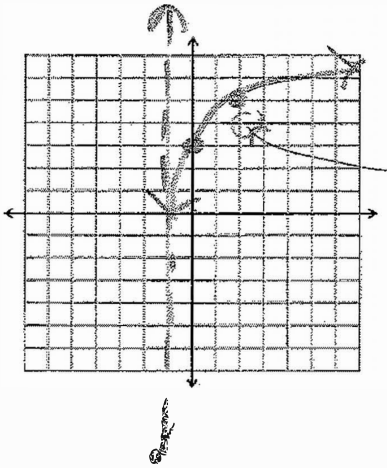
Stretched by 2

← 1

↑ 3

D: (-1, ∞)

R: (-∞, ∞)

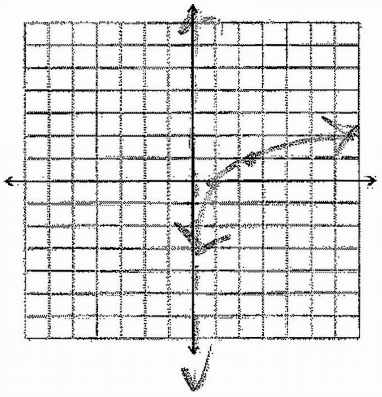


normally over 3 from asymptote, up 1! However, its up 2 because of the stretch.

**Exploration:** Use a table of values to sketch a graph of $y = \ln x$. Determine the domain and range.

✓

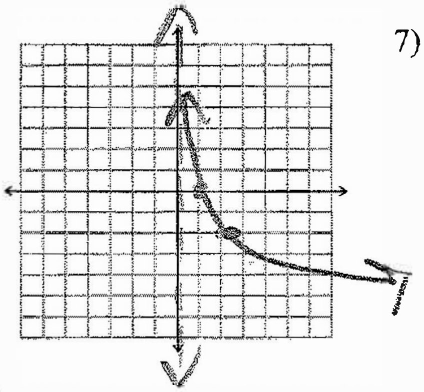
$y = \log_{2.71} x$



**Examples:** Graph the following functions, state the transformations from $y = \ln x$, and find the domain and range.

6) $f(x) = -2\ln x$

| | |
|---|---|
| 1 | $-2\ln(1) = -2(0) = 0$ |
| e | $-2\ln e = -2$ |



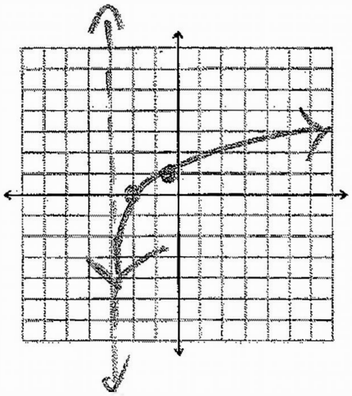
reflected
stretched by 2

D: (0, ∞)

R: (-∞, ∞)

7) $f(x) = \ln(x + 3)$

← 3



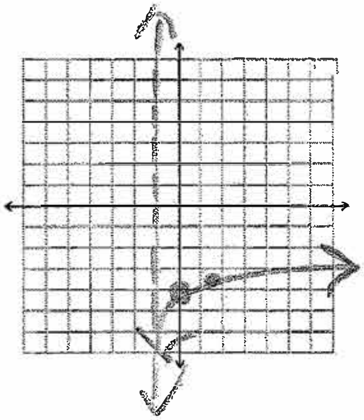
D: (-3, ∞)

R: (-∞, ∞)

8) $y = \frac{1}{2} \ln(x + 1) - 4$

← 1 ↓ 4

Compressed



9) Write the function $f(x) = \log_6 x$ after it has been compressed vertically by a factor of ¼, shifted up 3 units, and left 6 units.

$$f(x) = \frac{1}{4}\log_6(x+6) + 3$$

## *8.5 Notes: Apply Properties of Logarithms*

Product Property: $\log(xy) = \log x + \log y$

Quotient Property: $\log\left(\frac{x}{y}\right) = \log x - \log y$

Power Property: $\log(x^a) = a\log x$

**Examples:** Use properties of logarithms to evaluate the following without a calculator:

*Use* $\log_6 5 \approx 0.898$ *and* $\log_6 8 \approx 1.161$ *to evaluate the logarithm*

1) $\log_6\left(\frac{5}{8}\right)$

$\log_6 5 - \log_6 8$

$.898 - 1.161 =$

2) $\log_6 40 = \log_6 \frac{8}{5}$

$\log_6 8 - \log_6 5$

$1.161 - .898 =$

3) $\log_6 64 = \log_6 8^2$

$2\log_6 8$

$2(1.161) =$

**Examples:** Expand or condense each logarithmic expression.

4) Condense: ln 4 + 3 ln 3 – ln 12

$\ln 4 + \ln 3^3 - \ln 12$

$\ln\left(\frac{4 \cdot 3^3}{12}\right) = \boxed{\ln 9}$

5) Expand: $log_7 \frac{3x^2}{5y^3}$

$$\log_7 3 + \log_7 x^2 - \log_7 5 - \log_7 y^3$$

$$\log_7 3 + 2\log_7 x - \log_7 5 - 3\log_7 y$$

6) Condense: $\log_2 6x - 3\log_2(2y) + \log_2 24 - \log_2 3z$

$$\log_2\left(\frac{6x \cdot 24}{(2y)^3\, 3z}\right) = \log_2\left(\frac{6 \cdot \overset{8}{\cancel{24}}\, x}{\cancel{3} \cdot 8 y^3 z}\right)$$

$$= \boxed{\log_2\left(\frac{6x}{y^3 z}\right)}$$

7) Expand $ln\frac{2x^3y}{7z^4}$

$$\ln 2 + 3\ln x + \ln y - \ln 7 - 4\ln z$$

8) Condense: $ln5 + \frac{1}{3}lna - 2lnb - lnc$

$$\ln\left(\frac{5a^{1/3}}{b^2 c}\right)$$

## Change-of-Base Formula

$$\log_b a = \frac{\log a}{\log b} = \frac{\ln a}{\ln b}$$ (base doesn't matter)

**Examples:** Use the change-of-base formula to evaluate **ONE** of the logarithms. Give the exact solution and an approximate solution to 3 decimals.

a) $\log_5 8$ $\quad \frac{\log 8}{\log 5} = \frac{1.161}{.898} =$

b) $\log_8 14$ $\quad \frac{\log 14}{\log 8} =$

### *8.6 Notes: Solving Exponential Equations*

Property of Equality for Exponential Equations

- same base each side
- set exponents equal

**Examples:** Solve the following for *x*.

1) $3^{2x-1} = \cancel{81}\ 3^4$

$2x - 1 = 4$

$\quad +1 \quad +1$

$2x = 5$

$\boxed{x = 5/2}$

2) $e^{x+6} = e^{3x}$

$x + 6 = 3x$

$6 = 2x$

$\boxed{x = 3}$

3) $\cancel{100}^{7x+1} = \cancel{1000}^{\,3x-2}$

$10^2 \qquad 10^3$

$(10^2)^{7x+1} = (10^3)^{3x-2}$

$10^{14x+2} = 10^{9x-6}$

$14x + \cancel{2} = \cancel{9x} - 6$

$-9x \quad \cancel{-2} \quad \cancel{-9x} - 2$

$5x = -8$

$\boxed{x = -8/5}$

4) $\overset{(3/2)}{\cancel{1.5}} = \left(\frac{81}{16}\right)^{x+3}$

$\left(\frac{3}{2}\right)^1 = \left(\left(\frac{3}{2}\right)^4\right)^{x+3}$

$\left(\frac{3}{2}\right)^1 = \left(\frac{3}{2}\right)^{4x+12}$

$1 = 4x + 12$

$-11 = 4x$

$\boxed{x = -11/4}$

5) $\overset{7^{-3}}{\frac{1}{343}} = 7^{2x+5}$

$-3 = 2x + 5$

$-8 = 2x$

$\boxed{x = -4}$

6) $\overset{(5/2)}{\cancel{2.5}}^{8x-4} = \left(\frac{125}{8}\right)^{2x+4}$

$\left(\frac{5}{2}\right)^{8x-4} = \left[\left(\frac{5}{2}\right)^3\right]^{2x+4}$

$\left(\frac{5}{2}\right)^{8x-4} = \left(\frac{5}{2}\right)^{6x+12}$

$8x - \cancel{4} = \cancel{6x} + 12$

$-6x \cancel{+4} \quad \cancel{-6x} + 4$

$2x = 16$

$\boxed{x = 8}$

**Examples:** Solve the following equations by taking the logarithm of both sides

7) $4^x = 11$

$\log_4 4^x = \log_4 11$

$x = \boxed{\log_4 11}$

8) $2^{x-4} = 5.3$

$\log_2 2^{x-4} = \log_2 5.3$

$x - 4 = \log_2 5.3$

$\boxed{x = \log_2 5.3 + 4}$

9) $5 \cdot 3^{x-5} + 1 = 21$

$-1 \quad -1$

$\frac{5 \cdot 3^{x-5}}{5} = \frac{20}{5}$

$3^{x-5} = 4 \quad \to \quad \log_3 3^{x-5} = \log_3 4$

10) $4e^{-0.3x} - 7 = 13$

$+7 \quad +7$

$\frac{4e^{-.3x}}{4} = \frac{20}{4}$

$e^{-.3x} = 5$

$\ln e^{-.3x} = \ln 5$

$-.3x = \ln 5$

$\boxed{x = \frac{\ln 5}{-.3}}$

11) $3^{x-4} = 2^{3x+1} \Rightarrow$

$\log 3^{x-4} = \log 2^{3x+1}$

$(x-4)\log 3 = (3x+1)\log 2$

$x\log 3 + 4\log 3 = 3x\log 2 + \log 2$

$x\log 3 - 3x\log 2 = \log 2 + 4\log 3$

$\frac{x(\log 3 - \cancel{3}\log 2)}{\log 3 - \cancel{3}\log 2} = \frac{\log 2 + 4\log 3}{\log 3 - 3\log 2}$

$\boxed{x \approx -5.187}$

12) $\overset{3}{\cancel{4}}3 + 3 \cdot 2^x = 17 + 4 \cdot 2^x$

$-17 \quad -3 \cdot 2^x \quad -17 \quad -3 \cdot 2^x$

$26$

$26 = 4 \cdot 2^x - 3 \cdot 2^x$

$26 = 2^x \quad \to \quad \log_2 26 = \log_2 2^x$

$\boxed{\log_2 26 = x}$

**How could you solve an equation by graphing? (Check #12 on your calculator)**

13) How long would you have to invest \$30,000 in an account earning 6% interest compounded continuously so that you have a total of \$40,000?

$A = Pe^{rt}$

$\frac{40000}{30000} = \frac{30000e^{.06(t)}}{30000}$

$\frac{4}{3} = e^{.06t}$

$\ln(\frac{4}{3}) = \ln e^{.06t}$

$\boxed{\frac{\ln(\frac{4}{3})}{.06}} = \frac{\cancel{.06}t}{\cancel{.06}}$ $\boxed{t \approx 4.7947}$

14) Sally invests \$350 in an account earning 5% interest, compounded annually. How long will it take her to earn \$50 in interest?

$n=1$

$A = P(1+\frac{r}{n})^{nt}$

$\frac{400}{350} = \frac{350(1+.05)^t}{350}$

$\frac{40}{35} = 1.05^t$

$\log_{1.05}(\frac{40}{35}) = \log_{1.05} 1.05^t$

$\boxed{\frac{\log(40/35)}{\log(1.05)} = \boxed{t} \approx 2.7369}$

15) You want to have \$1000 in your savings account. Find the amount that you should deposit if the account pays 4% annual interest over a period of 5 years.

$n=1$

$\frac{1000}{1.04^5} = \frac{P(1.04)^5}{1.04^5} \Rightarrow P = \frac{1000}{1.04^5} = \boxed{821.93}$

16) The graph of an exponential function in the form $y = ab^x$ passes through the points $(3, 12)$ and $(7, 192)$. What is the value of $f(-2)$?

$12 = a\cdot b^3 \Rightarrow a = \frac{12}{b^3}$

$192 = a\cdot b^7$

$192 = \frac{12}{b^3}\cdot b^7$

$\frac{192}{12} = \frac{12b^4}{12}$

$16 = b^4$

$\sqrt[4]{16} = b$

$2 = b$

$a = \frac{12}{b^3} = \frac{12}{2^3} = \frac{12}{8} = \frac{3}{2}$

$y = \frac{3}{2}\cdot 2^x$

$f(-2) = \frac{3}{2}\cdot 2^{-2}$

$f(-2) = \frac{3}{2}\cdot\frac{1}{4} = \boxed{\frac{3}{8}}$

## *8.7 Notes: Solving Logarithmic Equations*

Property of Equality for Logarithmic Equations

If $\log_x a = \log_x b$, then $a=b$

**Examples:** Solve each logarithmic equation and check for extraneous solutions

1) $\cancel{\log_5}(4x-7) = \cancel{\log_5}(x+5)$

$$\underset{5}{\ } \qquad \underset{5}{\ }$$

$$4x\cancel{-7} = \cancel{x}+5$$
$$-x\ \cancel{+7} \quad \cancel{-x}\ +7$$
$$3x = 12$$
$$\boxed{x=4}$$

2) $\log_4(5x-1) = 3$

$$4 \qquad 4$$

$$5x-1 = 4^3$$
$$5x-1 = 64$$
$$5x = 65$$
$$\boxed{x=13}$$

3) $\ln(7x-4) = \ln(2x+11)$

$$7x\cancel{-4} = 2\cancel{x}+11$$
$$-2x\ \cancel{+4} \quad \cancel{-2x}\ +4$$
$$5x = 15$$
$$\boxed{x=3}$$

4) $\log_4(x+12) + \log_4 x = 3$

$$4^{\log_4\left[(x+12)(x)\right]} = 4^3$$

$$x^2+12x = 4^3$$
$$x^2+12x-64=0$$
$$(x+16)(x-4)=0$$
$$\boxed{x=\cancel{-16},4}$$

can't do log(−)
not valid #

5) $\ln(3x-2) = 2.8$

$e \qquad e$

$3x - 2 = e^{2.8}$

$\cancel{3}x = \frac{e^{2.8} + 2}{\cancel{3}}$

$3$

$\boxed{x \approx 6.148}$

6) $\log_6(x^2 - 16x) = 2$

$6 \qquad 6$

$x^2 - 16x = 36$

$x^2 - 16x - 36 = 0$

$(x - 18)(x + 2) = 0$

$\boxed{x = 18, -2}$

7) $\log x + \log(x + 5) = \log 24$

$10^{\log(x(x+5))} = 10^{\log 24}$

$x^2 + 5x = 24$

$x^2 + 5x - 24 = 0$

$(x + 8)(x - 3) = 0$

$\boxed{x = \cancel{-8}, 3}$

8) $\log x = 1 - \log(x - 3)$

$1 = \log x + \log(x+3)$

$10^1 = 10^{\log(x(x+3))}$

$10 = x^2 + 3x$

$0 = x^2 + 3x - 10$

$0 = (x + 5)(x - 2)$

$\boxed{x = \cancel{-5}, 2}$

9) $\log_3(x + 4) + \log_3 x = \log_3(5x) + \log_3(x - 8)$

$3^{\log_3((x+4)(x))} = 3^{\log_3((5x)(x-8))}$

$x^2 + 4x = 5x^2 - 40x$

$0 = 4x^2 - 44x$

$4x(x - 11) = 0$

$\boxed{x = 0, 11}$

10) The population of deer in a forest preserve can be modeled by the equation

P = 50 + 200 ln (*t* + 1), where *t* is the time in years from the present. In how many years will be deer population reach 500?

$500 = 50 + 200\ln(t+1)$

$\frac{450}{200} = \frac{\cancel{200}\ln(t+1)}{\cancel{200}}$

$\frac{45}{20} = \ln(t+1)$

$e^{45/20} = e^{\ln(t+1)}$

$e^{45/20} = t + 1$

$\boxed{t = e^{45/20} - 1}$

$\boxed{t \approx 8.488}$

11) One of the strongest earthquakes in recent history occurred in Mexico City in 1985 and measured 8.1 on the Richter scale. Find the amount of energy, E, released by this earthquake. Use the formula: $M = \frac{2}{3}\log\frac{E}{10^{11.8}}$

$$8.1 = \frac{2}{3}\log\left(\frac{E}{10^{11.8}}\right)$$

$$10^{\frac{3}{2}(8.1)} = 10^{\log\left(\frac{E}{10^{11.8}}\right)}$$

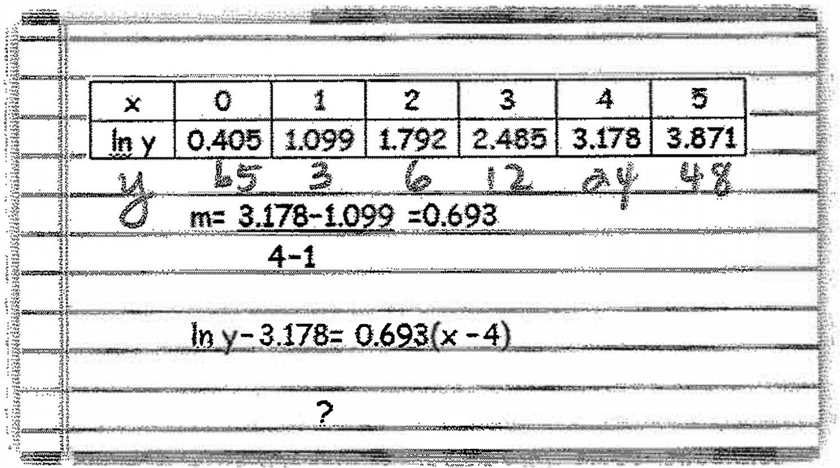
$$10^{\frac{3\cdot 8.1}{2}} = \frac{E}{10^{11.8}} \Rightarrow E = 10^{11.8}\cdot 10^{\frac{3(8.1)}{2}} \approx 8.9125$$

12)

A student is trying to find an exponential function in the form $y = ab^x$ that models a set of data given by the teacher. The student does not write down the original problem and only partially completes the work before going to a tutor for help. The student's work is shown below. Complete the work to find the exponential function.

$e^{.405} = 1.5$

$e^{1.099} = 3$



| x | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| ln y | 0.405 | 1.099 | 1.792 | 2.485 | 3.178 | 3.871 |
| y | 1.5 | 3 | 6 | 12 | 24 | 48 |

$m = \frac{3.178-1.099}{4-1} = 0.693$

$\ln y - 3.178 = 0.693(x-4)$

?

**A.** $y = 1.5(0.4)^x$

[✓ **C.**] $y = 1.5(2)^x$

**B.** $y = 0.4(1.5)^x$

**D.** $y = 2(1.5)^x$

$(0, 1.5)$ $\quad 1.5 = a\cdot b^0 \Rightarrow a = 1.5$

$(1, 3)$ $\quad 3 = 1.5b^1 \Rightarrow b = \frac{3}{1.5} = 2$

## *8.8 Notes: Modeling Logarithmic Equations*

All living things contain carbon-14. When a plant or animal dies, the carbon-14 in it begins to decay, or change to another substance. The process is very slow. It takes 5,730 years for just half of it to decay, then another 5,730 years for half the remaining amount to decay, and so on. By using a method similar to the one in this lesson, scientists can determine the amount of carbon-14 in a fossil and can use that amount to determine it age.

Radioactive substances decay to other substances over time. The half-life of a radioactive substance is the time it takes for one-half of the substance to decay. ***How can you determine the length of time it takes a given radioactive substance to decay to a specified percent?***

The isotope bismuth-210 has a half-life of 5 days. Complete the table showing the decay of a sample of bismuth-210.

| Number of half-lives | Number of Days ($t$) | Percent of isotope remaining ($p$) |
|---|---|---|
| 0 | 0 | 100 |
| 1 | 5 | 50 |
| 2 | 10 | 25 |
| 3 | 15 | 12.5 |
| 4 | 20 | 6.25 |

A. Write an exponential decay function for bismuth-210.
   a. What is the decay rate as a fraction: $\Omega = 1/2$
   b. Write an expression for the number of half-lives in $t$ days: $t/5$
   c. Write the exponential decay function that models this situation. The function $p(t)$ should give the percent of the isotope remaining after $t$ days.

$$p(t) = 100\left(\tfrac{1}{2}\right)^{t/5}$$

*Check that your model is correct by plugging in a $t$ value from the table above.

$$p(20) = 100(1/2)^{20/5} = 6.25 \checkmark$$

d. Every 5 days, the amount of bismuth-210 decreases by 50%. By what percent does the amount of bismuth-210 decrease *each day*? Explain.

$$\left(\frac{1}{2}\right)^{t/5} = \left(\left(\frac{1}{2}\right)^{1/5}\right)^{t} = (.871)^{t}$$

B. Convert the exponential decay function to a logarithmic function.

a. Write the inverse of the decay function by solving for *t*.

$$p = 100(.5)^{t/5}$$

$$\frac{p}{100} = .5^{t/5}$$

$$\log_{.5}\left(\frac{p}{100}\right) = \log_{.5} .5^{t/5}$$

$$\log_{.5}\left(\frac{p}{100}\right) = \frac{t}{5} \Rightarrow \boxed{5\cdot\log_{(.5)}\left(\frac{p}{100}\right) = t}$$

* Check that the logarithmic function is correct by substituting 50 for *p*. What is the resulting value of *t*? Compare it to the table above.

$$5\frac{\cancel{\log}(50/100)}{\cancel{\log}(1/2)} = 5 \quad ✓$$

C. Which equation would you use to determine how many days it would take to have 10% of the bismuth-210 remaining? How many days would it take?

$$t = \frac{5\log(10/100)}{\log(1/2)} \approx 16.610 \text{ days}$$

D. Which equation would you use it you wanted to know what percent of bismuth-210 is remaining after 40 days? What is the percent?

$$a = 100(.5)^{t/5} = 100(.5)^{40/5} \approx .391$$

E. Will there ever be 0% of the bismuth-210 remaining? Explain your reasoning.

no – there's a horizontal asymptote at $a=0$