The Imaginary Number

The four powers of i:

$$i = i^5 = i^9 = i^{13} = i$$

$$i^2 = i^6 = i^{10} = i^{14} = \left(\sqrt{-1}\right)^2 = -1$$

$$i^3 = i^7 = i^{11} = i^{15} = \left(\sqrt{-1}\right)^3 = -i$$

$$i^0 = i^4 = i^8 = i^{12} = i^{16} = \left(\sqrt{-1}\right)^4 = 1$$

So the pattern is:
$\{i, -1, -i, 1, i, -1, -i, 1, i, -1, -i, 1, \ldots\}$

How to simplify powers of i:

- Use exponent & divide by 4
- The remainder is your new power (will be 0,1,2,3)

Simplifying Radicals

Without imaginary numbers:

- Factor number(s) under radical
- Take out "pairs" or perfect squares of numbers AND variables
- Simplify outside
- Simplify inside

Ex:

$$\sqrt{48} = \sqrt{2 \cdot 2 \cdot 2 \cdot 2 \cdot 3}$$
$$= 2 \cdot 2 \cdot \sqrt{3}$$
$$= \mathbf{4\sqrt{3}}$$

<u>**Or**</u>

- Look for factors of radicand that are perfect squares (1, 4, 9, 16, etc.)
- Factor out and simplify

$$\sqrt{48} = \sqrt{16} \cdot \sqrt{3}$$
$$= \mathbf{4\sqrt{3}}$$

With imaginary numbers:

- If there's a negative under the radical, think of it as $\sqrt{-1}$
- Factor out the $\sqrt{-1} = i$ which means put an i on the outside of the radical instead

Multiplying Radical and Imaginary Numbers

Multiply radicals:

- Combine numbers inside the radicals and combine numbers outside the radicals
- Simplify normally

$$2\sqrt{10} \cdot 3\sqrt{20} = 2 \cdot 3 \cdot \sqrt{10 \cdot 20}$$
$$= 6\sqrt{2 \cdot 5 \cdot 5 \cdot 2 \cdot 2}$$
$$= 6 \cdot 5 \cdot 2\sqrt{2} = \mathbf{60\sqrt{2}}$$

Multiplying imaginary numbers:

- Combine i's by adding exponents.
- Simplify

$$-5i \cdot 3i = -15i^2 = -15 \cdot -1 = \mathbf{15}$$

Dividing Radicals and Imaginary Numbers

Steps to follow:

- Reduce fractions (inside radical and outside radical)
- Simplify the top.
- Simplify the bottom
- Rationalize
- Simplify again, if possible.

$$\frac{4\sqrt{4}}{2\sqrt{10}} = \frac{2\sqrt{2}}{\sqrt{5}} = \frac{2\sqrt{2}\cdot\sqrt{5}}{\sqrt{5}\cdot\sqrt{5}} = \frac{2\sqrt{10}}{5}$$

When dividing radicals, the finals solution is fully simplified if….

- The top and bottom are both simplified
- There isn't an i or $\sqrt{\ }$ in the denominator
- Fraction is reduced

How to rationalize:

- Identify the imaginary number or radical in the denominator that you want to get rid of
- Multiply the top and bottom by that number.
- Simplify.

The number i is the imaginary part.

$$\frac{2}{5i} = \frac{2\cdot i}{5i\cdot i} =$$

$$\frac{2i}{5i^2} = \frac{2i}{5\cdot -1} = -\frac{2i}{5}$$

Operations with Binomials and Complex Numbers

Binomials:

$5 + 3i$

Real part + imaginary part

Complex Numbers:

$5 + 3i$

Real part + imaginary part

Adding: Combine like terms

$$(3 + 2\sqrt{7}) + (-8 + \sqrt{7})$$

$$= (3 + -8) + (2\sqrt{7} + 1\sqrt{7})$$

$$= -5 + 3\sqrt{7}$$

Subtracting: Distribute the -1, and then combine like terms.

$$(3 + 2\sqrt{7}) - (-8 + \sqrt{7})$$

$$= (3 + 2\sqrt{7}) + (8 - 1\sqrt{7})$$

$$= 11 + 1\sqrt{7}$$

Multiplying: FOIL (use the distributive property twice), or use the box method to multiply.

$$(3 + 2\sqrt{7})(-8 + \sqrt{7})$$

$$= -24 + 3\sqrt{7} - 16\sqrt{7} + 2\cdot 7$$

$$= -24 - 13\sqrt{7} + 14$$

$$= -10 - 13\sqrt{7}$$

Dividing: Rationalize by multiplying the numerator and the denominator by the **conjugate** of the denominator.

Sample #1: $\frac{-3}{2+\sqrt{5}} = \frac{-3}{(2+\sqrt{5})}\cdot\frac{(2-\sqrt{5})}{(2-\sqrt{5})}$

$$= \frac{-6 + 3\sqrt{4}}{4 - 5}$$

$$= \frac{-6 + 3\sqrt{4}}{-1}$$

$$= 6 - 3\sqrt{4}$$

Sample #2: $\frac{4i}{3+2i} = \frac{4i}{(3+2i)}\cdot\frac{(3-2i)}{(3-2i)}$

$$= \frac{12i - 8i^2}{9 - 4i^2}$$

$$= \frac{12i + 8}{9 + 4}$$

$$= \frac{8 + 12i}{13}$$