| Day | Date | **Assignment** (Due the next class meeting) |
|---|---|---|
| Monday<br>Tuesday | 9/18/23 (A)<br>9/19/23 (B) | Matrices Lesson 1: Matrix Operations<br>**HW: Practice Problems Lesson 1** |
| Wednesday<br>Thursday | 9/20/23 (A)<br>9/21/23 (B) | Matrices Lesson 2: Vectors<br>**HW: Practice Problems Lesson 2** |
| Friday<br>Monday | 9/22/23 (A)<br>9/25/23 (B) | Matrices Lesson 3: Inverses, Determinants, Solving Systems<br>**HW: Practice Problems Lesson 3** |
| Tuesday<br>Wednesday | 9/26/23 (A)<br>9/27/23 (B) | Review<br>**HW: Practice Test** |
| Thursday<br>Friday | 9/28/23 (A)<br>9/29/23 (B) | **Matrix Unit Test** |

# Matrices Lesson 1 Practice Problems

**1.** In matrix *C*, the entries are the numbers of students in a chess club at a high school. Column 1 lists boys, column 2 lists girls, row 1 lists juniors, and row 2 lists seniors. What does the number in position $c_{21}$ represent?

$$C = \begin{bmatrix} 5 & 6 \\ 4 & 3 \end{bmatrix}$$

**2.** The rows in matrix *A* represent the prices of long-sleeved and short-sleeved shirts. The columns represent the fabrics: cotton, linen, and silk. If the sales tax rate is 5%, use scalar multiplication to list the sales tax for each shirt in matrix *S*. Express each entry as a decimal to the nearest hundredth. $A = \begin{bmatrix} 25 & 40 & 50 \\ 20 & 35 & 45 \end{bmatrix}$

**3.** Given matrices $X = \begin{bmatrix} 2 & -3 \\ -1 & 4 \end{bmatrix}$ and $Y = \begin{bmatrix} -4 & 1 \\ 3 & -2 \end{bmatrix}$, complete the matrix for each sum or difference: X + Y, X – Y, Y – X

**4.** What are the values of the variables in the matrix equation?

$$2\begin{bmatrix} 7.5 & 2x-2 & 6 \\ y+7 & 4z & 5 \end{bmatrix} = \begin{bmatrix} 15 & 20 & 12 \\ 2 & 20 & 10 \end{bmatrix}$$

**5.** If $A = \begin{bmatrix} 2 & 1 & -3 \\ 5 & -4 & -6 \end{bmatrix}, B = \begin{bmatrix} -2 & -1 & 3 \\ -5 & 4 & 6 \end{bmatrix}$, and $C = \begin{bmatrix} -2 & -5 \\ -1 & 4 \\ 3 & 6 \end{bmatrix}$, which statements about matrices *A*, *B*, and *C* are true? Select all that apply.

**A** Matrices *A* and *B* are additive inverses.
**B** Matrices *A* and *C* are additive inverses.
**C** Matrices *B* and *C* cannot be combined using addition or subtraction.
**D** Matrices *A* and *B* cannot be combined using addition or subtraction.
**E** $A + B = B + A$
**F** $A - B = B - A$

**6.** Find the additive inverse of the matrix $X = \begin{bmatrix} 2 & -5 \\ -6 & 3 \end{bmatrix}$

**7.** $\overline{EF}$ has endpoints (2, 4) and (4, 5).

**a.** Use matrices to translate $\overline{EF}$ 2 units right and 4 units down to $\overline{YZ}$. What are the coordinates of *Y* and *Z*?

**b.** Use matrices to dilate $\overline{EF}$ to $\overline{UV}$ by a scale factor of 4, centered at the origin. What are the coordinates of *U* and *V*?

8. Solve for x and y: $\begin{bmatrix} 5x & 7 \\ -4 & 2x \end{bmatrix} + \begin{bmatrix} y & 3 \\ 13 & y \end{bmatrix} = \begin{bmatrix} 13 & 10 \\ 9 & 7 \end{bmatrix}$

***Lesson 1 Practice Problems Continued on next page…***

**For # – 13 use the matrices to perform the following operations. If not possible, explain why.**

$$A = \begin{bmatrix} 8 & 7 & -4 \\ 1 & -4 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} -9 & -7 \\ -2 & 2 \\ 10 & 9 \end{bmatrix}, \quad C = \begin{bmatrix} -1 & -5 \\ -2 & 8 \end{bmatrix}, \quad D = \begin{bmatrix} -10 \\ 1 \\ 7 \end{bmatrix}, \quad E = [10 \quad -5 \quad -5]$$

**9.** $A \cdot B$ **10.** $B \cdot A$ **11.** $A \cdot C$ **12.** $2B \cdot C$ **13.** $C \cdot D$

**14.** Find $IQ$, if $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $Q = \begin{bmatrix} 3 & -5 & 0 \\ -7 & 4 & 8 \\ 2 & 6 & -3 \end{bmatrix}$

**For #15-16, determine whether each equation is true for the following matrices:**

$$A = \begin{bmatrix} 1 & 2 \\ 0 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} -4 & 0 \\ -1 & 8 \end{bmatrix}, \quad C = \begin{bmatrix} 5 & 1 \\ 7 & -2 \end{bmatrix}$$

**15.** $(A + B)C = AC + BC$ **16.** $A(BC) = (AB)C$

**17.** Raul owns and operates two souvenir stands. At his baseball park stand, sweatshirts cost $45 and T-shirts cost $20. At his football stadium stand, sweatshirts cost $50 and T-shirts cost $15. Today Raul sold 20 sweatshirts and 25 T-shirts at each stand. Use matrix multiplication to find the total amount in daily sales at each souvenir stand.

**For #18-19:** A drama teacher assigns final grads in her class based on the weighted system shown below. The matrix G represents the grades for Kiyo and his two friends, Rachel and Leo.

$$G = \begin{matrix} tests \\ proj \\ part \end{matrix} \begin{bmatrix} 90 & 83 & 78 \\ 94 & 88 & 96 \\ 98 & 94 & 89 \end{bmatrix}$$

Drama Syllabus
Tests 45%
Projects 30%
Participation 25%

**18.** Write matrix $W$ as a 1x3 matrix to represent the weighted grading system.

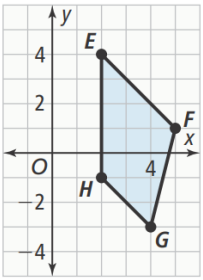
**19.** Perform matrix multiplication to find the final grades for each of the three students.

**20.** Refer to quadrilateral EFGH for questions a – c.

**a.** Create matrix A to represent the coordinates of quadrilateral EFGH.

**b.** Multiply matrix A by $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

**c.** Graph the quadrilateral represented by the resulting matrix and describe the movement of the quadrilateral in the coordinate plane.




# Matrices Lesson 2 Practice Problems

**Add each vector pair.**

**1.** $\overrightarrow{MN} = \langle 10, 5 \rangle$ and $\overrightarrow{NO} = \langle -2, 5 \rangle$

**2.** $\overrightarrow{MN} = \langle -3, 7 \rangle$ and $\overrightarrow{NO} = \langle -1, -2 \rangle$

**Subtract each vector.**

**3.** $\vec{s} = \langle 2, -6 \rangle$, $\vec{t} = \langle -1, 4 \rangle$

**4.** $\vec{s} = \langle 4, 7 \rangle$, $\vec{t} = \langle 0, -1 \rangle$

**Multiply each vector by the given scalar.**

**5.** $\vec{t} = \langle 2, 3 \rangle$ scalar = 8

**6.** $\vec{t} = \langle -4, 8 \rangle$ scalar = 6

**7.** Reflect $\overrightarrow{EF} = \langle 5, 3 \rangle$ across the *x*-axis using a matrix.

**8.** Reflect $\overrightarrow{GH} = \langle 2, 1 \rangle$ across the *y*-axis using matrices.

**10.** Describe how the magnitude and the direction of $\vec{t} = \langle x, y \rangle$ is affected when $\vec{t}$ is multiplied by a scalar of *z*. a scalar of −*z*?

**~~11. SKIP~~** ~~Emelia is paddling a kayak in the ocean at 5 mph headed 20° north of west. The current of the ocean is 3 mph at a direction that is 20° east of south. What are the magnitude and direction of the path of her kayak as she paddles across the ocean?~~

**12**. Which of the following operations will reflect the vector $\langle 3, -2\rangle$ across the y-axis?

**A.** $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ -2 \end{bmatrix}$ **B.** $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ -2 \end{bmatrix}$

**C.** $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ -2 \end{bmatrix}$ **D.** $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ -2 \end{bmatrix}$

**13.**

Three matrices are given below.

$X = \begin{bmatrix} -2 & 0 \\ 5 & 7 \end{bmatrix}$ $Y = \begin{bmatrix} -1 & 3 \\ -5 & 8 \end{bmatrix}$ $Z = \begin{bmatrix} 11 & 3 \\ 5 & -7 \end{bmatrix}$

Which of the following statements are true? Select all that apply.

**F.** $(X + Y)Z = XZ + YZ$

**G.** $XY = YX$

**H.** $-5(XY) = (-5X)Y$

**I.** $X(Y + Z) = XY + XZ$

**J.** $X + Y = Y + X$

**K.** $Y - Z = Z - Y$

## Matrices Lesson 3 Practice Problems

**For 1 – 5, find the determinants of the following matrices, or if it's not possible, explain why**:

1. $\begin{bmatrix} -8 & -9 \\ 5 & -10 \end{bmatrix}$

2. $\begin{bmatrix} -2 & -4 \\ 10 & -10 \end{bmatrix}$

3. $\begin{bmatrix} 7 & 2 & -6 \\ 10 & 5 & -4 \end{bmatrix}$

4. $\begin{bmatrix} -5 & 9 & 9 \\ -10 & -8 & 4 \\ 10 & 3 & -4 \end{bmatrix}$

5. $\begin{bmatrix} -9 & 10 & -6 \\ -2 & -8 & -5 \\ 7 & -3 & 2 \end{bmatrix}$

**For #6-10, Does each given matrix have an inverse? If so, find it.**

6. $P = \begin{bmatrix} 1 & -3 \\ -1 & 4 \end{bmatrix}$

7. $R = \begin{bmatrix} -2 & 8 & -5 \\ 3 & -11 & 7 \\ 9 & -34 & 21 \end{bmatrix}$

8. $Q = \begin{bmatrix} -6 & -9 \\ -4 & -6 \end{bmatrix}$

9. $S = \begin{bmatrix} -24 & 18 & 5 \\ 20 & -15 & -4 \\ -5 & 4 & 1 \end{bmatrix}$

10. Are $\begin{bmatrix} 8 & 4 \\ 4 & -2 \end{bmatrix}$ and $\begin{bmatrix} \frac{1}{16} & \frac{1}{8} \\ \frac{1}{8} & -\frac{1}{4} \end{bmatrix}$ inverses? Explain how you know.

~~11. SKIP A student sketches a triangle with vertices at (5, 12), (8, 9), and (3, 6). Using vectors, what is the area of the triangle?~~

**For #12-13, Solve the matrix $A \cdot X = B$ for the following matrices:**

12. $A = \begin{bmatrix} 8 & -7 \\ -6 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 11 \\ -12 \end{bmatrix}$

13. $A = \begin{bmatrix} 2 & 8 & 4 \\ 1 & -1 & -3 \\ -3 & 2 & -9 \end{bmatrix}$, $B = \begin{bmatrix} 26 \\ -2 \\ 37 \end{bmatrix}$

**For #14-17, Solve the following systems of equations using invers matrices, if possible. Show the setup of your matrices.**

14. $\begin{cases} -x + 2y = 8 \\ -3x + 6y = -12 \end{cases}$

15. $\begin{cases} 2x + y + 2z = 18 \\ x - y + 2z = 9 \\ x + 2y - z = 6 \end{cases}$

16. $\begin{cases} -3x + 4y = -4 \\ \frac{1}{2}x - 3y = -11 \end{cases}$

17. $\begin{cases} 2x + \frac{2}{3}y + z = -8 \\ x + 2y - \frac{1}{3}z = 6 \\ -\frac{1}{2}x + 3y - 2z = 22 \end{cases}$

**For #18-19:** Luke had some quarters and dimes in his pocket. The quarters and dimes are worth $2.55. He has 3 times as many quarters as dimes.

18. Write a matrix equation to find the number of quarters, $x$, and dimes, $y$, Luke has.
19. How many quarters and dimes does Luke have?
20. The coordinates $(x, y)$ of a point in a plane are the solution of the matrix equation $\begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -5 \\ 2 \end{bmatrix}$. In what quadrant is the point located?

## Matrix Unit Practice Test

**Part I: No Graphing Calculators! You must show your work!**

Let $A = \begin{bmatrix} 2 & -3 \\ 3 & -3 \end{bmatrix}$ $B = \begin{bmatrix} 3 & 0 \\ -6 & 2 \end{bmatrix}$ $C = \begin{bmatrix} 2 & 1 & -3 \\ 0 & 4 & 1 \end{bmatrix}$ $D = \begin{bmatrix} 2 & 0 \\ -3 & 1 \\ 5 & -2 \end{bmatrix}$ $E = \begin{bmatrix} -1 & -1 & 2 \\ 2 & 2 & -3 \end{bmatrix}$

#1 – 9: Perform the indicated operation, if possible.

1) -2A 2) C + E 3) B + D 4) BA 5) E – ½ C 6) $B^{-1}$

7) $C^{-1}$ 8) DET (A) 9) Find $\begin{vmatrix} 10 & 7 \\ -3 & -2 \end{vmatrix}$

10) Solve for x and y: $2\begin{bmatrix} x & 3 \\ 9 & y-2 \end{bmatrix} = \begin{bmatrix} 14 & 6 \\ 18 & 5 \end{bmatrix}$

11) Find $\begin{vmatrix} -4 & 2 & 1 \\ 3 & 1 & 0 \\ 0 & 2 & -1 \end{vmatrix}$

12) A segment has endpoints (5, -3) and (2, 4) and the matrix form is $\begin{bmatrix} 5 & 2 \\ -3 & 4 \end{bmatrix}$. The segment is translated using the matrix operation $\begin{bmatrix} 5 & 2 \\ -3 & 4 \end{bmatrix} + \begin{bmatrix} -2 & -2 \\ 5 & 5 \end{bmatrix}$. Describe the translation.

13) Describe how $\overrightarrow{MN} = \langle -2, 9 \rangle$ is transformed when (a) it is multiplied by the matrix $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ (b) it is multiplied by $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$?

14) Given $\vec{v} = \langle 3, -7 \rangle$ and $\vec{w} = \langle -6, 4 \rangle$: (a) Find $\vec{v} + \vec{w}$ (b) Find $\vec{v} - \vec{w}$

15) Write a matrix that is the additive inverse of $A = \begin{bmatrix} -4 & -3 \\ 15 & -12 \\ 20 & 0 \end{bmatrix}$

**Part II: Graphing Calculators are okay. *Work must be shown!***

Solve the following systems. Set up matrices for each problem.

16) $\begin{cases} 4x - 2y = -6 \\ 3x + y = -7 \end{cases}$ 17) $\begin{cases} 5x + 2y = 1 \\ x + 2y = 5 \end{cases}$ 18) $\begin{cases} x + y + z = 7 \\ 2x - 3y - z = -1 \\ 3x + 2y - 2z = -4 \end{cases}$

19) $\begin{cases} x - y + 4z = 0 \\ 3x - 2y + z = -5 \\ 2x - y + 3z = 1 \end{cases}$ 20) $\begin{cases} 2a + 4b + c = 4 \\ a + 3b + 2c = 12 \\ 3a + b + c = -2 \end{cases}$

## Matrices Lesson 1 Answers

**1.** 4 boys who are seniors

**2.** $S = \begin{bmatrix} 1.25 & 2.00 & 2.50 \\ 1.00 & 1.75 & 2.25 \end{bmatrix}$

**4.** $x = 6, y = -6, z = 2.5$

**3.** $X + Y = \begin{bmatrix} -2 & -2 \\ 2 & 2 \end{bmatrix}$ $X - Y = \begin{bmatrix} 6 & -4 \\ -4 & 6 \end{bmatrix}$ $Y - X = \begin{bmatrix} -6 & 4 \\ 4 & -6 \end{bmatrix}$

**5.** **A, C, E**

**6.** $X = \begin{bmatrix} -2 & 5 \\ 6 & -3 \end{bmatrix}$

**7.** **a.** $Y$: (4, 0); $Z$: (6, 1)

**b.** $U$: (8, 16); $V$: (16, 20)

**8.** $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

**9.** $\begin{bmatrix} -126 & -78 \\ -1 & -15 \end{bmatrix}$

**10.** $\begin{bmatrix} -79 & -35 & 36 \\ -14 & -22 & 8 \\ 89 & 34 & -40 \end{bmatrix}$

**11.** Not possible

**12.** $\begin{bmatrix} 46 & -22 \\ -4 & 52 \\ -56 & 44 \end{bmatrix}$

**13.** Not possible

**14.** $\begin{bmatrix} 3 & -5 & 0 \\ -7 & 4 & 8 \\ 2 & 6 & -3 \end{bmatrix}$

**15.** Yes, both = $\begin{bmatrix} -1 & -7 \\ 37 & -13 \end{bmatrix}$

**16.** Yes, both = $\begin{bmatrix} 82 & -38 \\ -102 & 34 \end{bmatrix}$

**17.** $\begin{bmatrix} 1400 \\ 1375 \end{bmatrix}$

**18.** $[.45 \quad .30 \quad .25]$

**19.** $[93.2 \quad 87.25 \quad 86.15]$

**20.** **a.** $\begin{bmatrix} 2 & 5 & 4 & 2 \\ 4 & 1 & -3 & -1 \end{bmatrix}$

**b.** $\begin{bmatrix} -4 & -1 & 3 & 1 \\ 2 & 5 & 4 & 2 \end{bmatrix}$

**c.** 90° counterclockwise rotation

## Matrices Lesson 2 Answers

**1.** $\langle 8, 10 \rangle$

**2.** $\langle -4, 5 \rangle$

**3.** $\langle 3, -10 \rangle$, 10.44, −73.30°

**4.** $\langle 4, 8 \rangle$, 8.94, 63.43°

**5.** $\langle 16, 24 \rangle$, 28.84, 56.31°

**6.** $\langle -24, 48 \rangle$, 53.67, 116.57°

**7.** $T \cdot \overrightarrow{EF} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 5 \\ 3 \end{bmatrix} = \begin{bmatrix} 5+0 \\ 0+(-3) \end{bmatrix} = \begin{bmatrix} 5 \\ -3 \end{bmatrix}$

**8.** $T \cdot \overrightarrow{GH} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 2+0 \\ 0+1 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

**9.** **C**

**12.** **C**

**13.** **F, H, I, J**

**10.** The magnitude for $z \cdot \vec{t}$ increases by a factor of $z$, while the direction remains the same. The magnitude of $-z \cdot \vec{t}$ increases by a factor of $|-z|$, or $z$. The direction of $-z \cdot \vec{t}$ would be the direction of $\vec{t} + 180°$.

**~~11.~~** 3.83 mph; 196.8° or 16.8° south of west

## Matrices Lesson 3 Answers

1. 125 2. 60 3. Not possible 4. 350 5. -403

6. $\begin{bmatrix}4 & 3\\1 & 1\end{bmatrix}$ 7. 1 8. Does Not Exist 9. 1 10. Yes, the product equals the identity.

~~11. 12 units$^2$~~

12. $\begin{bmatrix}4\\3\end{bmatrix}$ 13. $\begin{bmatrix}-3\\5\\-2\end{bmatrix}$ 14. No solution 15. $\begin{bmatrix}5\\2\\3\end{bmatrix}$ 16. $\begin{bmatrix}8\\5\end{bmatrix}$ 17. $\begin{bmatrix}-2\\3\\-6\end{bmatrix}$

18. $\begin{bmatrix}.25 & .10\\1 & -3\end{bmatrix}\cdot\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}2.55\\0\end{bmatrix}$ 19. $9\ quarters, 3\ dimes$ 20. $IV$

## Practice Test Answers

1. $\begin{bmatrix}-4 & 6\\-6 & 6\end{bmatrix}$ 2. $\begin{bmatrix}1 & 0 & -1\\2 & 6 & -2\end{bmatrix}$ 3. Not Possible 4. $\begin{bmatrix}6 & -9\\-6 & 12\end{bmatrix}$

5. $\begin{bmatrix}-2 & -3/2 & 7/2\\2 & 0 & -7/2\end{bmatrix}$ 6. $\begin{bmatrix}1/3 & 0\\1 & 1/2\end{bmatrix}$ 7. Not Possible 8. 3 9. 1

10. x = 7, y = 9/2 11. 16 12. Left 2, up 5 13. (a) $180°$ Rotation (b) reflect across the y-axis

14. (a) (-3, -3) (b) (9, -11) 15. $\begin{bmatrix}4 & 3\\-15 & 12\\-20 & 0\end{bmatrix}$ 16. (-2, -1)

17. (-1, 3) 18. (2, 0, 5) 19. (2, 6, 1) 20. (-3, 1, 6)