Algebra 2 Ch 3 Notes

### Chapter 3 Calendar

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Day	Date	Assignment (Due the next class meeting)
Tuesday	9/13/22	3.1 Worksheet
Wednesday	9/14/22	Simplifying Radicals, real and imaginary
Thursday	9/15/22	3.2 Worksheet
Friday	9/16/22	Dividing radicals
Monday	9/19/22	3.3 Worksheet
Tuesday	9/20/22	Operations with radicals and complex numbers
Wednesday	9/21/22	3.4 Worksheet
Thursday	9/22/22	Division with binomials and complex numbers
Friday	9/23/22	3.5 Worksheet
Monday	9/26/22	Solving equations with complex roots
Tuesday	9/27/22	Ch 2 Prestign Test
Wednesday	9/28/22	Ch 3 Practice Test
Thursday	9/29/22	Ch 2 Tost
Friday	9/30/22	Ch 3 Test

- \* Be prepared for daily quizzes.
- \* Every student is expected to do every assignment for the entire unit.
- \* Students who complete *every assignment* for this semester are eligible for a 2% semester grade bonus and a pizza lunch paid by the math department if there are no late assignments.
- \* Try www.khanacademy.org or www.mathguy.us (Earl's website) if you need help.

## 3.1 Notes: Simplifying Radicals (Real and Imaginary)

Evaluate the following square roots:

$$\sqrt{49}$$

$$\sqrt{144}$$

$$\sqrt{81}$$

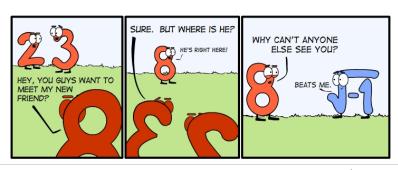
$$\sqrt{196}$$

$$\sqrt{25}$$

Explain in words how you knew the square roots of those values.

Does  $\sqrt{-36}$  have a solution? Why or why not?

The imaginary number *i*:



**Evaluate the following expressions:** 

$$\sqrt{-49}$$

$$\sqrt{-144}$$

$$\sqrt{-81}$$

$$\sqrt{-196}$$

$$\sqrt{-25}$$

Explore the powers of i below:

If  $i = \sqrt{-1}$ , then what are the values of the following expressions?

i	$i^5$
$i^2$	$i^6$
$i^3$	$ i^7 $
$i^4$	$ i^8 $

What pattern exists with the powers of i?

How would you find the value of  $i^{15}$ ?

What is the value of  $i^{37}$ ?

Find the value of  $i^{22}$ .

## **Simplifying Radicals**

**Examples:** Simplify the following expressions. If needed, write your answer in terms of i.

1) 
$$\sqrt{54}$$

2) 
$$-2\sqrt{48x^3}$$

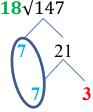
3) 
$$\sqrt{-75}$$

4) 
$$\sqrt{-56}$$

## **Multiplying Radicals**

Consider the following example to help you multiply radical expressions:

Example 7) 
$$6\sqrt{7} \cdot 3\sqrt{21}$$
  
=  $18\sqrt{147}$ 



$$= 18 \cdot 7\sqrt{3}$$
$$= 126\sqrt{3}$$

Multiply the following radical expressions:

5) 
$$4\sqrt{12} \cdot 5\sqrt{8}$$

6) 
$$2\sqrt{10} \cdot 3\sqrt{-5}$$

What portions were multiplied?

How were the factors of 147 found?

7) 
$$\sqrt{-4} \cdot \sqrt{-9}$$

8) 
$$-4\sqrt{-6} \cdot 2\sqrt{-18}$$

Why isn't the answer positive,  $since -4 \cdot -9 = 36$ ?

9) 
$$7x\sqrt{15x} \cdot \sqrt{6x}$$

#### **Multiplying Imaginary Numbers**

Use the example below of multiplying imaginary numbers, and then try the following problems on your own!

Multiply: 
$$-5i \cdot 3i$$
  
=  $-15i^2$   
=  $-15(-1)$   
= 15

Multiply:

10) 
$$-2i \cdot 12i$$
 11)  $-8i \cdot -7i$ 

11) 
$$-8i \cdot -7i$$

12) 
$$6i \cdot 2i^2$$

13) Create a multiplication problem involving 2 radicals that gives an answer of -20.

Review: Factor the following trinomials.

14) 
$$3x^2 - 2x - 5$$

14) 
$$3x^2 - 2x - 5$$
 15)  $2x^2 + 11x + 14$  16)  $4x^2 - 7x - 2$ 

16) 
$$4x^2 - 7x - 2$$

## 3.2 Notes: Dividing Radicals

Simplify the following expressions:

**Examples:** 

1) 
$$\frac{\sqrt{49}}{\sqrt{25}}$$

2) 
$$\frac{\sqrt{18}}{\sqrt{200}}$$

You try:

3) 
$$\frac{\sqrt{64}}{\sqrt{169}}$$

4) 
$$\frac{\sqrt{48}}{\sqrt{24}}$$

**Examples:** 

5) 
$$\frac{\sqrt{60}}{\sqrt{3}}$$

6) 
$$\frac{\sqrt{-36}}{\sqrt{-121}}$$

7) 
$$\frac{\sqrt{-8}}{\sqrt{18}}$$

You try:

8) 
$$\frac{\sqrt{-180}}{\sqrt{-20}}$$

9) 
$$\frac{\sqrt{-98}}{\sqrt{8}}$$

10) 
$$\frac{\sqrt{-15}}{\sqrt{3}}$$

11) 
$$\frac{3i}{5i}$$

$$12) \ \frac{i\sqrt{2}}{6i}$$

Multiply each:

a) 
$$\sqrt{6} \cdot \sqrt{6}$$

b) 
$$\sqrt{3} \cdot \sqrt{3}$$

c) 
$$\sqrt{5} \cdot \sqrt{5}$$

### Rationalizing the denominator:

The following article is taken from West Texas AM University's website: http://www.wtamu.edu/academic/anns/mps/math/mathlab

When a radical contains an expression that is not a perfect root, for example, the square root of 3 or cube root of 5, it is called an <u>irrational number</u>. So, in order to <u>rationalize</u> the denominator, we need to get rid of all radicals that are in the denominator.

# Step 1: Multiply numerator and denominator by a radical that will get rid of the radical in the denominator.

If the radical in the denominator is a square root, then you multiply by a square root that will give you a perfect square under the radical when multiplied by the denominator.

Note that the phrase "perfect square" means that you can take the square root of it.

Keep in mind that as long as you multiply the numerator and denominator by the exact same thing, the fractions will be equivalent.

Ex: Rationalize  $\frac{1}{\sqrt{10}}$ 

### Step 2: Make sure all radicals are simplified.

Some radicals will already be in a simplified form, but make sure you simplify the ones that are not.

**Ex**: Simplify then rationalize  $\frac{\sqrt{10}}{\sqrt{15}}$ 

### Step 3: Simplify the fraction if needed.

Be careful. You cannot cancel out a factor that is on the outside of a radical with one that is on the inside of the radical. In order to cancel out common factors, they have to be both inside the same radical or be both outside the radical.

**Ex**: Rationalize and simplify  $\frac{4}{\sqrt{6}}$ 

**Examples:** Divide the following expressions. Rationalize, if needed.

1) 
$$\frac{1}{\sqrt{5}}$$

2) 
$$\frac{7}{\sqrt{-11}}$$

You try!

3) 
$$\frac{5}{\sqrt{6}}$$

4) 
$$\frac{1}{\sqrt{-7}}$$

**Examples:** 

5) 
$$\frac{\sqrt{6}}{\sqrt{-13}}$$

6) 
$$\frac{\sqrt{20}}{\sqrt{7}}$$

7) 
$$\frac{2}{9i}$$

You try!

8) 
$$\frac{\sqrt{8}}{\sqrt{3}}$$

9) 
$$\frac{\sqrt{5}}{\sqrt{-20}}$$

10) 
$$\frac{3}{5i}$$

$$11) \ \frac{\sqrt{2}}{6i}$$

## 3.3 Notes: Operations with Radicals and Complex Numbers

Matching! Try to match each problem with its simplified expression.

**Problems** 

1) 
$$(5+3\sqrt{2})+(-3+4\sqrt{2})$$

2) 
$$(5+3\sqrt{2})-(-3+4\sqrt{2})$$

3) 
$$(5+3\sqrt{2})(-3+4\sqrt{2})$$

Perform the indicated operations:

1) 
$$(14 + 7\sqrt{3}) + (-10 + 2\sqrt{3})$$

3) 
$$(8 + \sqrt{11}) - (-12 + 7\sqrt{11})$$

A) 
$$8 - \sqrt{2}$$

B) 
$$9 + 11\sqrt{2}$$

C) 
$$2 + 7\sqrt{2}$$

2) 
$$(-27 + 9i) + (-13 - 6i)$$

4) 
$$(-10+3i)-(15-4i)$$

5) 
$$3(4-2i)-6(3+4i)$$

6) 
$$3i + 2i(1-i) - (7i + 8)$$

7) 
$$(6-3\sqrt{5})(2+4\sqrt{5})$$

8) 
$$(3+8i)(3-8i)$$

9) 
$$(2 - 3\sqrt{7})^2$$

10) 
$$(9+4i)^2$$

11) If you square a complex number, a + bi, how many terms will you get in your answer? Create an example, and show the solution for squaring the binomial.

12) If you multiply two complex numbers, will you always get a binomial? Why or why not? Create two examples with complex numbers that justify your answer.

Challenge: Can you multiply two **binomials with radicals** and have a product with four terms? Try to create an example where this happens.

13)

$$4x^2 - 9 = (px + t)(px - t)$$

In the equation above, p and t are constants. Which of the following could be the value of p?

- A) 2
- B) 3
- C) 4
- D) 9

## 3.4 Notes: Division with Binomials and Complex Numbers

For each binomial below, multiply it by another binomial so that the final answer has only one term:

1) 
$$7 - \sqrt{2}$$

2) 
$$8 + 5i$$

The pairs of binomials multiplied above are called \_\_\_\_\_\_.

Why could it be useful to have binomials that multiply out to a single term that is an integer?

Examples: Simplify the following expressions. Rationalize if needed.

1) 
$$\frac{1}{3-\sqrt{2}}$$

2) 
$$\frac{3}{4+5i}$$

You try!

$$3) \qquad \frac{3}{1+\sqrt{3}}$$

$$4) \qquad \frac{5}{2-4i}$$

**Examples:** 

5) 
$$\frac{5i}{2-9i}$$

6) 
$$\frac{3+\sqrt{5}}{4-\sqrt{2}}$$

You try!

$$7)\ \frac{\sqrt{7}}{8+\sqrt{3}}$$

8) 
$$\frac{1+2i}{5-4i}$$

**Review:** 

A motor powers a model car so that after starting from rest, the car travels s inches in t seconds, where  $s = 16t\sqrt{t}$ . Which of the following gives the average speed of the car, in inches per second, over the first t seconds after it starts?

- A)  $4\sqrt{t}$
- B) 16√*t*
- C)  $\frac{16}{\sqrt{t}}$
- D) 16t

## 3.5 Notes: Solving equations with complex roots

Review: For each equation below, work with a partner to solve for x using square roots.

1) 
$$5x^2 = 45$$

2) 
$$2x^2 - 10 = 88$$

$$3) -3x^2 + 5 = 5$$

With your partner, list the steps to solve for x:

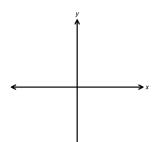
How many solutions does each example have above?

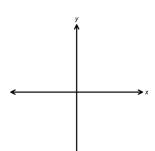
Draw a quadratic that has:

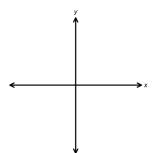
2 real solutions

1 real solution

0 real solutions







A quadratic that has no *real* solutions has 2 *complex* solutions.

**Examples**: solve for x to find the solutions (real or complex).

4) 
$$x^2 + 50 = 34$$

$$(x+5)^2 + 8 = 4$$

You try!

6) 
$$10(x-3)^2 = -90$$

7) 
$$-2x^2 + 5 = 13$$

**Examples**:

8) 
$$2(x-1)^2 - 9 = -7$$

9) 
$$-(x-2)^2-2=6$$

You try!

$$10) -5(x+6)^2 = 30$$

11) 
$$2(x-7)^2 - 7 = 13$$