Algebra 1 Ch 5 Notes and Calendar — Piecewise Functions

Name _____

Day	Date	Assignment (Due the next class meeting)
Tuesday Wednesday	11/12/19 (A) 11/13/19 (B)	• 5.1 Worksheet: Absolute Value Functions • Ch 4 Test Corrections
Thursday Friday	11/14/19 (A) 11/15/19 (B)	• 5.2 Worksheet: Piecewise Functions
Monday Tuesday	11/18/19 (A) 11/19/19 (B)	• 5.3 Worksheet: More Piecewise Functions
Wednesday Thursday	11/20/19 (A) 11/21/19 (B)	• 5.4 Worksheet: Extra Topics
Friday Monday	11/22/19 (A) 11/25/19 (B)	• 5.5 Worksheet: Spiral Review
Tuesday Monday	11/26/19 (A) 12/02/19 (B)	• Ch 5 Extra Practice Wk
Tuesday Wednesday	12/03/19 (A) 12/04/19 (B)	• Ch 5 Practice Test
Thursday Friday	12/05/19 (A) 12/06/19 (B)	• **Ch 5 Test** • Semester Review Worksheet #1
Monday Tuesday	12/09/19 (A) 12/10/19 (B)	• Semester Review Worksheet #2
Wednesday Thursday	12/11/19 (A) 12/12/19 (B)	• Practice Final
Friday Monday	12/13/19 (A) 12/16/19 (B)	• Review Activity in class • STUDY for final exam
Tuesday Wednesday Thursday	12/17/19 12/18/19 12/19/19	• Tues: Per 1 and Per 2 final exam • Wed: Per 3 and Per 4 final exam • Thurs: Per 5 and Per 6 final exam

NOTE: Be prepared for daily quizzes.

Students with 100% homework completion AND no late/missing homework for the semester will be rewarded with a pizza party. Students with 100% homework completion and no missing homework for the semester will get a 2% grade increase.

Do you need a worksheet or a copy of the teacher notes? Go to www.washoeschools.net/DRHSmath

Online textbook information: Go to www.washoeschools.net

- Click on Student and Parent
- Click on Envision
- Click on Sign In – sign in using washoe\studentID#, and then your school computer password. (Note: use a back slash not a forward slash.)

5.1 Notes: The Absolute Value Function

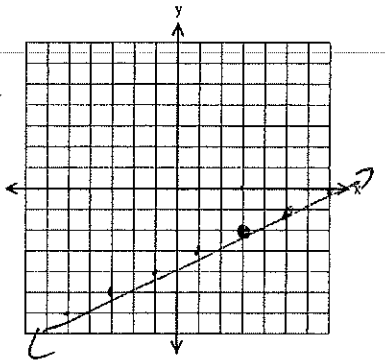
Objectives:

- Students will be able to graph absolute value functions.
- Students will be able to identify domain, range, vertex, and transformations of absolute value functions.

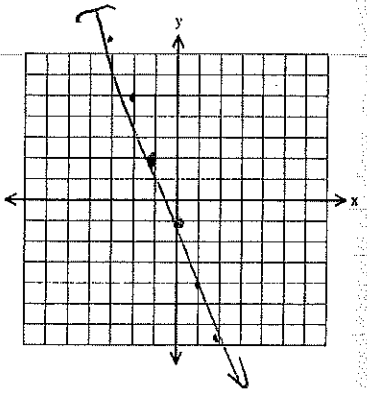
Warm up: For #1-2, graph each line using the point (h, k) and the slope.

1) $y = \frac{1}{2}(x-3) - 2$

slope point



2) $y = -3(x+1) + 2$



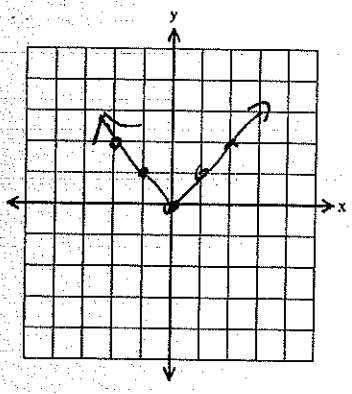
A **parent function** is the most basic form of a family of functions.

The parent function for Absolute Value functions is $y = |x|$

Example 1: Use a table of values to graph the functions $y = |x|$ and $y = -|x|$.

a) $y = |x|$

x	$y = \|x\|$
-2	2
-1	1
0	0
1	1
2	2



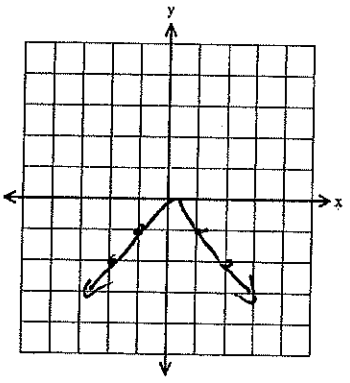
Domain: $\mathbb{R}$

Vertex: (0,0)

Range: $y \geq 0$

b) $y = -|x|$

x	$y = -\|x\|$
-2	-2
-1	-1
0	0
1	-1
2	-2



Domain: $\mathbb{R}$

Vertex: (0,0)

Range: $y \leq 0$

You can find the vertex of an absolute value function from the equation in *(h,k)* form the same way you found a point from an equation in *(h,k)* form.

OPP sign

$$y = a|x - h| + k$$

Slope (on each side)

Vertex: (h, k)

For Examples #2 – 7: Find the vertex of the following absolute value functions.

2) $y = 3|x + 1| - 2$ (-1, -2)

3) $y = -\frac{2}{3}|x| - 4$ (0, -4)

4) $y = 2|x - 3|$ (3, 0)

You try!

5) $y = \frac{3}{5}|x - 7| + 3$ (7, 3)

6) $y = \frac{1}{3}|x + 5|$ (-5, 0)

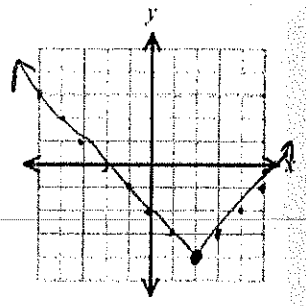
7) $y = -3|x| - 2$ (0, -2)

To graph an absolute value function in (h,k) form – also called Vertex form:

$$y = a|x - h| + k$$

1) Find the vertex (h,k) and plot that point.
2) Use the slope a to graph another point.
3) Draw the other side of the V – shape, making it symmetrical.

$y = |x - 2| - 4$

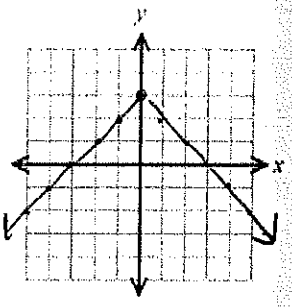


Reflections in the x-axis:

An absolute value function is **reflected in the x-axis** when the value of a negative.

$y = a|x - h| + k$ reflected in the x axis when a is negative.

$y = -|x| + 3$

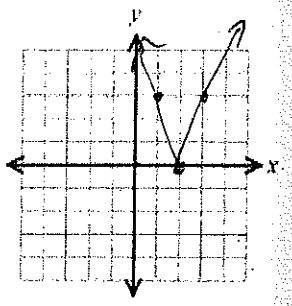


Vertical Stretches:

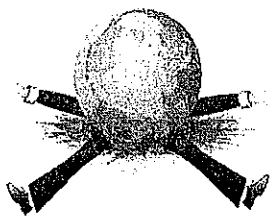
An absolute value function is **vertically stretched** when the value of a is greater than one, or less than negative one.

$y = a|x - h| + k$ is ***stretched*** when $a > 1$ *or* $a < -1$

$y = 3|x - 2|$



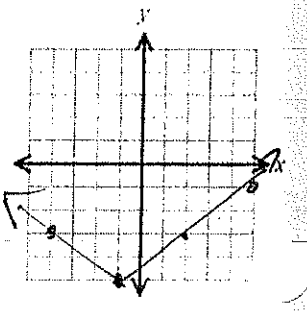
Vertical Compressions:



An absolute value function is **vertically compressed** when the value of a is between -1 and 1 $(a \neq 0)$.

$y = a|x - h| + k$ is ***compressed*** when $-1 < a < 1, a \neq 0$.

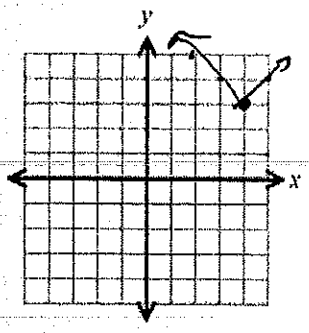
$y = \frac{2}{3}|x + 1| - 5$



For #8 – 15: Graph each equation below and identify the vertex, domain, and range. Describe the transformation from the parent function $y = |x|$.

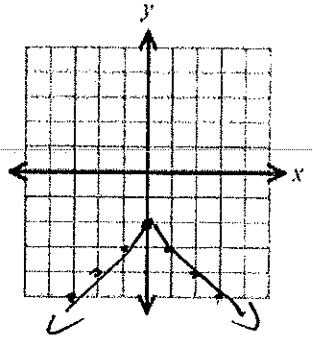
8) $y = |x - 4| + 3$

Vertex: (4, 3)
Domain: ℝ
Range: $y \geq 3$
Transformations:
Right 4
Up 3



9) $y = -|x| - 2$

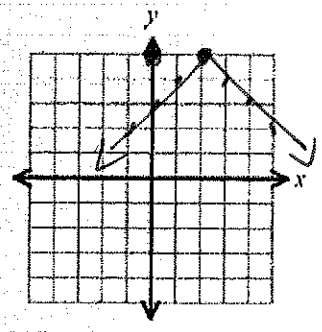
Vertex: (0, -2)
Domain: ℝ
Range: $y \leq -2$
Transformations:
Down 2
Reflection



You try!

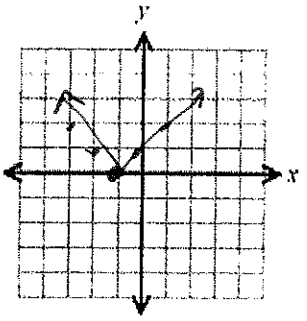
10) $y = -|x - 2| + 5$

Vertex: (2, 5)
Domain: ℝ
Range: $y \leq 5$
Transformations:
Reflection
up 5, Right 2



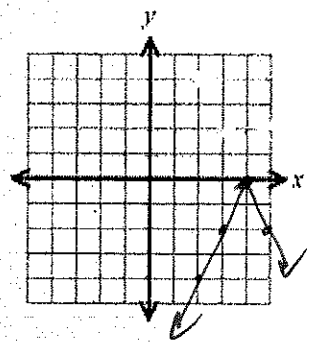
11) $y = |x + 1|$

Vertex: (-1, 0)
Domain: ℝ
Range: $y \geq 0$
Transformations:
Left 1



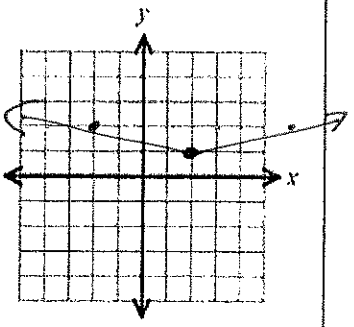
12) $y = -2|x - 4|$

Vertex: (4, 0)
Domain: ℝ
Range: $y \leq 0$
Transformations:
Stretch
Right 4
Reflect



13) $y = \frac{1}{4}|x - 2| + 1$

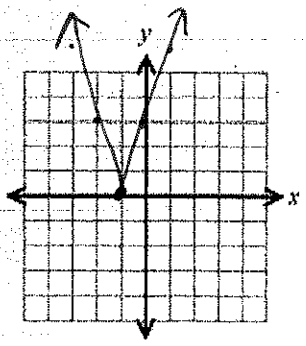
Vertex: (2, 1)
Domain: ℝ
Range: $y \geq 1$
Transformations:
Compress
Right 2
Up 1



You try!

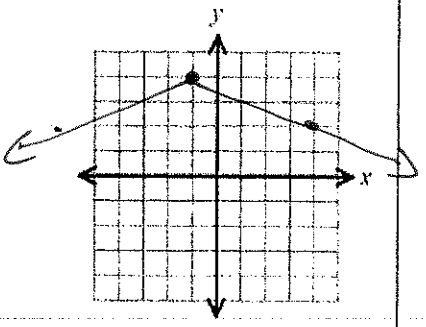
14) $y = 3|x + 1|$

Vertex: (-1, 0)
Domain: ℝ
Range: $y \geq 0$
Transformations:
- Left 1
- Stretch



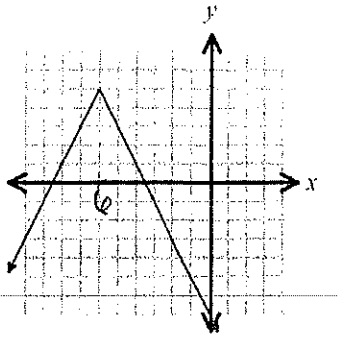
15) $y = -\frac{2}{5}|x + 1| + 4$

Vertex: (-1, 4)
Domain: ℝ
Range: $y \leq 4$
Transformations:
- Left 1
- Up 4
- Reflect
- Compress

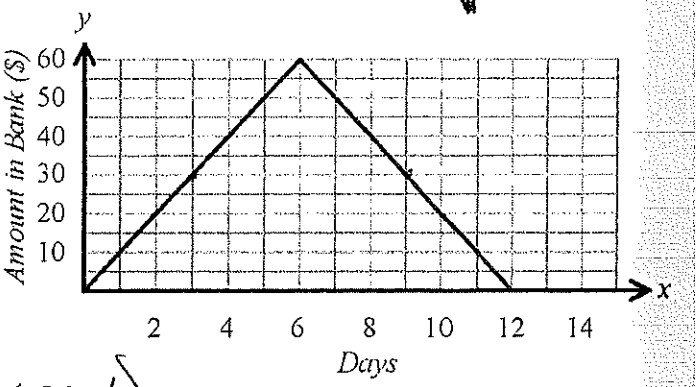


16) Which equation below matches the graph shown to the right?

A) $y = 2|x - 6| + 5$
B) $y = -2|x - 6| + 5$
C) $y = 2|x + 6| + 5$
D) $y = -2|x + 6| + 5$



For #17 – 19: Oliver wants to put \$10 in the bank per day for 6 days. He will then spend \$10 per day for another 6 days. The graph below models the amount of money Oliver has in the bank each day.



17) On which days will he have \$30 in the bank?

Day 3 and 9

18) What is the total amount that Oliver will spend?

\$60 (Decrease is spend)

19) What is the maximum amount that Oliver will have in the bank? On which day will he have this maximum?

60\$ Day 6

20) a) The equation $y = |x|$ is reflected in x-axis, vertically compressed by a factor of $\frac{1}{3}$, translated right 2 units and up 4 units. What is the equation of the new graph?

$$y = -\frac{1}{3}|x - 2| + 4$$

You try! b) The equation $y = |x|$ is reflected in x-axis, vertically stretched by a factor of 5, translated left 3 units and down 1 unit. What is the equation of the new graph?

$$y = -5|x + 3| - 1$$

21) a) Translate the graph of $f(x) = |x - 5| - 2$ four units to the left and one unit up. Write the function after the translation.

$$y = |x - 1| - 1$$

You try! b) Translate the graph of $f(x) = |x + 3| + 1$ two units to the right and 3 units down. Write the function after the translation.

$$f(x) = |x + 1| - 2$$

5.2 Notes: Piecewise Functions

Objectives

- Students will be able to graph lines on a restricted domain.
- Students will be able to graph piecewise functions.

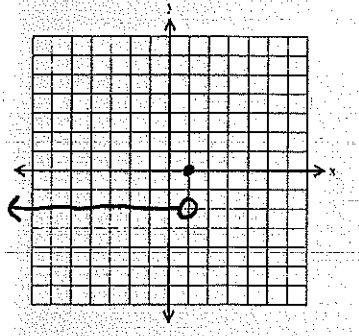
Key Vocabulary

Restricted Domain:

when a specific Domain is given

Examples 1 – 3: Graph each function on the restricted domain given.

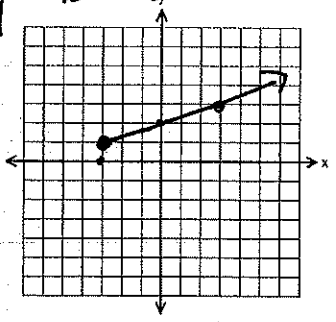
1) $y = -2;\ x < 1$



2) $y = \frac{1}{3}x + 2;\ x \geq -3$

(3,1) closed

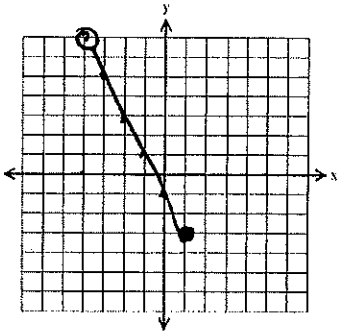
$\frac{1}{3}(-3) + 2 = -1 + 2 = 1$



3) $y = -2x - 1;\ -4 \leq x \leq 1$

open

close



Piecewise Function:

A piece of the function; multiple sub functions. ~~given Restricted Domain for 2 or more functions~~ each function applying to certain restriction.

-4

$y = -2(-4) - 1$

$y = 8 - 1$

$y = 7$ (-4, 7)

1

$y = -2(1) - 1$

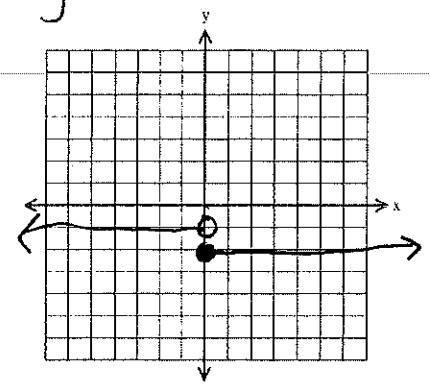
$y = -2 - 1$

$y = -3$ (1, -3)

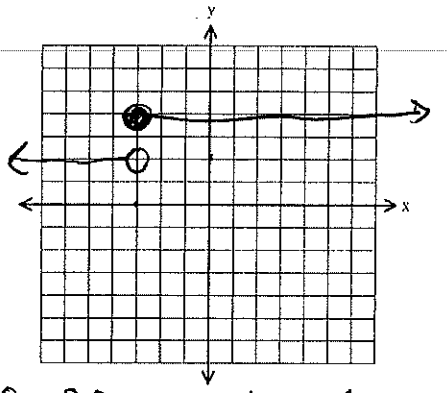
5.2 Day 1 Piecewise Activity in class!

For #4 – 5: Graph each part of the function over its given domain. Graph both parts of the function on the same coordinate plane.

4) $f(x) = \begin{cases} -1 & if\ x \le 0 \\ 2 & if\ x \ge 0 \end{cases}$

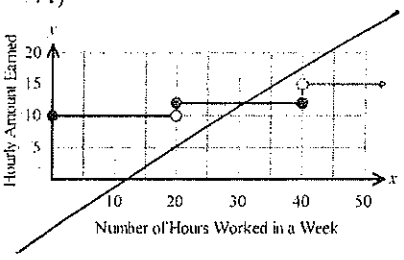


You try! 5) $y = \begin{cases} 2 & if\ x < -3 \\ 5 & if\ x \ge -3 \end{cases}$

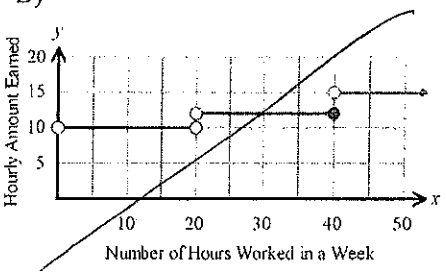


6) Jalissa earns \$10 per hour at her job if she works up to and including 20 hours. If she works more than 20 hours per week but less than or equal to 40 hours, then she earns \$12 per hour. If she works more than 40 hours per week, then she earns \$15 per hour. Which graph below models this situation?

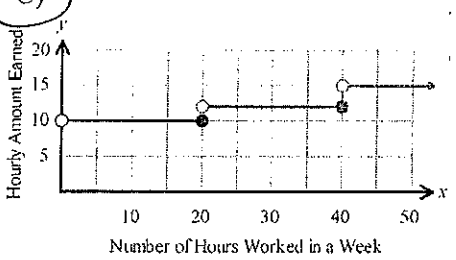
A)



B)

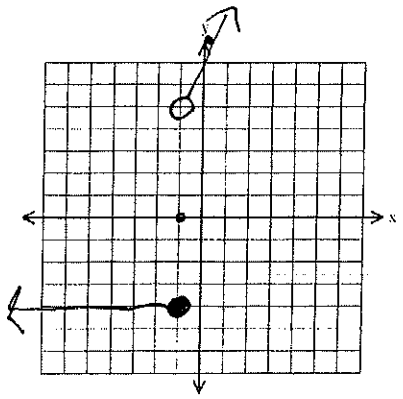


C)



For #7 – 12: Graph each piecewise function.

7) $f(x) = \begin{cases} -4 & if\ x \le -1 \\ -2x + 3 & if\ x > -1 \end{cases}$



$-2(-1)+3$

$2+3$

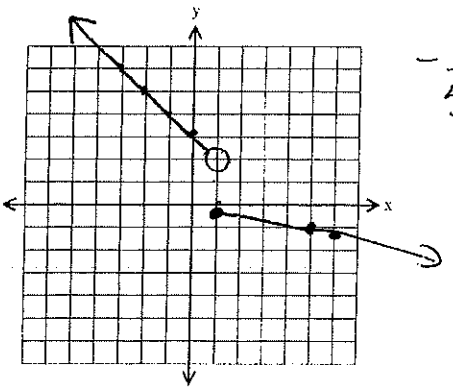
5

$(-1, 5)$

8) $y = \begin{cases} -x + 3 & if\ x < 1 \\ -\frac{1}{5}x & if\ x \ge 1 \end{cases}$

$-1+3=2$

$(1, 2)$

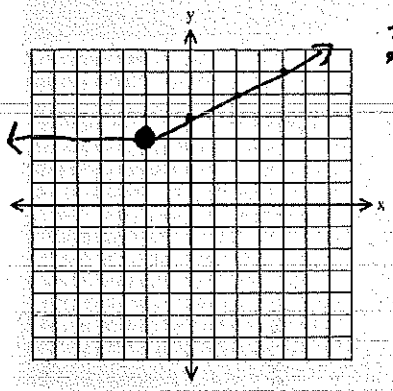


$-\frac{1}{5}(1) = -\frac{1}{5}$

$(1, -\frac{1}{5})$

You try! Graph each piecewise function.

9) $y = \begin{cases} 3 & \text{if } x \le -2 \\ \frac{1}{2}x + 4 & \text{if } x > -2 \end{cases}$

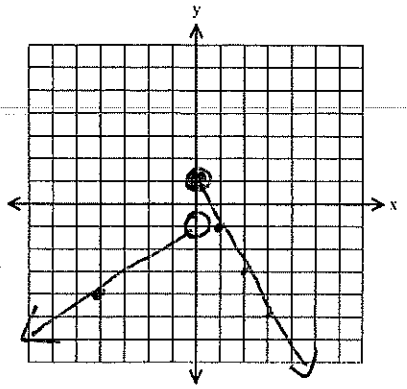


$\frac{1}{2}(-2)+4$

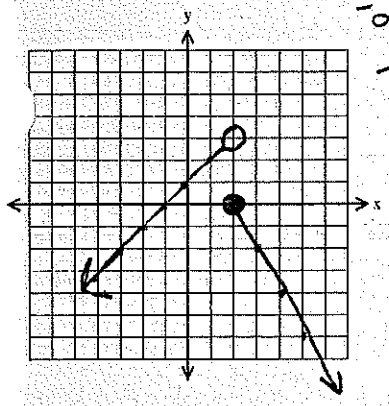
$-1+4$

3

10) $h(x) = \begin{cases} \frac{3}{4}x - 1 & \text{if } x < 0 \\ -2x + 1 & \text{if } x \ge 0 \end{cases}$



11) $y = \begin{cases} x + 1 & \text{if } x < 2 \\ -2x + 4 & \text{if } x \ge 2 \end{cases}$ $2+1=3$

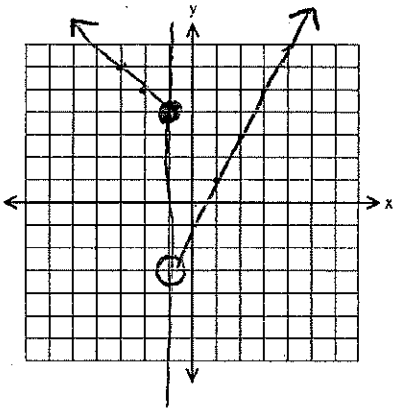


$-2(2)+4$

$-4+4$

0

12) $h(x) = \begin{cases} -x + 3 & \text{if } x \le -1 \\ 2x - 1 & \text{if } x > -1 \end{cases}$



$-(-1)+3$

$1+3=4$

$2(-1)-1$

$-2-1$

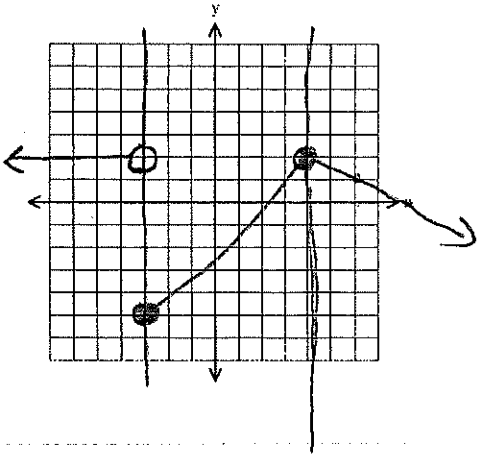
-3

Challenge Problem:

$y = \begin{cases} 2 & \text{if } x < -3 \\ x - 2 & \text{if } -3 \le x < 4 \\ -\frac{1}{2}x + 4 & \text{if } x \ge 4 \end{cases}$

$-3-2=-5$

$4-2=2$



$-\frac{1}{2}(4)^2+4$

$-2+4=2$

5.3: More Piecewise Functions

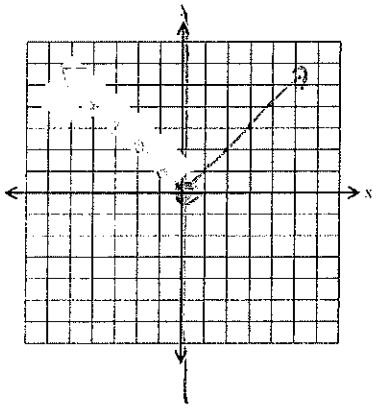
Objectives

- Students will be able to graph absolute value functions as piecewise functions.
- Students will be able to match absolute value equations and piecewise functions.

5.3 Activity in class…

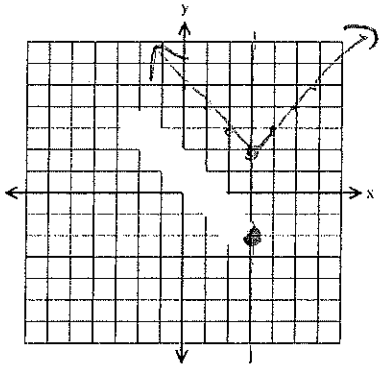
Examples 1 and 2: Graph the piecewise function. Then write the equation of the absolute function that is represented.

1) $f(x) = \begin{cases} -x & if\ x \le 0 \\ x & if\ x > 0 \end{cases}$



equation: $y = |x|$

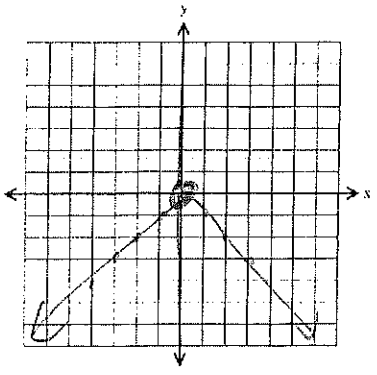
2) $f(x) = \begin{cases} -x+5 & if\ x \le 3 \\ x-1 & if\ x > 3 \end{cases}$ $-3+5$ $+2$



equation: $y = |x-3|+2$

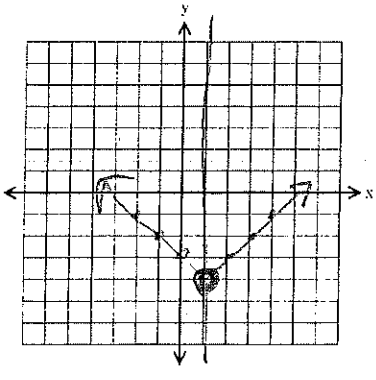
You try! Examples 3 and 4: Graph the piecewise function. Then write the equation of the absolute function that is represented.

3) $f(x) = \begin{cases} x & if\ x \le 0 \\ -x & if\ x > 0 \end{cases}$



equation: $y = -|x|$

4) $f(x) = \begin{cases} -x-3 & if\ x \le 1 \\ x-5 & if\ x > 1 \end{cases}$ $-1-3=-4$ $1-5=-4$



equation: $y = |x-1|-4$

Piecewise Functions

Example 5: Graph the piecewise function. Then write the equation of the absolute function that is represented.

$$f(x) = \begin{cases} 2x+7 & \text{if } x \le -2 \\ -2x-1 & \text{if } x > -2 \end{cases}$$

$2(-2)+7$

$-4+7=3$

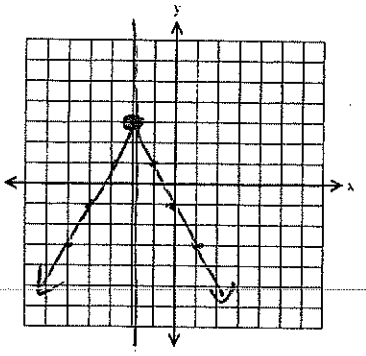
$-2(-2)-1$

$4-1$

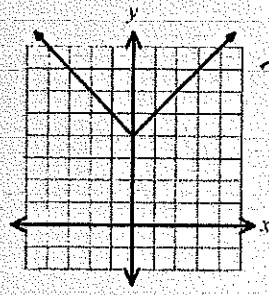
3

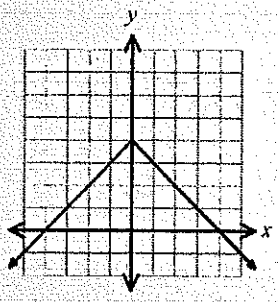
Equation:

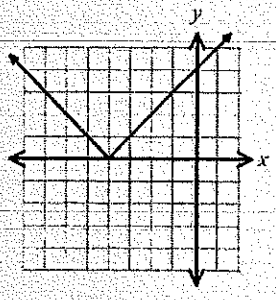
$y = -2|x+2|+3$



For #6 – 8, work with a partner. Match each graph below with a piecewise function **and** an absolute value equation.

6)  D, E

7)  B, F

8)  A, H

Choices for Absolute Value Functions:

A) $y = |x+4|$

B) $y = -|x|+4$

C) $y = -|x+4|$

D) $y = |x|+4$

Choices for piecewise functions:

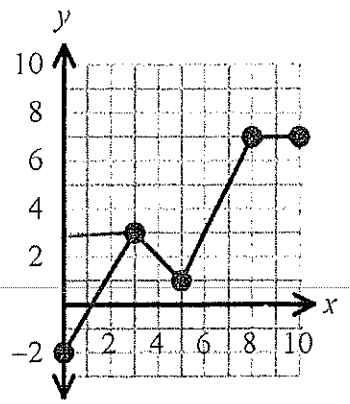
E) $f(x) = \begin{cases} -x+4, & x<0 \\ x+4, & x \ge 0 \end{cases}$

F) $f(x) = \begin{cases} x+4, & x<0 \\ -x+4, & x \ge 0 \end{cases}$

G) $f(x) = \begin{cases} x+4, & x<-4 \\ -x-4, & x \ge -4 \end{cases}$

H) $f(x) = \begin{cases} -x-4, & x<-4 \\ x+4, & x \ge -4 \end{cases}$

Examples 9 – 12: Consider the piecewise function shown, which shows the elevation (in *km*) of hikers as they travel in a mountainous region. The *x*-axis shows the horizontal distance traveled (in *km*).



9) For what horizontal distance(s) did the hikers reach an elevation of 3 *km*?

3 km

10) What is the starting elevation of the hikers?

-2 km

11) What is the total change in elevation of the hikers?

9 km

12) Which of the following is the piecewise function that models the graph?

A) $h(x) = \begin{cases} \frac{5}{3}x + 2 & \text{if } 0 \le x < 3 \\ -x - 6 & \text{if } 3 \le x < 5 \\ -2x - 9 & \text{if } 5 \le x < 8 \\ 7 & \text{if } 8 \le x < 10 \end{cases}$

B) $(x) = \begin{cases} \frac{5}{3}x - 2 & \text{if } 0 \le x < 3 \\ -x + 6 & \text{if } 3 \le x < 5 \\ 2x - 9 & \text{if } 5 \le x < 8 \\ 7 & \text{if } 8 \le x < 10 \end{cases}$

C) $(x) = \begin{cases} -\frac{5}{3}x - 2 & \text{if } 0 \le x < 3 \\ x + 6 & \text{if } 3 \le x < 5 \\ 2x - 9 & \text{if } 5 \le x < 8 \\ 7 & \text{if } 8 \le x < 10 \end{cases}$

D) $(x) = \begin{cases} \frac{5}{3}x - 2 & \text{if } 0 \le x < 3 \\ -x - 6 & \text{if } 3 \le x < 5 \\ 2x + 9 & \text{if } 5 \le x < 8 \\ 7 & \text{if } 8 \le x < 10 \end{cases}$

5.4 Notes: Extra Topics

Objectives

- Students will be able to solve proportions.
- Students will be able to solve absolute value equations graphically and confirm their answer algebraically (review).
- Students will be able to graph piecewise functions.

Proportions:

Solving Proportions:

Cross Multiplication $\frac{a}{b} \times \frac{c}{d}$ $ad = cb$

Examples 1 – 3: Solve each proportion for the variable.

1) $\frac{x}{4} = \frac{3}{7}$

$7x = 12$

$x = \frac{12}{7}$

2) $\frac{6}{5y} = \frac{11}{y+1}$

$6y + 6 = 55y$

$\frac{6}{49} = \frac{49y}{49}$

$\frac{6}{49} = y$

3) $\frac{d-2}{-3} = \frac{5d+2}{8}$

$8d - 16 = -15d - 6$

$+15d \quad +16 \quad +15d \quad +16$

$23d = 10$

$d = \frac{10}{23}$

You Try! #4 – 6: Solve each proportion for the variable.

4) $\frac{4}{a} = \frac{3}{10}$

$40 = 3a$

$\frac{40}{3} = a$

5) $\frac{-4}{b+3} = \frac{9}{b}$

$-4b = 9b + 27$

$-9b \quad -9b$

$-13b = 27$

$b = \frac{-27}{13}$

6) $\frac{2}{x+3} = \frac{5}{x-1}$

$2x - 2 = 5x + 15$

$-2x \quad -15 \quad -2x \quad -15$

$-17 = 3x$

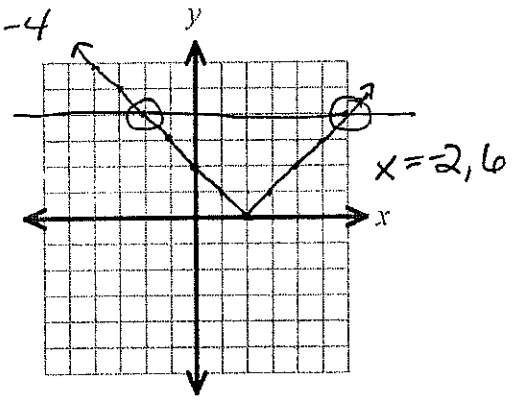
$\frac{-17}{3} = x$

Solving Absolute Value Equations by Graphing

Example 7: Solve $|x - 2| = 4$ by graphing. Check your solution by solving algebraically.

$x-2=4$ $x-2=-4$

$x=6$ $x=-2$



$x=-2,6$

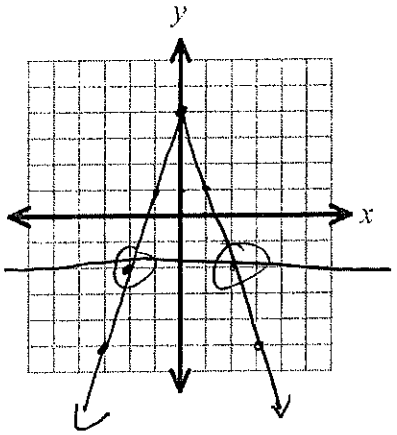
$x-2=4$ $x-2=-4$

$x=6$ $x=-2$

For Examples 8 – 9, solve each absolute value equation by graphing. Check your solutions by solving algebraically.

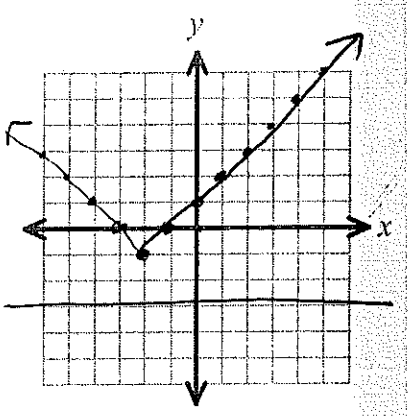
8) $-3|x| + 4 = -2$

$x=-2,2$



9) $|x + 2| - 1 = -3$

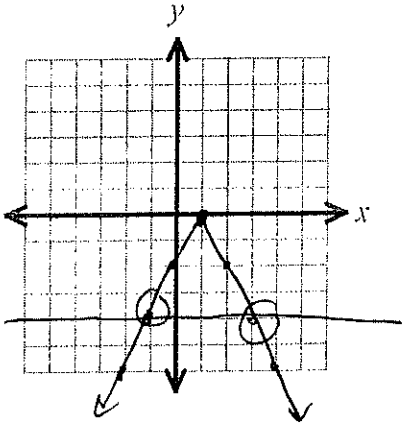
No Solution



You Try! #10 – 11: Solve each absolute value equation by graphing. Check your solutions by solving algebraically.

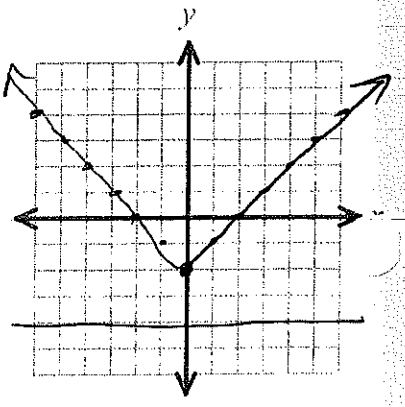
10) $-2|x - 1| = -4$

$x=-1,3$



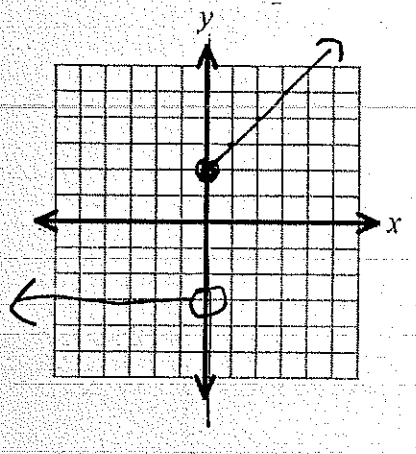
11) $|x| - 2 = -4$

No Solution



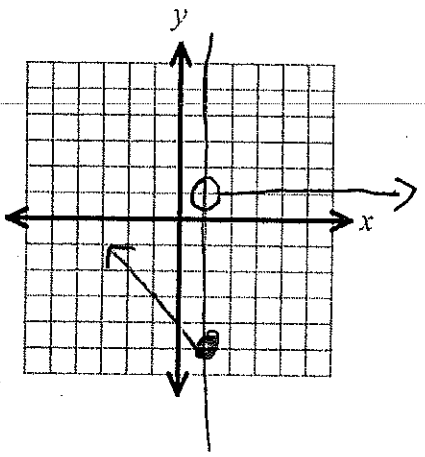
Review: For #12 – 15: Graph each piecewise function shown below.

12) $f(x) = \begin{cases} -3 & \text{if } x < 0 \\ x + 2 & \text{if } x \geq 0 \end{cases}$



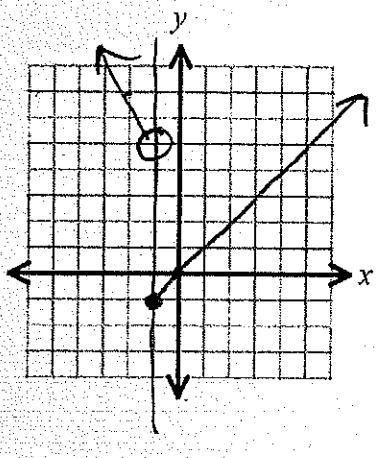
13) $y = \begin{cases} -x - 4 & \text{if } x \leq 1 \\ 1 & \text{if } x > 1 \end{cases}$

$-1-4$
-5



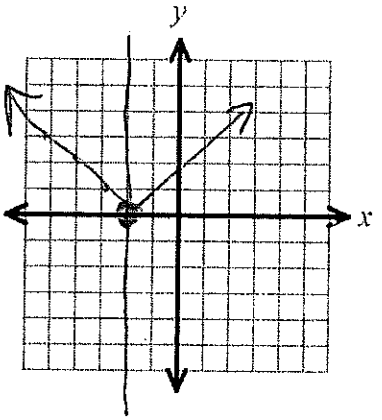
14) $g(x) = \begin{cases} -2x + 3 & \text{if } x < -1 \\ x & \text{if } x \geq -1 \end{cases}$

$-2(-1)+3$
$2+3$
5



15) $a(x) = \begin{cases} -x - 2 & \text{if } x \leq -2 \\ x + 2 & \text{if } x > -2 \end{cases}$

$-(-2)-2$
$2-2$
0



16) What is the equation of the absolute value function graphed in #15?

$y = |x + 2|$

5.5 Notes: Spiral Review

Warm-up: Consider the given arithmetic sequence: 6, -1, -8, -15, -22, …

1) Which of the following is the correct explicit formula?

A) $a_n = -7 + 6n$
B) $a_n = 6 - 7n$
C) $a_n = -7 + 13n$
D) $a_n = 13 - 7n$

2) Which of the following is the correct recursive formula?

A) $a_1 = 6; a_n = a_{n-1} - 7$
B) $a_1 = -7; a_n = a_{n-1} + 6$
C) $a_1 = 6; a_n = a_{n-1} - 13$
D) $a_1 = 13; a_n = a_{n-1} - 7$

3) Use your explicit formula from #1 to find the 50th term in the sequence.

$a_n = 13 - 7(50)$ $a_{50} = -337$

Arithmetic Sequence Formulas can be written in both sequence notation and function notation.

Recursive Formulas

Using Sequence Notation	Using Function Notation
$a_1 = ?$	$f(1) = ?$
$a_n = a_{n-1} + \boldsymbol{d}$	$f(n) = f(n-1) + \boldsymbol{d}$

Explicit Formulas

Using Sequence Notation	Using Function Notation
$a_n = dn + a_0$	$f(n) = dn + f(0)$

NOTE: In Unit 3 we used mostly sequence notation, but now we should expand our horizons to both!

Example 1: Which of the following formulas below correctly describe the sequence? Choose all that apply.
-5, -2, 1, 4, 7, …

A) $a_n = -5 + 3n$
B) $a_n = -8 + 3n$
C) $f(n) = -5 + 3n$
D) $f(n) = -8 + 3n$
E) $f(1) = -5; f(n) = f(n-1) + 3$
F) $f(1) = -8; f(n) = f(n-1) + 3$
G) $a_1 = -5; a_n = a_{n-1} + 3$
H) $a_1 = -8; a_n = a_{n-1} + 3$

Example 2: Which sequence below matches the explicit formula? $f(n) = 11 + 2n$

A) 11, 13, 15, 17, … B) 9, 11, 13, 15, … C) 2, 13, 24, 36, … D) 13, 15, 17, 19, …

Properties for solving equations

- **Distributive Property:** You can multiply all items in parenthesis by the term that is multiplying the parenthesis.

  Sample: $2(5x - 7) = 10x - 14$

- **Addition Property of Equality:** The same value can be added to both sides of an equation.

- **Subtraction Property of Equality:** The same value can be subtracted from both sides of an equation.

- **Multiplication Property of Equality:** The same value can be multiplied to both sides of an equation.

- **Division Property of Equality:** both sides of an equation can be divided by the same value.

- **Associative Property (of addition or multiplication):** Terms can be grouped together differently with addition or multiplication. Note: the order of the numbers is not changed.

  Sample: $(3 + 5) + 2 = 3 + (5 + 2)$

  Sample: $(3 \cdot 5)2 = 3(5 \cdot 2)$

- **Commutative Property (of addition or multiplication):** Terms that are added (or multiplied) can be written in any order.

  Sample: $3 + 2 = 2 + 3$

  Sample: $3 \cdot 2 = 2 \cdot 3$

Example 6: The equation below is solved step-by-step. Write the property that describes each step.

Solution	Property used
$\frac{4(-2+3x)}{7} - 1 = 8$	Given equation
$4(-2+3x) - 7 = 56$	2 properties here! Multiplication & Distributive
$-8 + 12x - 7 = 56$	Distributive
$12x - 8 - 7 = 56$	Commutative
$12x - 15 = 56$	Associative Prop of addition
$12x = 71$	Addition Prop.
$x = \frac{71}{12}$	Division Prop

Could this problem have been solved in a different order? In less steps? Explain.

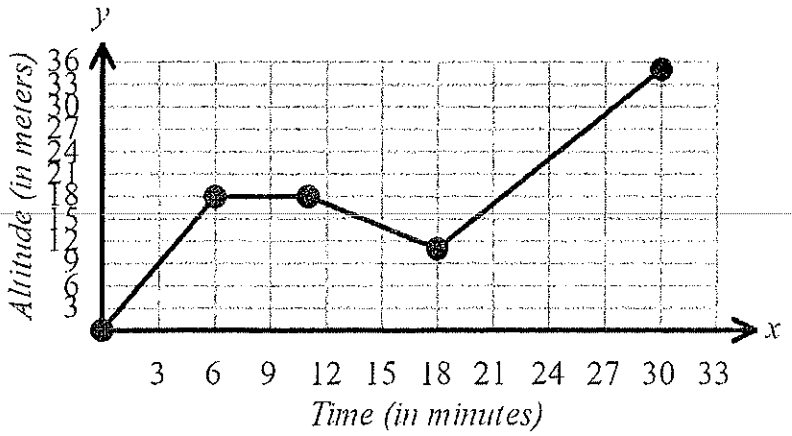
Example 7: What properties were used below?

Given equation: $3(2x + 8) + 5x = 2$ Distribution

$(24 + 6x) + 5x = 2$

$24 + (6x + 5x) = 2$ Associative

Example 8: Cora took a morning run in the hills around her house. Use the graph to answer the questions below.



a) For what time interval(s) was she running uphill?

0 - 6 min
18 - 30 min

b) At what time did she first start running downhill?

11 min

c) Which of the following statements is true? Choose all that apply.

A) The rate of change of her altitude is negative when $11 \leq x \leq 18$.
B) The rate of change of her altitude is positive when $18 \leq x \leq 30$.
C) The rate of change of her altitude is 0 from time $x = 6$ min to $x = 11$ min.
D) Cora ends her run after 30 minutes.