7.1: Properties of Exponents

Do the first part of the reading (Pg 1-3)! There are questions on the W drive if time.

Examples: Simplify the following expressions.

a) $\frac{(-2x^5y^3z^{-4})^4}{(4x^2y^7z^2)^3}$

$$\frac{\cancel{2^4}^{\,16}\, x^{20}\, y^{12}\, z^{-16}}{\cancel{4^3}^{\,64}\, x^6\, y^{21}\, z^6}$$

$$= \frac{\cancel{16}^{\,1}\, x^{14}}{\cancel{64}_{\,4}\, y^9\, z^6\, z^{16}}$$

$$= \boxed{\frac{x^{14}}{4y^9z^{22}}}$$

b) $(8a^3b^{-4})^{-2}$

$$8^{-2}a^{-6}b^8 = \frac{b^8}{8^2a^6}$$

$$= \boxed{\frac{b^8}{64a^6}}$$

c) $(14y^2z^{-5})(-3yz^2)^3$ $= 14y^2z^{-5}(-27)y^3z^6$

add $2+3=5$

add $6-5=1$

$$\boxed{378y^5z}$$

Example 4: Betelgeuse is one of the stars found in the constellation Orion. Its radius is about 1500 times the radius of the sun. How many times as great as the sun's volume is Betelgeuse's volume? Use $V=\frac{4}{3}\pi r^3$.

$$V_B = \frac{4}{3}\pi(1500r_s)^3 = \frac{4\cdot 1500^3}{3}\pi r_s^3$$

$$= 4{,}500{,}000{,}000\pi r_s^3$$

$4{,}500{,}000{,}000\pi$ times as great

Nth root: $\sqrt[n]{x}$ or $x^{1/n}$

Index of a radical: in this case, n is the index

Rational Exponents: Let *a* be a real number, and let *m* and *n* be positive integers with n > 1.

$a^{m/n} = \sqrt[n]{a^m}$ **Example:** $9^{1/2} = \sqrt[2]{9^1} = \sqrt{9} = \boxed{3}$

$a^{-m/n} = \sqrt[n]{a^{-m}} = \sqrt[n]{\frac{1}{a^m}}$ or $\frac{1}{\sqrt[n]{a^m}}$ **Example:** $16^{-1/2} = \sqrt[2]{16^{-1}} = \sqrt{\frac{1}{16}} = \frac{1}{\sqrt{16}} = \boxed{\frac{1}{4}}$

Examples: Simplify each expression. Assume all variables are positive values.

1) $\sqrt[3]{27}$

$\sqrt[3]{3\cdot 3\cdot 3}$

3

instead of taking out pair, you take out groups of 3.

2) $\sqrt[4]{32x^8}$

$\sqrt[4]{2\cdot 2\cdot 2\cdot 2\cdot 2\, xxxxxxxx}$

$= 2\cdot x\cdot x\sqrt[4]{2}$

$= \boxed{2x^2\sqrt[4]{2}}$

3) $\sqrt[3]{-64a^5}$

$\sqrt[3]{-2^6a^5}$

$-2^2a\sqrt[3]{a^2}$

← can take 2 sets of 3.

← can take 1 set of 3 w/ 2 left in $\sqrt[3]{}$

if index is odd, − comes out normally.

4) $8^{1/3}$

$(2^3)^{1/3}$

$2^1 = \boxed{2}$

or

$\sqrt[3]{8} = \sqrt[3]{2\cdot 2\cdot 2} = \boxed{2}$

5) $(\sqrt[4]{16})^5 = \left(\sqrt[4]{2^4}\right)^5$

2^4

$2^5 = \boxed{32}$

6) $8^{-4/3} =$ $(2^3)^{-4/3}$

2^3

$= 2^{-4} = \boxed{\frac{1}{16}}$

7) $(9h^6)^{3/2}$

$(3^2h^6)^{3/2}$

$3^3h^9 = \boxed{27h^9}$

8) $(\sqrt[3]{-54x^{11}})^4 = \sqrt[3]{(3^3\cdot 2x^{11})^4}$

$54 = 3\cdot 3\cdot 3\cdot 2$

$= \sqrt[3]{3^{12}\cdot 2^4x^{44}}$

$= 3^4\cdot 2x^{14}\sqrt[3]{2x^2}$

$= \boxed{162x^{14}\sqrt[3]{2x^2}}$

Example 9: The population P of a certain animal species after t months can be modeled by $P = C(1.21)^{\frac{t}{3}}$, where C is the initial population. Find the population after 19 months if the initial population was 75.

$P = 75(1.21)^{19/3} = \boxed{250.824}$

Example 10: A study determined that the weight w (in grams) of a coral cod near Palawan Island, Philippines, can be approximated using the model $w = 0.0167l^3$, where l is the coral cod's length (in centimeters). Estimate the weight of a coral cod with a length of 13 cm.

$w = .0167(13)^3 = \boxed{36.690}$

7.2: Simplifying Rational Exponents and Radicals

Do second part of reading (page 4)!

Examples: Simplify the following expressions. Assume all variables are positive values.

1) $9^{\frac{1}{2}} \cdot 9^{\frac{3}{4}}$

$(3^2)^{1/2} \cdot (3^2)^{3/4}$

$3^1 \cdot 3^{3/2}$

$3^{1+3/2} = \boxed{3^{5/2}}$

$= \sqrt[5]{3^2}$ or

$\boxed{\sqrt[5]{9}}$

2) $\frac{3^{5/6}}{3^{1/3}}$

$3^{5/6 - 1/3}$

$3^{5/6 - 2/6}$

$3^{3/6} = \boxed{3^{1/2}}$

$= \boxed{\sqrt{3}}$

3) $\left(\frac{16^{2/3}}{4^{2/3}}\right)^4 =$

$\left(\frac{(2^4)^{2/3}}{(2^2)^{2/3}}\right)^4$

$\left(\frac{2^{8/3}}{2^{4/3}}\right)^4 = \left(2^{4/3}\right)^4$

$= \boxed{2^{16/3}}$ or

$= 2^{5\,1/3} = 2^5 2^{1/3} = \boxed{32\sqrt[3]{2}}$

4) $\sqrt[5]{27} \cdot \sqrt[5]{9}$

$3^3 \quad 3^2$

$\sqrt[5]{3^5} = \boxed{3}$

or

$(3^3)^{1/5} \cdot (3^2)^{1/5}$

$(3^5)^{1/5} = 3$

5) $\dfrac{\sqrt[3]{192}}{\sqrt[3]{3}}$

$\sqrt[3]{\dfrac{\cancel{3} \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 2}{\cancel{3}}}$

$\boxed{3\sqrt[3]{2}}$

6) $\sqrt[6]{x^7} \cdot \sqrt[3]{x^2}$

$x^{7/6} \cdot x^{2/3}$

$= x^{\frac{7}{6} + \frac{4}{6}} = \boxed{x^{11/6}}$

or

$\sqrt[6]{x^{11}} = \boxed{x\sqrt[6]{x^5}}$

or

x

7) $\sqrt[4]{\dfrac{a^2}{b^8}}$

$\dfrac{a^{2/4}}{b^{8/4}} = \boxed{\dfrac{a^{1/2}}{b^2}}$

or

$= \boxed{\dfrac{\sqrt{a}}{b^2}}$

8) $10\boxed{\sqrt[5]{y}} - 6\boxed{\sqrt[5]{y}}$

like terms

$10x - 6x = 4x$

$= \boxed{4\sqrt[5]{y}}$

or

$\boxed{4y^{1/5}}$

can write it either way

9) $3\boxed{a^2b^{1/4}} + 4\boxed{a^2b^{1/4}}$

like terms

$\boxed{7a^2b^{1/4}}$

or

$\boxed{7a^2\sqrt[4]{b}}$

10) $7\sqrt[3]{2a^5} - a\sqrt[3]{16a^2}$

2^4

$7a\sqrt[3]{2a^2} - a \cdot 2\sqrt[3]{2a^2}$

$7a\sqrt[3]{2a^2} - 2a\sqrt[3]{2a^2}$

$\boxed{5a\sqrt[3]{2a^2}}$

11) $\dfrac{2^{\frac{2}{3}} \cdot 8^{\frac{2}{3}}}{4^{\frac{4}{3}}}$

change 8 to 2^3

change 4 to 2^2

change bases so they all have the same base of 2.

$\dfrac{2^{2/3} \cdot (2^3)^{2/3}}{(2^2)^{4/3}} = \dfrac{2^{2/3} \cdot 2^{6/3}}{2^{8/3}}$

$= \dfrac{2^{8/3}}{2^{8/3}} = 2^0 = \boxed{1}$

12) $\dfrac{\sqrt[3]{c^{12}} \cdot \sqrt[3]{c^4}}{\sqrt[3]{c^{10}}}$

$\sqrt[3]{\dfrac{c^{12} \cdot c^4}{c^{10}}} = \sqrt[3]{c^6}$

$= \boxed{c^2}$

or

$\dfrac{c^{12/3} \cdot c^{4/3}}{c^{10/3}}$ ($12/3 = 4$)

$= c^{4 + 4/3 - 10/3} = c^2$

Rationalizing with Roots:

Simplify the following expressions by rationalizing:

13) $\dfrac{3}{\sqrt[3]{25}} = \dfrac{3}{\sqrt[3]{5^2}}$

$\dfrac{3 \cdot \sqrt[3]{5}}{\sqrt[3]{5^2} \cdot \sqrt[3]{5}} = \dfrac{3\sqrt[3]{5}}{5}$

14) $\dfrac{9^{1/4}}{5 \cdot 8^{1/4}}$ $\dfrac{(3^2)^{1/4}}{5(2^3)^{1/4}}$

$= \dfrac{3^{1/2} \cdot 2^{1/4}}{5 \cdot 2^{3/4} \cdot 2^{1/4}} = \dfrac{3^{1/2} \cdot 2^{1/4}}{5 \cdot 2}$

$= \boxed{\dfrac{\sqrt[2]{3} \cdot \sqrt[4]{2}}{10}}$

15) $\dfrac{-2x}{\sqrt[5]{16x^3}} = \dfrac{-2x}{\quad}$

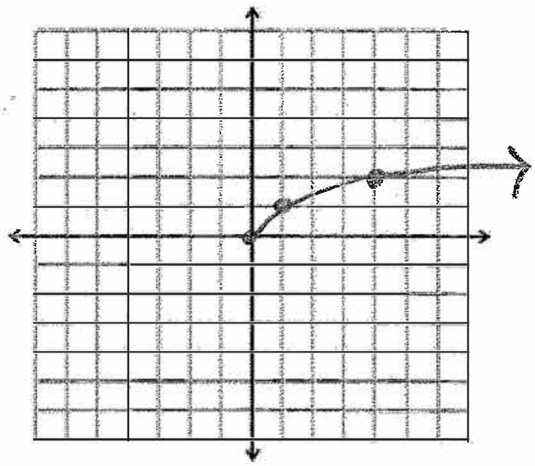
$\dfrac{-2x \cdot \sqrt[5]{2x^2}}{\sqrt[5]{2^4x^3} \cdot \sqrt[5]{2x^2}} = \dfrac{\quad}{\sqrt[5]{2^5x^5}}$

$= \boxed{\dfrac{-2x\sqrt[5]{2x}}{2x}}$

16) Create an expression that would simplify to $2m\sqrt[3]{3}$. Include two different variables.

7.3: Key Features of Radical Functions

Find the key features of the parent radical function: $y = \sqrt{x}$



Endpoint: $(0,0)$ (h,k)!

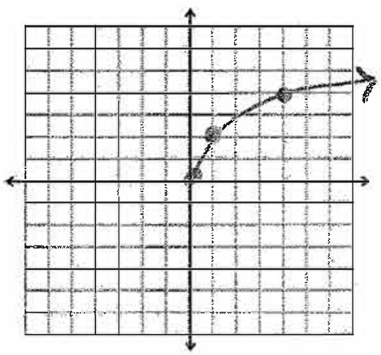
Domain: $\leftrightarrow$ $[0,\infty)$

Range: $\updownarrow$ $[0,\infty)$

End Behavior: as $x \to \infty$, $y \to \infty$
(only one end so no behavior on left side)

Examples: For each radical function, describe the transformation from the parent function $y = \sqrt{x}$, identify the domain and range, and then sketch the graph.

1) $y = 2\sqrt{x}$



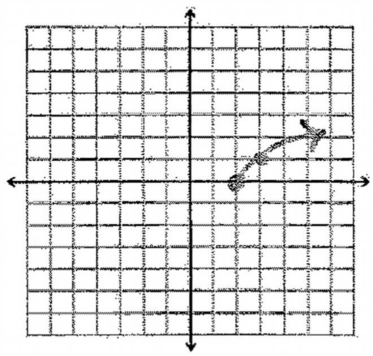
-stretched by 2

D: $\{x \mid x \geq 0\}$ or $[0,\infty)$

R: $\{y \mid y \geq 0\}$ or $[0,\infty)$

2) $y = \sqrt{x-2}$

$(2,0)$

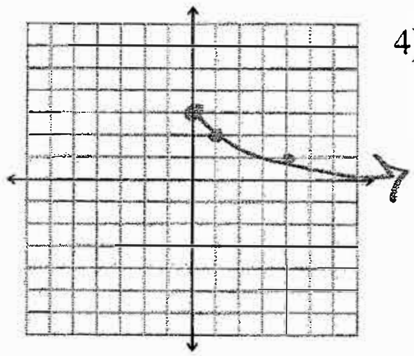


-right two

D: $\{x \mid x \geq 2\}$ or $[2,\infty)$

R: $\{y \mid x \geq 0\}$ or $[0,\infty)$

3) $y = -\sqrt{x} + 3$



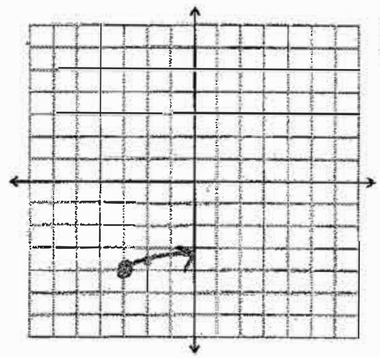
- ↑3
- reflected

D: $\{x \mid x \geq 0\}$ or $[0, \infty)$

R: $\{y \mid y \leq 3\}$ or $(-\infty, 3]$

4) $y = \frac{1}{5}\sqrt{x+3} - 4$

$(-3, -4)$



- ← 3
- ↓ 4
- compressed by 1/5

D: $[-3, \infty)$

R: $[-4, \infty)$

Find the key features of the parent cube root function: $y = \sqrt[3]{x}$

"Center" (h, k) at $(0, 0)$

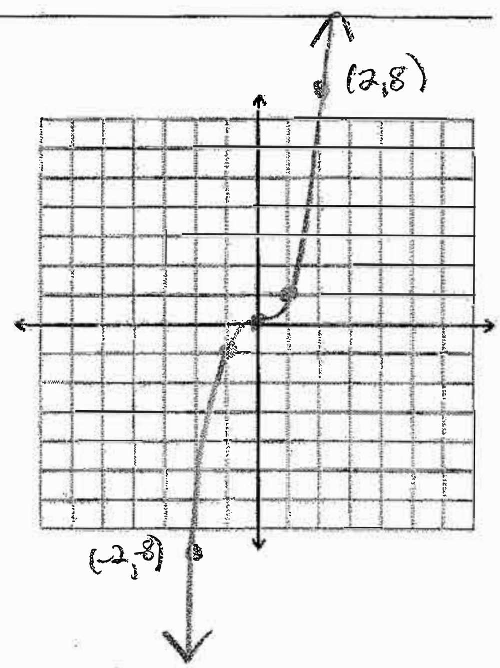
Domain: $\{x \mid x \in \mathbb{R}\}$ or $(-\infty, \infty)$

Range: $\{y \mid y \in \mathbb{R}\}$ or $(-\infty, \infty)$

End Behavior:

as $x \to \infty$, $y \to \infty$

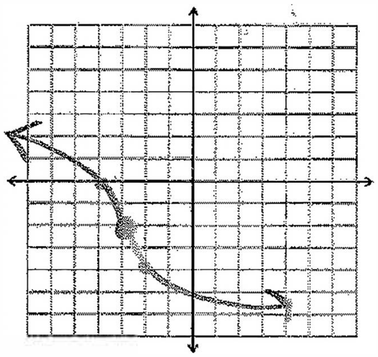
as $x \to -\infty$, $y \to -\infty$



Examples: Describe the transformation from the parent function, identify the domain and range, and then sketch a graph of the following.

5) $y = -2\sqrt[3]{x+3} - 2$

$(-3, -2)$



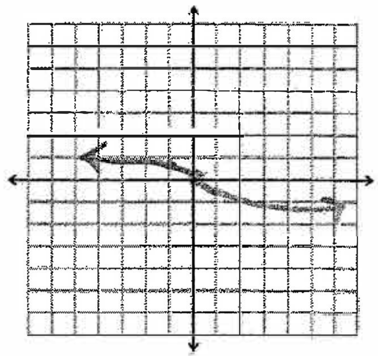
- reflected
- stretched by 2
- $\leftarrow 3$
- $\downarrow 2$

D: $\{x \mid x \in \mathbb{R}\}$ or $(-\infty, \infty)$

R: $\{y \mid y \in \mathbb{R}\}$ or $(-\infty, \infty)$

6) $y = \frac{-1}{2}\sqrt[3]{x}$

- reflected
- compressed by $1/2$



D: $(-\infty, \infty)$

R: $(-\infty, \infty)$

7) Create a function that has end behavior *as* $x \to \infty, f(x) \to -\infty$ and a range of $(-\infty, 6.5]$.

$y = -\sqrt{x} + 6.5$

7.4 Solving Equations with Exponents and Radicals

Step 1: isolate $\sqrt[n]{\ }$ or exponent

Step 2: $(\sqrt{x})^2$ or $\sqrt{x^2}$ or $(\sqrt[3]{x})^3$ or $\sqrt[5]{x^5}$

Step 3: isolate variable

Examples: Solve the following equations. Check for extraneous solutions.

1) $\left(\sqrt{x+6}\right)^2=(3)^2$

$x+6=9$
$-6 \quad -6$

$\boxed{x=3}$

check

$\sqrt{3+6}=3$

$\sqrt{9}=3$ ✓

2) $\sqrt[3]{x-5}+1=-1$
$-1 \quad -1$

$\sqrt[3]{x-5}=-2$

not ok to have (−) with $\sqrt{\ }$ or $\sqrt[4]{\ }$ or $\sqrt[6]{\ }$, etc., but ok w/ odd indices

$\left(\sqrt[3]{x-5}\right)^3=(-2)^3$

$x-5=-8$
$+5 \quad +5$

$\boxed{x=-3}$

check

$\sqrt[3]{-3-5}+1=-1$

$\sqrt[3]{-8}+1=-1$

$-2+1=-1$ ✓

3) $\frac{2x^6}{2}=\frac{1458}{2}$

$x^6=729$

$\sqrt[6]{x^6}=\pm\sqrt[6]{729}$

$\boxed{x=\pm 3}$

check

$2(-3)^6=1458$ ✓

$2(3)^6=1458$ ✓

If you take a $\sqrt{\ }$, $\sqrt[4]{\ }$, $\sqrt[6]{\ }$, $\sqrt[8]{\ }$, etc., you need to do ±.

4) $(x+4)^3=12$

$\sqrt[3]{(x+4)^3}=\sqrt[3]{12}$

$x+4=\sqrt[3]{12}$ simplify if possible

$\boxed{x=-4+\sqrt[3]{12}}$

check

$(x+4)^3=12$

$(\cancel{-4}+\sqrt[3]{12}\cancel{+4})^3=12$

$\left(\sqrt[3]{12}\right)^3=12$ ✓

5) $(y-3)^4 = 625$

$\sqrt[4]{(y-3)^4} = \pm\sqrt[4]{625}$

$y-3 = \pm 5$

$y = 3 \pm 5 = \boxed{8, -2}$

check 8

$(8-3)^4 = 625?$

$5^4 = 625$ ✓

check −2

$(-2-3)^4 = 625?$

$(-5)^4 = 625$ ✓

6) $(3x+4)^{\frac{2}{3}} = 16$

$\left[(3x+4)^{2/3}\right]^{3/2} = (16)^{3/2}$

$3x+4 = (2^4)^{3/2}$

$3x+4 = \cancel{2^6}\ 64$

$3x = 60$

$\boxed{x = 20}$

check

$(3(20)+4)^{2/3} = 16$

used calc this time :)

7) $2\sqrt{6x-7} + 14 = 4$

$-14 \quad -14$

$\frac{2\sqrt{6x-7}}{2} = \frac{-10}{2}$

$\sqrt{6x-7} = -5$

$\sqrt{\ } = -$ so will be no solution

$(\sqrt{6x-7})^2 = (-5)^2$

$6x-7 = 25$

$+7 \quad +7$

$\frac{6x}{6} = \frac{32}{6}$

$\boxed{\cancel{x = \frac{16}{3}}}$

$\boxed{\text{no sol}}$

check

$2\sqrt{6 \cdot \frac{16}{3} - 7} + 14 = 4?$

$2\sqrt{32-7} + 14 = 4?$

$2\sqrt{25} + 14 = 4?$

$2(5) + 14 = 4?$ no!

8) $x-2 = \sqrt{x+10}$

$(x-2)^2 = (\sqrt{x+10})^2$

$x^2 - 4x + 4 = x + 10$

$-x \quad -10 \quad -x \quad -10$

$x^2 - 5x - 6 = 0$

$(x-6)(x+1) = 0$

$\boxed{x = 6, \cancel{-1}}$

check 6

$6-2 = \sqrt{6+10}?$

$4 = \sqrt{16}?$ ✓

check −1

$-1-2 = \sqrt{-1+10}$

$-3 = \sqrt{9}$

no!

9) $\left(\sqrt{x+6}+2\right)^2=\left(\sqrt{10-3x}\right)^2$

$(\sqrt{x+6}+2)(\sqrt{x+6}+2)=10-3x$

$(\sqrt{x+6})^2+2\sqrt{x+6}+2\sqrt{x+6}+4=10-3x$

$x+\cancel{6}+4\sqrt{x+6}\cancel{+4}=\cancel{10}-3x$

$-x \qquad -x$

$4\sqrt{x+6}=-4x$

$\sqrt{x+6}=-x$

$(\sqrt{x+6})^2=(-x)^2$

$x+6=x^2$

$x^2-x-6=0$

$(x-3)(x+2)=0$

$\boxed{x=\cancel{3},-2}$

Check 3

$\sqrt{3+6}+2\overset{?}{=}\sqrt{10-3(3)}$

$\sqrt{9}+2=\sqrt{1}$ no!

check -2

$\sqrt{-2+6}+2=\sqrt{10-3(-2)}$

$\sqrt{4}+2=\sqrt{16}$ ✓

(~~If time show~~ them how to do #9 in a calculator by creating a system of equations $f(x)=\sqrt{x+6}+2$ and $g(x)=\sqrt{10-3x}$ and then graphing to see where they intersect.)

10) The population P of a certain animal species after *t* months can be modeled by $P=C(1.04)^{\frac{t}{3}}$, where C is the initial population. Find the initial population if, after 9 months, the total population was 3462.

P t

$3462=C(1.04)^{9/3}$

$C=\dfrac{3462}{1.04^3}=3077.71$

Example 11: A study determined that the weight *w* (in grams) of a coral cod near Palawan Island, Philippines, can be approximated using the model $w=0.0167l^3$, where l is the coral cod's length (in centimeters). Estimate the length of a coral cod with a weight of 9280 grams.

$\dfrac{9280}{.0167}=\dfrac{\cancel{.0167}\,l^3}{\cancel{.0167}}$

$\dfrac{9280}{.0167}=l^3$

$\sqrt[3]{\dfrac{9280}{.0167}}=\sqrt[3]{l^3}$

$\boxed{82.21=l}$

7.5: Compositions and Inverses

Exploration: Work with a partner to perform the indicated operations given that
$$f(x) = 8x - 12, \quad g(x) = 3x^2, \quad \text{and } h(x) = 2$$

1) $f(x) + h(x)$

$(8x-12) + (2)$

$\boxed{8x-10}$

2) $g(x) - f(x)$

$3x^2 - (8x-12)$

$\boxed{3x^2 - 8x + 12}$

3) $g(x) \cdot f(x)$

$3x^2(8x-12)$

$\boxed{24x^3 - 36x^2}$

4) $\frac{f(x)}{h(x)}$

$\frac{8x-12}{2} = \boxed{4x-6}$

5) $f(g(x))$

$8(3x^2) - 12$

$\boxed{24x^2 - 12}$

6) $g(g(x))$

$3(3x^2)^2$

$3 \cdot 9x^4$

$\boxed{27x^4}$

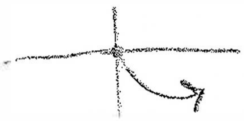
Compositions: The domain of a composition of functions f and g consists of the x-values that are in the domains of ___________________. Additionally, the domain of a quotient does not include x-values for which the denominator = 0.

Example 1: Find the following if $f(x) = 3x^{\frac{1}{2}}$ and $g(x) = -5x^{\frac{1}{2}}$. Also, name the domain of each composition.

a) $f(x) + g(x)$

$3x^{1/2} + -5x^{1/2}$

$\boxed{-2x^{1/2}}$

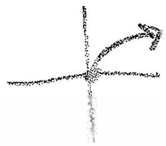


$D = [0, \infty)$

b) $f(x) - g(x)$

$3x^{1/2} - -5x^{1/2}$

$\boxed{8x^{1/2}}$



$D: [0, \infty)$

c) $\dfrac{f(x)}{g(x)}$

$\dfrac{3x^{1/2}}{-5x^{1/2}} = \boxed{\dfrac{-3}{5}}$

Example 2: Find the following if $f(x) = 7x$ and $g(x) = x^{\frac{1}{6}}$. Also, find the domain of each composition.

a) $f(x) \bullet g(x)$

$(7x)(x^{1/6})$

$\boxed{7x^{7/6}}$

b) $\dfrac{f(x)}{g(x)}$

$\dfrac{7x}{x^{1/6}} = 7x^{1-1/6} = \boxed{7x^{5/6}}$

Example 3: Let $f(x) = 6x^{-1}$ and $g(x) = 3x + 5$. Find the following compositions and their domains.

a) $f(g(x))$

$6(3x+5)^{-1}$

$\boxed{\frac{6}{3x+5}}$

b) $g(f(x))$

$3\left(\frac{6}{x}\right)+5$

$\boxed{\frac{18}{x}+5}$

c) $f(f(x))$

$6(6x^{-1})^{-1}$

$\frac{6}{\frac{6}{x}} = 6 \div \frac{6}{x} = 6 \cdot \frac{x}{6}$

$= \boxed{x}$

Inverse functions: A function that "reverses" another function.

Three ways to tell if relations are inverses:

- If $f(g(x)) = x$ and $g(f(x)) = x$ then $f(x)$ and $g(x)$ are inverses.
- If two functions are inverses, their graph are reflections over $y = x$.
- A table of values will switch the input and output values.

To find the inverse of a function:

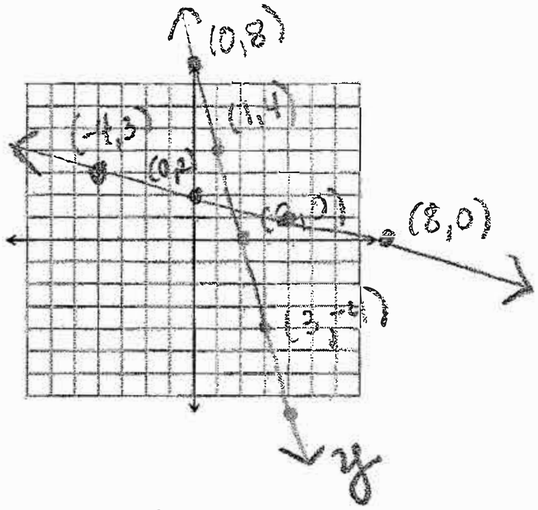
Switch x, y

(then solve for y)

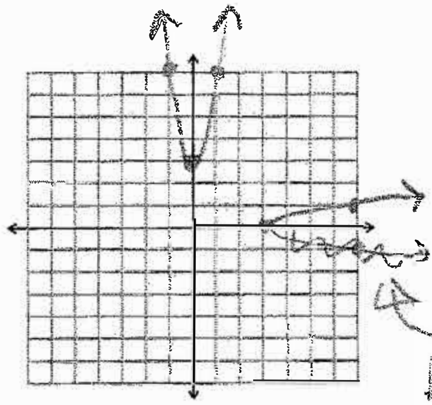
Notation: $f(x)$ → function $f^{-1}(x)$ → inverse function

Examples) Find the inverse of each function. Graph the function and its inverse on the same coordinate grid.

1) $y = -4x + 8$

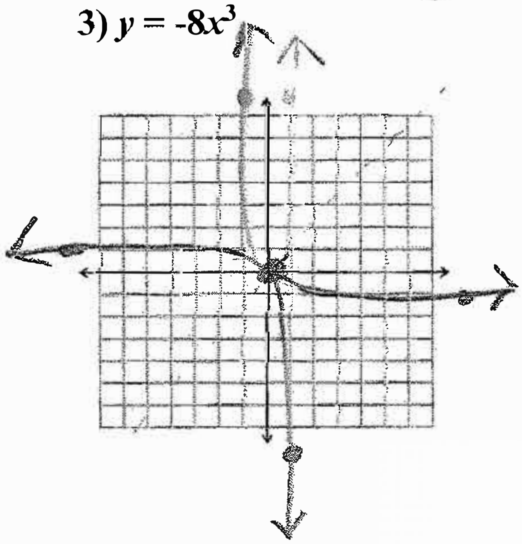


2) $f(x) = 4x^2 + 3$

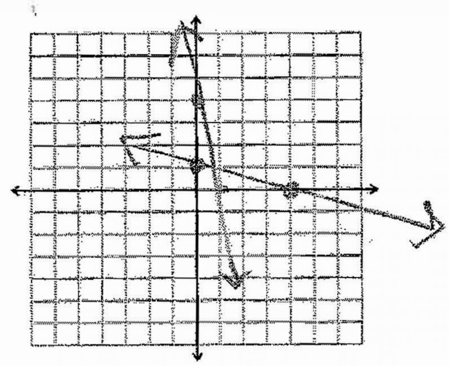


leave off bottom - its square root function

3) $y = -8x^3$



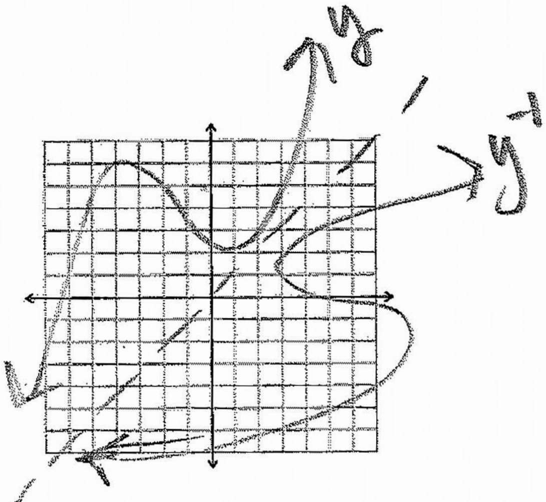
4) $f(x) = -\frac{1}{4}x + 1$



Horizontal Line Test:

y^{-1} not a function (vertical line test)

y is a function but doesn't pass horizontal line test



Example 5: Find the inverse of the function $h(x) = 4x^2$ if $x \geq 0$. Then graph both relations.

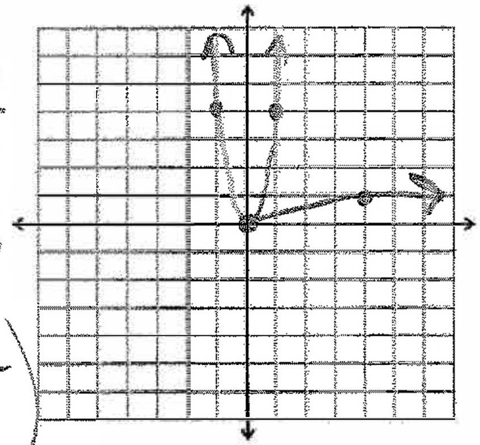
$y = 4x^2 \leftarrow$ function

$x = 4y^2 \leftarrow$ inverse is x, y switched

then solve for y just so it's normal equation

$\frac{x}{4} = \frac{4y^2}{4}$

$\sqrt{y^2} = \sqrt{\frac{x}{4}} \Rightarrow \boxed{y = \frac{\sqrt{x}}{2}}$ (only take $+\sqrt{\ }$ not $\pm\sqrt{\ }$)



Example 6: Determine if the following relations are inverses: $f(x) = 27x^3$ and $g(x) = \frac{1}{3}x^{\frac{1}{3}}$. Explain your reasoning.

$f(g(x)) = 27\left(\frac{1}{3}x^{1/3}\right)^3$

$27 \cdot \frac{1}{27}x$

$f \circ g = x$

$g(f(x)) = \frac{1}{3}\left(\overset{3^3}{\cancel{27}}x^3\right)^{1/3}$

$\cancel{\frac{1}{3}}\,\cancel{3^1}\,x^1$

$g \circ f = x$

$\boxed{\text{yes, they are inverses}}$

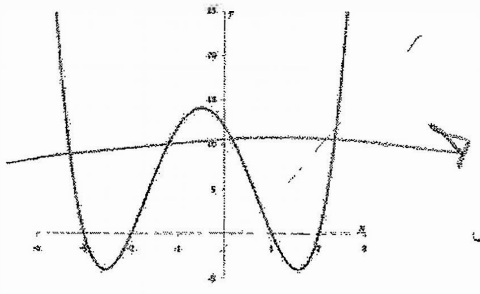
Example 7: Determine if the following relations are inverses: $f(x) = 625x^4$ and $g(x) = \frac{1}{5}x^{-4}$. Explain your reasoning.

$f(g(x)) = \overset{5^4}{\cancel{625}}\left(\frac{1}{5}x^{-4}\right)^4$

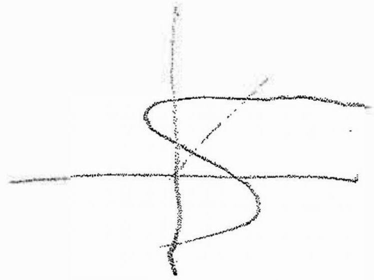
$f(g(x)) = \cancel{5^4 \cdot \frac{1}{5^4}}\, x^{-16}$

$f(g(x)) = x^{-16} \leftarrow$ not ~~an~~ the inverse

Example 8: The graph of $b(x)$ is given. Is b^{-1} a function?



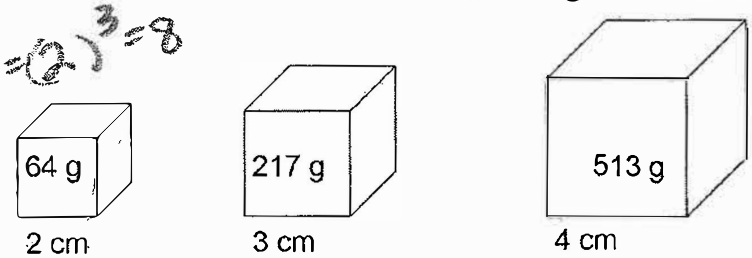
horizontal line test fail so $\boxed{\text{NO}}$



7.6: Modeling with Rational Functions

The side lengths and masses of three stainless steel cubes are given below. Use this information to write a model for the radius of a spherical stainless steel ball bearing as a function of its mass. What is the radius of a stainless steel ball bearing with a mass of 100 grams?

$v_1 = (2)^3 = 8$



A: Find the density of stainless steel.

The density of a material is a measure of its mass per unit of volume. $D = m/v$

To find the density of each stainless steel cube, divide its mass in grams by its volume in cubic centimeters. Round each density to the nearest whole number.

Cube 1: $D_1 = \dfrac{m_1}{v_1} = \dfrac{64g}{8cm^3} = 8\,g/cm^3$

Cube 2: $D_2 = \dfrac{m_2}{v_2} = \dfrac{217g}{(3cm)^3} = 8\,g/cm^3$

Cube 3: $D_3 = \dfrac{m_3}{v_3} = \dfrac{513g}{(4cm)^3} = 8\,g/cm^3$

a. What do you notice about the relationship among the densities you calculated?

They are about equally dense.

so the density of stainless steel is 8 g/cm^3

b. How can you find the mass in grams of a stainless steel cube if you know it edge length in centimeters?

$m = D \cdot V = D \cdot l^3$

$V = l^3$

B: Write a model for the mass of a stainless steel sphere as a function of its radius.

a. What is the formula for the volume of a sphere?

$V = \frac{4}{3}\pi r^3$

$\rightarrow D = 8\ g/cm^3$

b. Write a function $m(r)$ for the mass in grams of a stainless steel ball bearing with a radius of r centimeters.

$m = D \cdot V = 8 \cdot \frac{4}{3}\pi r^3 \rightarrow \boxed{m(r) = \frac{32}{3}\pi r^3}$

c. What are the reasonable domain and range of $m(r)$?

D: $r \geq 0$

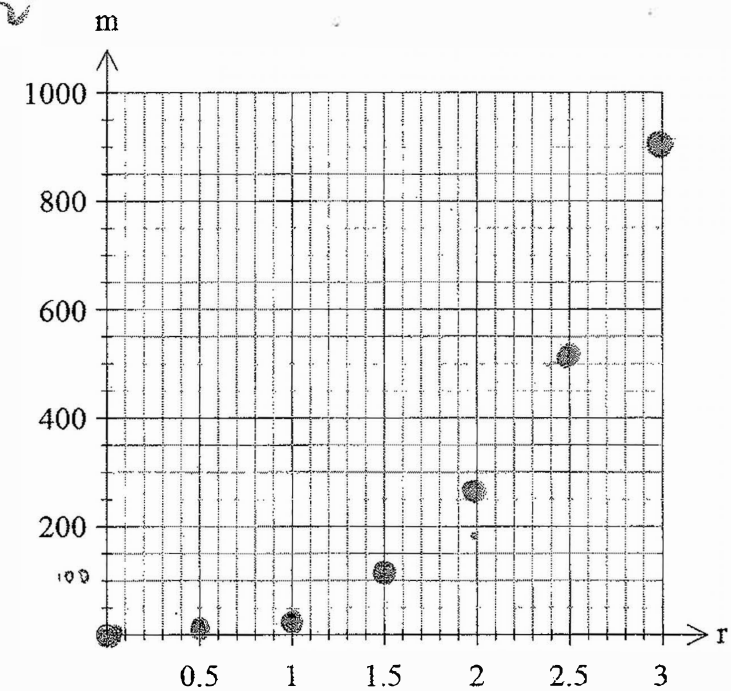
R: $m(r) \geq 0$

C: Graph and write the inverse function $r(m)$

a. Complete the table of values and use it to graph $m(r)$ for $r \geq 0$. Round the function values to the nearest whole number.

$= \frac{32}{3}\pi r^3$

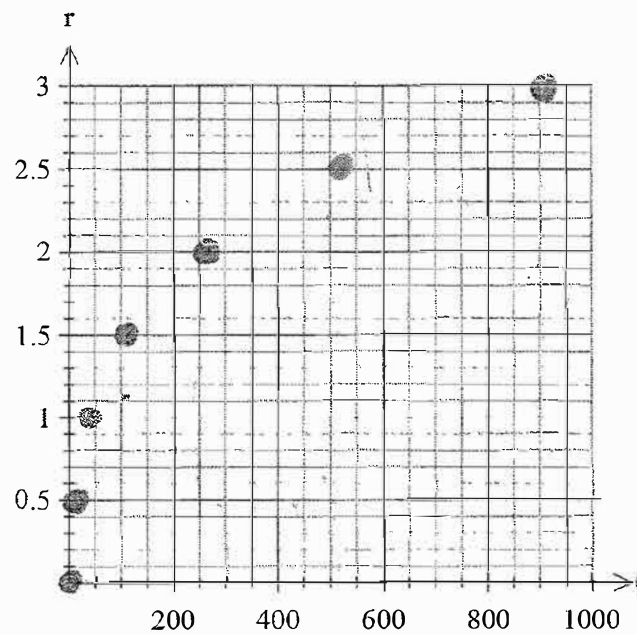
r	$m(r)$
0	0
0.5	4.
1	34
1.5	113
2	268
2.5	523
3	905



b. Reverse the input and output for each of the ordered pairs in part a above. Record the coordinates of the points below.

c. Plot the points from part b above and draw a smooth curve through them to graph the inverse function, $r(m)$.

(0,0)
(4,.5)
(34,1)
(113,1.5)
(268,2)
(523,2.5)
(905,3)



d. Find the equation for the inverse function, $r(m)$

$m(r) = \frac{32}{3}\pi r^3 \Rightarrow \frac{3m}{32\pi} = r^3$

$r = \sqrt[3]{\frac{3m}{32\pi}}$

e. What type of function is $m(r)$? What type of function is its inverse $r(m)$?

$\downarrow$ Cubic

$\downarrow$ Cube root

f. The function $m(r)$ models the mass in grams of a stainless steel ball bearing as a function of its radius in centimeters. What does $r(m)$ model?

the radius in cm as a function of the mass (g/cm³)

D: Find the radius of a stainless steel ball bearing with a mass of 100 grams

$r = \sqrt[3]{\frac{3m}{32\pi}} = \sqrt{\frac{3(100)}{32\pi}} = 1.44$ cm

7.7: Factoring with Rational Exponents

Factor:

1) $2(x+4)^3 - 4(x+4)^2$

$2☺^3 - 4☺^2$

$2☺^2(☺ - 2)$

$2(x+4)^2((x+4)-2)$

$2(x+4)^2(x+2)$

2) $6x(x-3)^2 + 5(x-3)^3$

$(x-3)^2(6x+5(x-3))$

$(x-3)^2(6x+5x-15)$

$(x-3)^2(11x-15)$

3) $4(x-1)^4 - 5(x-1)^3$

$(x-1)^3\left(4(x-1)-5\right)$

$(x-1)^3(4x-4-5)$

$(x-1)^3(4x-9)$

4) $3x^{\frac{1}{2}} - 4x^{-\frac{1}{2}}$

$\frac{3\sqrt{x}\sqrt{x}}{\sqrt{x}} - \frac{4}{\sqrt{x}}$

$\boxed{\frac{1}{\sqrt{x}}(3x-4)}$

or

$\boxed{x^{-1/2}(3x-4)}$

different way to show it:

$3x^{1-\frac{1}{2}} - 4x^{-1/2}$

$3x^1x^{-1/2} - 4x^{-1/2}$

$x^{-1/2}(3x-4)$

5) $5(x-1)^{\frac{1}{2}} + 3(x-1)^{-\frac{1}{2}}$

$5(x-1)^{1-\frac{1}{2}} + 3(x-1)^{-\frac{1}{2}}$ $\quad \frac{5x-2}{\sqrt{x-1}}$

$5(x-1)(x-1)^{-1/2} + 3(x-1)^{-1/2}$

$(x-1)^{-1/2}\left(5(x-1)+3\right)$

$(x-1)^{-1/2}(5x-5+3)$

$\boxed{(x-1)^{-1/2}(5x-2)}$

or

$\boxed{\frac{5x-2}{\sqrt{x-1}}}$

6) $7(x+3)^{-\frac{1}{2}} - 4(x+3)^{\frac{1}{2}}$

$7(x+3)^{-\frac{1}{2}} - 4(x+3)^{1-\frac{1}{2}}$

$7(x+3)^{-\frac{1}{2}} - 4(x+3)^1(x+3)^{-1/2}$

$(x+3)^{-1/2}\left(7-4(x+3)\right)$

$(x+3)^{-1/2}(7-4x-21)$

$(x+3)^{-1/2}(-4x-14)$

$\boxed{(x+3)^{-1/2}(-2)(2x+7)}$

or

$\boxed{\frac{-2(2x+7)}{\sqrt{x+3}}}$