7.1: Properties of Exponents

Example 4: Betelgeuse is one of the stars found in the constellation Orion. Its radius is about 1500 times the radius of the sun. How many times as great as the sun's volume is Betelgeuse's volume? Use $V = \frac{4}{3}\pi r^3$.

n^{th} root:

Index of a radical:

Rational Exponents: Let *a* be a real number, and let *m* and *n* be positive integers with n > 1.

$a^{\frac{m}{n}} =$

Example: $9^{\frac{1}{2}} =$

$a^{-\frac{m}{n}} =$

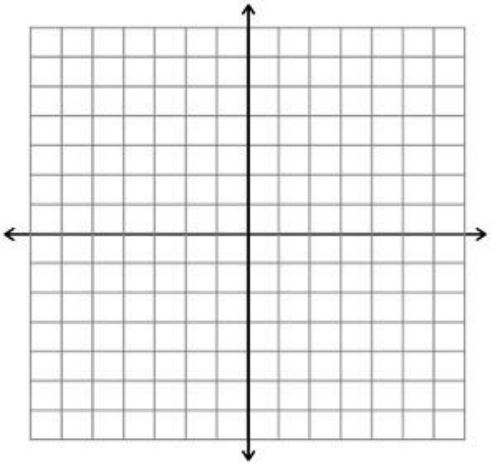
Example: $16^{-\frac{1}{2}} =$

Example 13: The population P of a certain animal species after *t* months can be modeled by $P = C(1.21)^{\frac{t}{3}}$, where *C* is the initial population. Find the population after 19 months if the initial population was 75.

Example 14: A study determined that the weight *w* (in grams) of a coral cod near Palawan Island, Philippines, can be approximated using the model $w = 0.0167l^3$, where *l* is the coral cod's length (in centimeters). Estimate the weight of a coral cod with a length of 13 cm.

7.3: Key Features of Radical Functions

Find the key features of the parent radical function: $y = \sqrt{x}$



Endpoint:

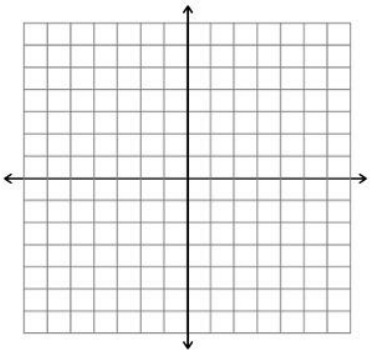
Domain:

Range:

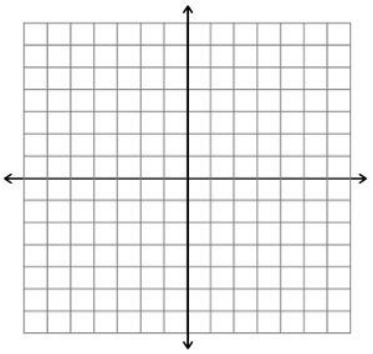
End Behavior:

Examples: For each radical function, describe the transformation from the parent function $y = \sqrt{x}$, identify the domain and range, and then sketch the graph.

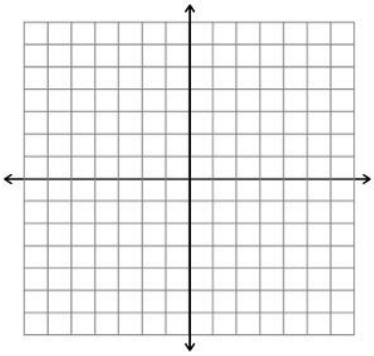
1) $2\sqrt{x}$



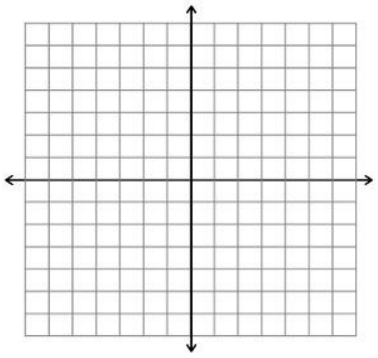
2) $y = \sqrt{x-2}$



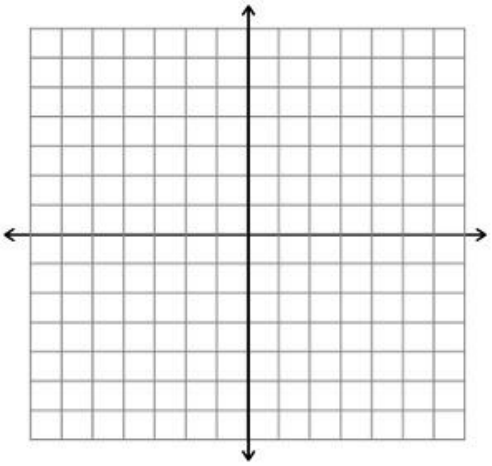
3) $y = -\sqrt{x} + 3$



4) $y = \frac{1}{5}\sqrt{x+3} - 4$



Find the key features of the parent cube root function: $y = \sqrt[3]{x}$



"Center"

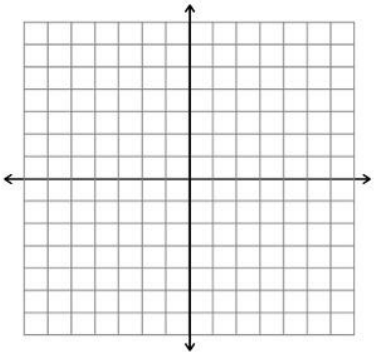
Domain:

Range:

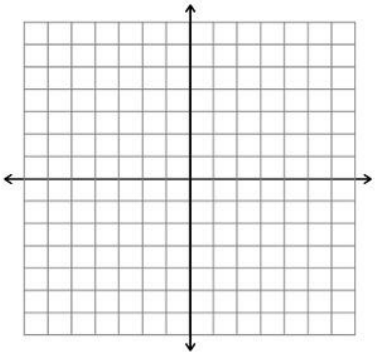
End Behavior:

Examples: Describe the transformation from the parent function, identify the domain and range, and then sketch a graph of the following.

5) $y = -2\sqrt[3]{x+3} - 2$



6) $y = -\frac{1}{2}\sqrt[3]{x}$



7.4 Solving Equations with Exponents and Radicals

Example 10: The population P of a certain animal species after *t* months can be modeled by $P = C(1.04)^{\frac{t}{3}}$, where *C* is the initial population. Find the initial population if, after 9 months, the total population was 3462.

Example 11: A study determined that the weight *w* (in grams) of a coral cod near Palawan Island, Philippines, can be approximated using the model $w = 0.0167l^3$, where *l* is the coral cod's length (in centimeters). Estimate the length of a coral cod with a weight of 9280 grams.

7.5: Solving Inequalities, Compositions, and Inverses

Exploration: Work with a partner to perform the indicated operations given that

$$\boldsymbol{f(x) = 8x - 12,\ \ g(x) = 3x^2,\ \ \text{and } h(x) = 2}$$

1) $f(x) + h(x)$ 2) $g(x) - f(x)$ 3) $g(x) \cdot f(x)$

4) $\frac{f(x)}{h(x)}$ 5) $f(g(x))$ 6) $g(g(x))$

Compositions: The domain of a composition of functions *f* and *g* consists of the *x*-values that are in the domains of _____. Additionally, the domain of a quotient does not include *x*-values for which the denominator = 0.

Inverse functions: A function that "reverses" another function.

Three ways to tell if relations are inverses:

- If $f(g(x)) = x$ and $g(f(x)) = x$ then $f(x)$ and $g(x)$ are inverses.
- If two functions are inverses, their graph are reflections over $y = x$.
- A table of values will switch the input and output values.

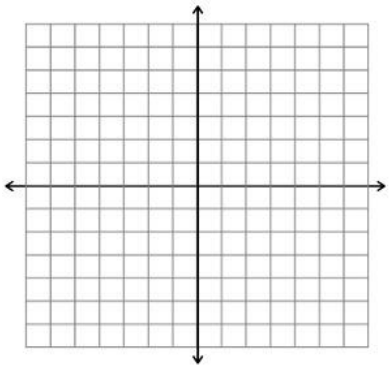
To find the inverse of a function:

Notation: Given $f(x)$ is a function, then $f^{-1}(x)$ represents the **inverse function**

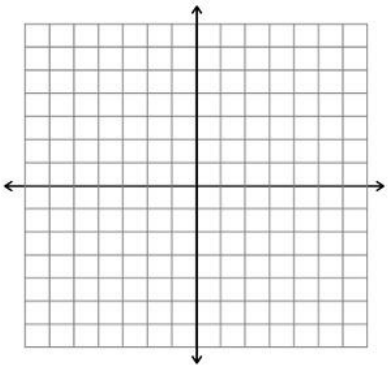
Note: The inverse of a function is NOT always a function.

Examples: Find the inverse of each function. Graph the function and its inverse on the same coordinate grid. Is the inverse also a function?

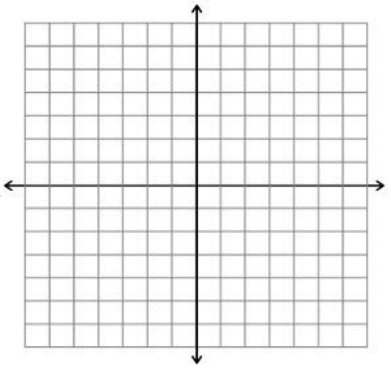
1) $y = -4x + 8$



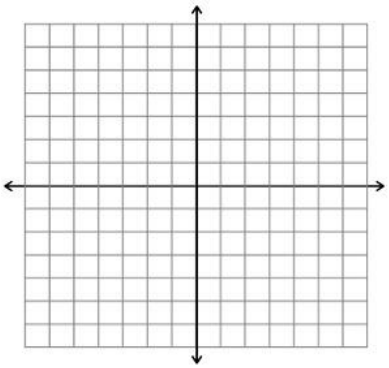
2) $f(x) = 4x^2 + 3$



3) $y = -8x^3$



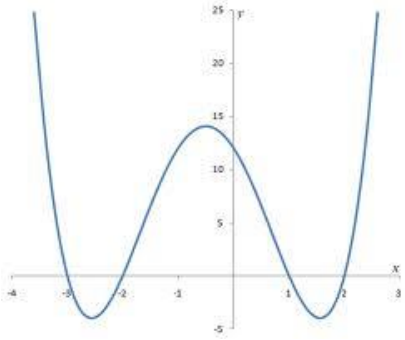
4) $f(x) = -\frac{1}{4}x + 1$



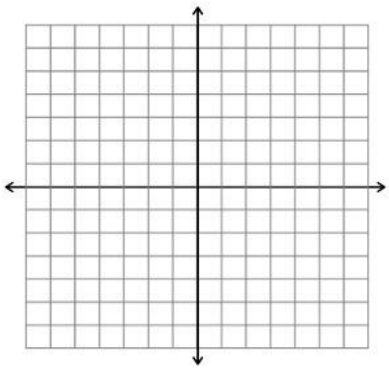
Example 6: Determine if the following relations are inverses: $f(x) = 27x^3$ and $g(x) = \frac{1}{3}x^{\frac{1}{3}}$. Explain your reasoning.

Example 7: Determine if the following relations are inverses: $f(x) = 624x^4$ and $g(x) = \frac{1}{5}x^{-4}$. Explain your reasoning.

Example 8: The graph of $b(x)$ is given. Is b^{-1} a function?

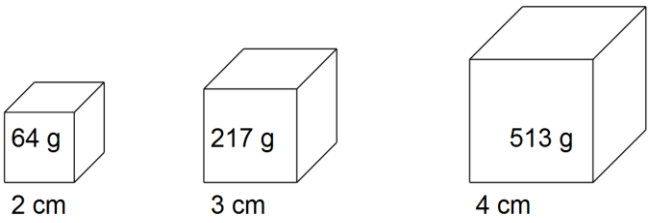


Horizontal Line Test:



7.6: Modeling with Rational Functions

The side lengths and masses of three stainless steel cubes are given below. Use this information to write a model for the radius of a spherical stainless steel ball bearing as a function of its mass. What is the radius of a stainless steel ball bearing with a mass of 100 grams?



I: Find the density of stainless steel.

The density of a material is a measure of its mass per unit of volume.

To find the density of each stainless steel cube, divide its mass in grams by its volume in cubic centimeters. Round each density to the nearest whole number.

Cube 1:

Cube 2:

Cube 3:

a. What do you notice about the relationship among the densities you calculated?

b. How can you find the mass in grams of a stainless steel cube if you know the edge length in centimeters?

II: Write a model for the mass of a stainless steel sphere as a function of its radius.

a. What is the formula for the volume of a sphere?

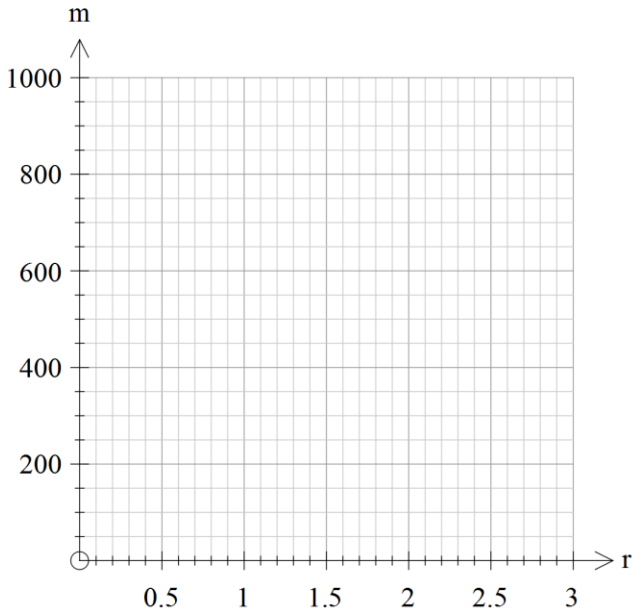
b. Write a function $m(r)$ for the mass in grams of a stainless steel ball bearing with a radius of r centimeters.

c. What are the reasonable domain and range of $m(r)$?

III: Graph and write the inverse function *r*(*m*)

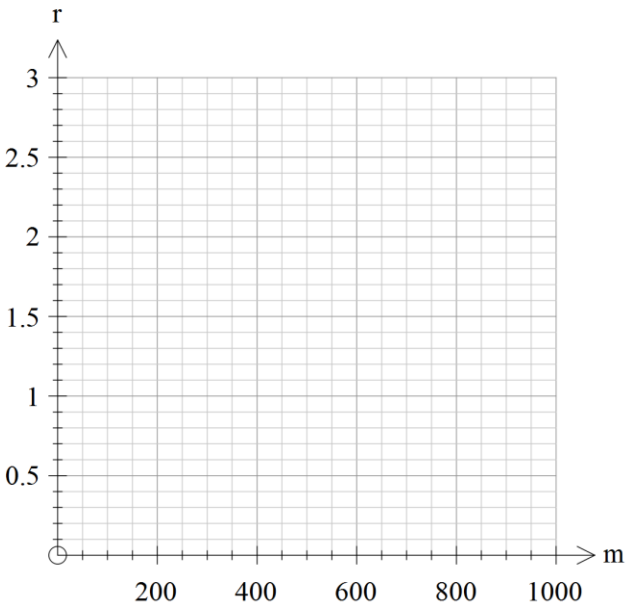
a. Complete the table of values and use it to graph $m(r)$ for $r \geq 0$. Round the function values to the nearest whole number.

r	$m(r)$
0	
0.5	
1	
1.5	
2	
2.5	
3	



b. Reverse the input and output for each of the ordered pairs in part a above. Record the coordinates of the points below.

c. Plot the points from part b above and draw a smooth curve through them to graph the inverse function, $r(m)$.



d. Find the equation for the inverse function, $r(m)$.

e. What type of function is $m(r)$? What type of function is its inverse $r(m)$?

f. The function $m(r)$ models the mass in grams of a stainless steel ball bearing as a function of its radius in centimeters. What does $r(m)$ model?

IV: Find the radius of a stainless steel ball bearing with a mass of 100 grams.