Unit 12 Assignments for Prob/Stat/Discrete
Chapter 15: Graph Theory

Day	Date	**Assignment** (Due the next class meeting)
		12.1 Worksheet
		12.2 Worksheet
		12.3 Worksheet
		12.4 Worksheet
		12.5 Worksheet
		Unit 12 Practice Test
		Unit 12 Test

NOTE: You should be prepared for daily quizzes.

HW reminders:

- If you cannot solve a problem, get help **before** the assignment is due.
- Help is available before school, during lunch, or during IC.
- For extra practice, visit www.interactmath.com
- Don't forget that you can get 24-hour math help from www.smarthinking.com or mathguy.us

Prob/Stat/Discrete **Name**______________________________
Unit 12 Guided Notes

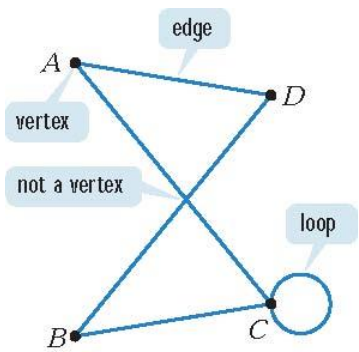
12.1: Graphs, Paths, and Circuits

Objectives

1. Can you model relationships using graphs?
2. Can you use the vocabulary of graph theory?

VOCABULARY

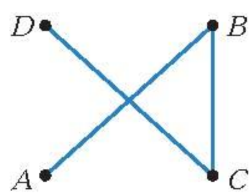
- ____________________: a finite set of points (singular is *vertex*)
- Line segments or curves, called __________________.

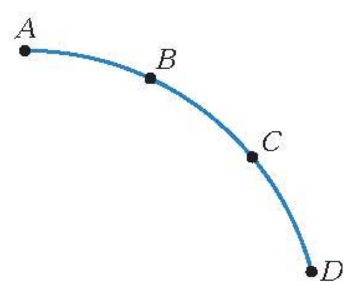


- An edge that starts and ends at the same vertex is called a _________________.
- A _________________ consists of vertices and edges that start and end at vertices.

- Equivalent Graphs: graphs with the same _________________ connected by the same number of ________________.

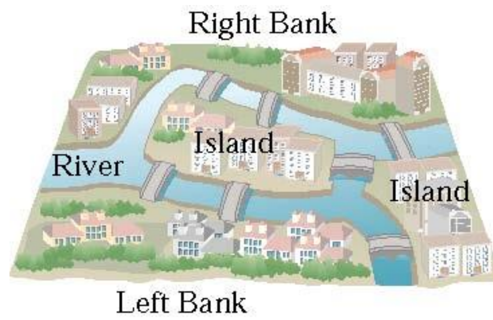
Example: Are the two graphs shown equivalent? Explain your answer.





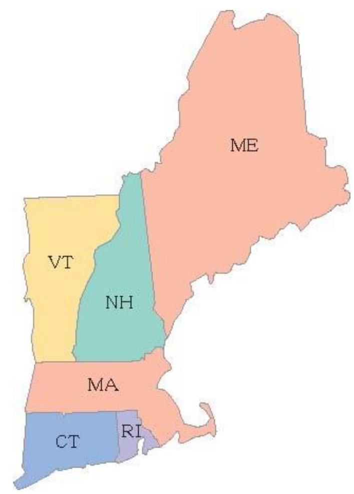
Modeling Graphs: We can use graphs to represent relationships in a variety of situations.

1. **Layout of a city**
   Example: In the early 1700's, the city of Königsberg, Germany, was located on both banks and two islands of the Pregel River. The figure shows that the town's sections were connected by seven bridges.

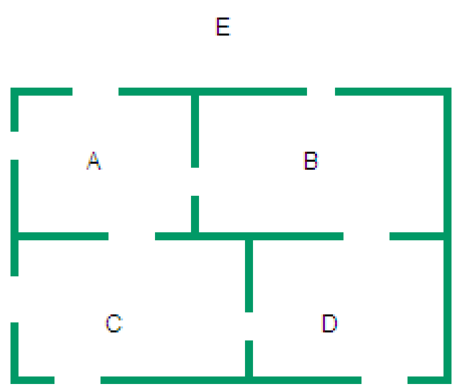
   

   Draw a graph that models the layout of Königsberg. Use vertices to represent the land masses and edges to represent the bridges.

2. **Bordering Relationships**
   Example: The map of New England states are given. Draw a graph that models which New England states share a common border. Use vertices to represent the states and edges to represent common borders.

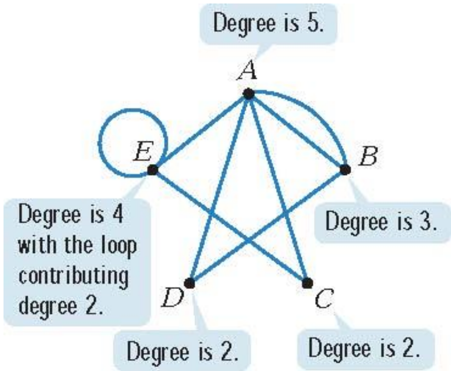
   

3. **Floor plan of a building**
   Example: The floor plan of a four-room house is shown. The rooms are labeled A, B, C, and D, and the outside of the house is labeled as E. The openings represent doors. Draw a graph that representing the connecting relationships, using vertices as rooms and the outside, and edges to model the connecting doors.

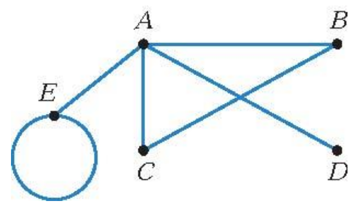
Vocabulary of Graph Theory

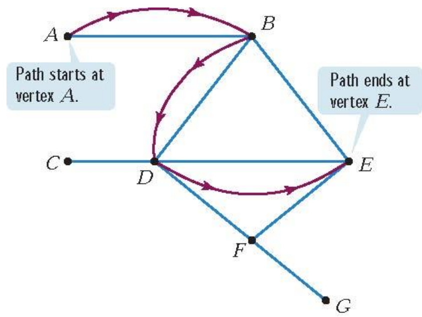
- The ___________________ ***of a vertex*** is the number of edges at that vertex.



- A vertex with an even number of edges attached to it is an _____________ ***vertex***.
- A vertex with an odd number of edges attached to it is an ______________ ***vertex***.
- Two vertices in a graph are said to be _______________________ ***vertices*** if there is at least one edge connecting them.

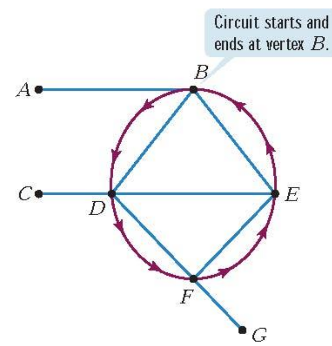
Example: List the pairs of adjacent vertices for the graph shown.





A ________________ in a graph is a sequence of adjacent vertices and the edges connecting them.

Notice the path along this graph is represented by sequential arrows.



A ____________________ is a path that begins and ends at the same vertex.

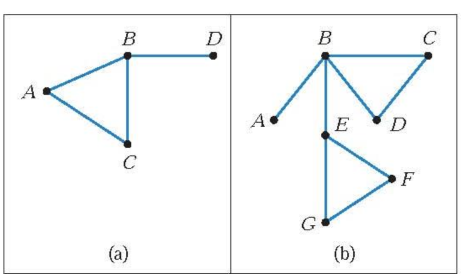
A graph is _____________________ if for any two of its vertices there is at least one path connecting them.

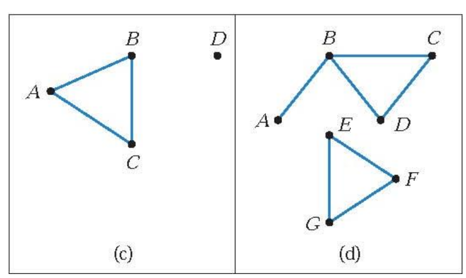
A graph that is not connected is called ________________________________.

A disconnected graph is made of pieces that are connected by themselves, called __________________________of the graph.

A ______________ is an edge of a connected graph that, if removed, would leave behind a disconnected graph.

Example: label the graphs below as connected or disconnected.





12.2 Notes: Euler Paths and Circuits

Objectives

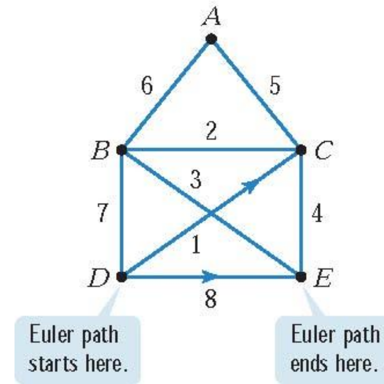
1. **Can you compare and contrast an Euler path and an Euler circuit?**
2. **Can you summarize Euler's Theorem?**
3. **Can you solve problems using Euler's Theorem?**
4. **Can you use Fleury's Algorithm to find possible Euler paths and Euler circuits?**

Vocabulary

- An _________________ ______________ is a path that travels through every edge of a graph once and only once. Each edge must be traveled and no edge can be retraced.

  A graph with an Euler Path has _______ odd vertices and the rest are _________ vertices.

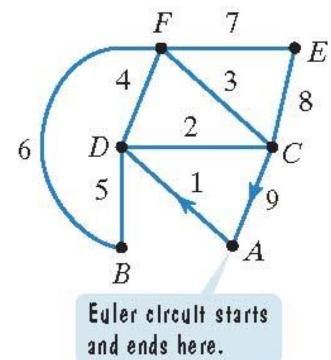
  An Euler Path must start and end at the two odd vertices of the graph. Use your pencil to darken the edges and trace an Euler Path.



- An ________________ ________________ is a circuit that travels through every edge of a graph once and only once. Like all circuits, an Euler circuit must begin and end at the same vertex.

  A graph with an Euler Circuit has all __________ vertices.

  An Euler Circuit may start at any vertex. Use your pencil to trace an Euler Circuit starting and ending at vertex A.



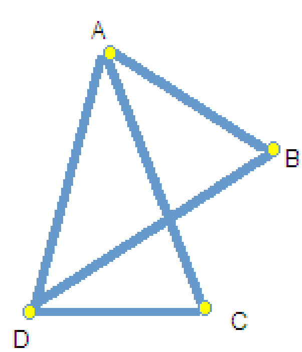
Euler's Theorem: The following are true for connected graphs…

1. If a graph has _____________________ _______ __________ vertices, then it has at least one Euler path, but no Euler circuit.
2. If a graph has no odd vertices (__________ ___________________ vertices), it has at least one Euler circuit.
3. If a graph has _____________ _______________ two odd vertices, then it has no Euler paths and no Euler circuits.

Example:

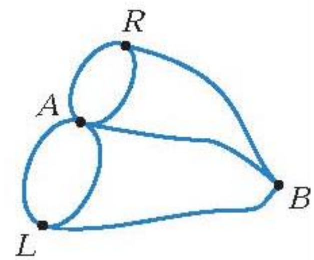
- Explain why the graph in the figure has at least one Euler path.

- Use trial and error to name one Euler path.



Example: Recall the graph that models the layout of Königsberg.

1) Use Euler's theorem to determine whether or not a person can walk across all of the bridges in Königsberg exactly one time without re-crossing any bridge.



2) Use Euler's Theorem to decide whether or not a person can start at one land mass of Königsberg, cross all bridges exactly once, and return to the original land mass.

Fleury's Algorithm to find Euler Paths and Circuits

To find an Euler Path:

1. Choose one of the two ________________ vertices as the starting point. The other odd vertex will be the ending point.

2. List the edges as you trace through the graph according to the following rules:
    - As you travel over an edge, draw hash marks to show that edge has been crossed. Label each edge with a consecutive number.
    - When faced with a choice of edges to trace, first choose any edge that is not a bridge. Travel over a bridge only if there is no alternative.

To find an Euler Circuit:

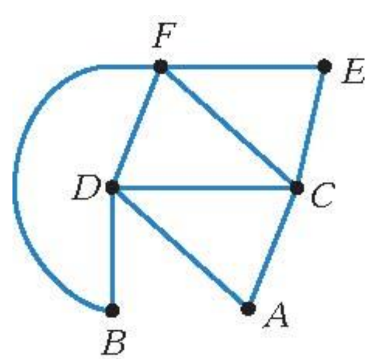
1. Choose any vertex as the ________________ point.

2. List the edges as you trace through the graph according to the following rules:
    a. As you travel over an edge, draw hash marks to show that edge has been crossed. Label each edge with a consecutive number.
    b. When faced with a choice of edges to trace, first choose any edge that is not a bridge.

3. The starting vertex is also the ______________ point.

Example:

a) Explain why the graph shown has an Euler circuit.



b) Find one by using Fleury's Algorithm.

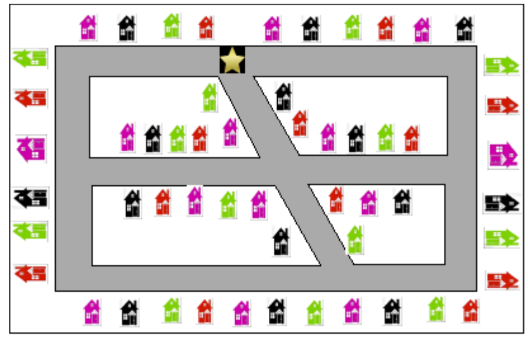
12.2 Notes continued: Euler Paths and Circuits

Objectives

1. **Can you determine whether a situation is asking you to find an Euler Path or Circuit?**
2. **Can you use Fleury's Algorithm in application problems?**

Examples:

Use the graph shown for #1 – 5.



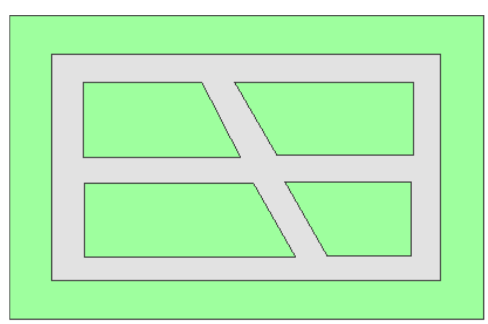
1) A mail carrier parks her truck at the intersection starred in the figure and then walks to deliver mail to each of the houses. The streets on the outside of the neighborhood have houses on side of the street only. Many interior streets have houses on both sides of the street. On those streets, the mail carrier must walk down the street twice, covering each side of the street separately. Draw a graph of this situation.

2) Is it possible for the mail carrier to start at the starred intersection, deliver mail to each house without retracing her route, and then return to the starred intersection? Explain your reasoning.

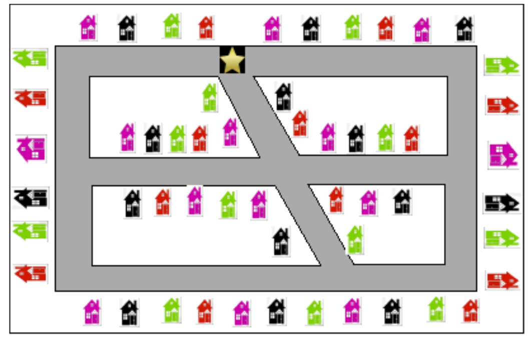
3) Does this situation describe an Euler Path, an Euler Circuit, or neither or these?

4) Use Fleury's Algorithm to list the route the mail carrier would take for the situation described in #2.

5) Use your answer from #4 to show the mail carrier the route she should follow on the map of the neighborhood. Be sure the route is clearly designated.



For # 6 – 9, use the map of the neighborhood shown.



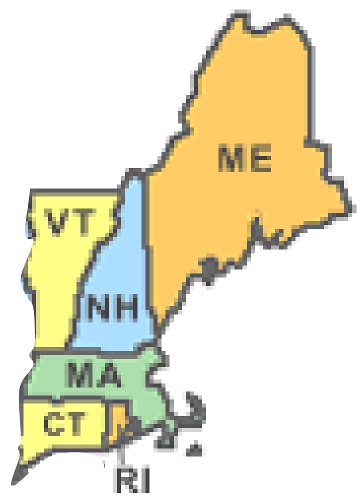
6) A security guard patrols the streets of the neighborhood. Unlike the mail carrier, the guard is to walk down each street once, whether or not the street has houses on both sides. Draw a graph that models the neighborhood walked by the security guard.

7) Will the residents in the neighborhood be able to establish a route for the security guard so that each street is walked exactly once? If so, where would the security guard begin the walk? Explain your answer.

8) Does the situation in #7 describe an Euler Path, an Euler Circuit, or neither of these?

9) Use the graph you drew in #6 to show the route the security guard will need to walk, if this situation is possible.

For # 10 – 14, use the map shown.

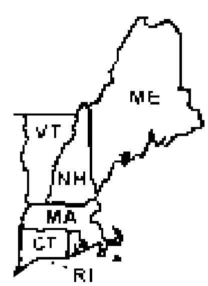


10) Create a graph of the map shown, using each state as a vertex and each border as an edge.

11) A family would like to travel to each state shown on the map while only crossing each common state border exactly once. Does this describe an Euler Path, and Euler Circuit, or neither?

12) Use Euler's Theorem to explain why it is possible for the family to do travel in the way described in #11.

13) Use Fleury's Algorithm to find this route on the graph you drew in #10.



14) Use your answer from #13 to show the route on the map of the New England states shown.

12.3 Notes: Hamilton Paths and Hamilton Circuits

Objectives

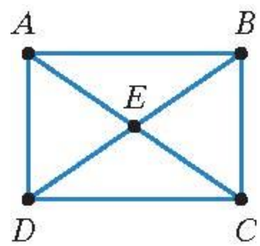
1. **Can you define Hamilton paths & Hamilton circuits?**
2. **Can you find the number of Hamilton circuits in a complete graph?**
3. **Can you use weighted graphs to solve problems?**

Vocabulary:

- A path that passes through each vertex of a graph exactly once is called a ________________ ________________.
- If a Hamilton path begins and ends at the same vertex and passes through all other vertices exactly once, it is called a ________________________ ___________________.
- A _______________ ______________ is a graph that has an edge between each pair of its vertices.

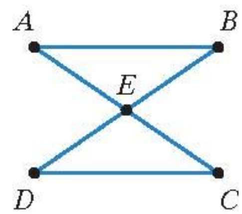
Examples:

1) Find a Hamilton Path.

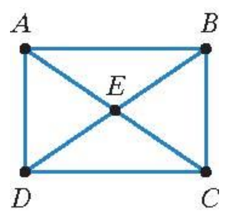


2) Find a Hamilton Circuit.

3) Is the graph shown connected? Complete?



4) Is the graph below connected? Complete?

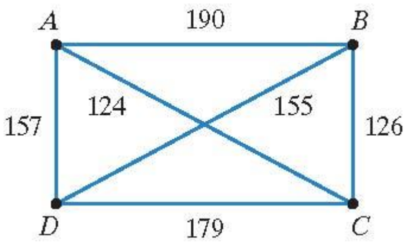


Note: The number of Hamilton circuits in a complete graph with n vertices is __________________.

5) How many Hamilton circuits are there in the graph from #4? List them!

Weighted Graphs

- A complete graph whose edges have numbers attached to them is called a ______________________ _____________________.
- The numbers shown along the ________________ of a weighted graph are called the *weights* of the edges.



Examples: The graph shown is a model graph for one-way airfares for cities *A*, *B*, *C*, *D*.

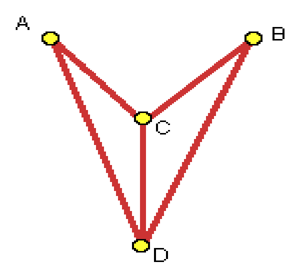
1) Find the weight of edge AC.

2) Use the weighted graph to find the cost of the trip for the Hamilton circuit *A, B, D, C, A*.

3) Use the weighted graph to find the cost of the trip for the Hamilton circuit *A, D, C, B, A*.

4) Use the weighted graph to find the cost of the trip for the Hamilton circuit *A, C, B, D, A*.

5) Which Hamilton circuit would be the least expensive?



Example: Use the graph shown for # 6 – 8.
6) Modify the graph by adding the least number of edges so that the resulting graph is complete. Determine the number of Hamilton Circuits for the graph.

7) Give two Hamilton Circuits for your new graph.

8) Modify the original graph by removing the least number of edges so that the resulting graph has an Euler Circuit.

9) Find an Euler Circuit for the graph drawn in #8.

12.3 Continued: Methods of Solving Traveling Salesperson Problems

Objectives

1. Can you use the Brute Force Method to solve traveling salesperson problems?
2. Can you use the Nearest Neighbor Method to approximate solutions to traveling salesperson problems?
3. Can you describe the benefits and weaknesses of the Nearest Neighbor Method?

The Traveling Salesperson Problem (TSP)

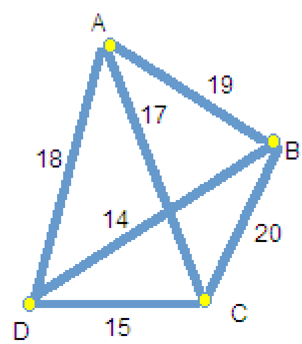
The *traveling salesperson problem* is the problem of finding a _______________ circuit in a complete weighted graph for which the sum of the weights of the edges is a ___________________.

Note: Such a Hamilton circuit is called the *optimal Hamilton circuit* or the *optimal solution*.

The Brute Force Method: One way to find the optimal solution for a TSP.

- Model the problem with a _________________, weighted graph.
- Make a _____________ of all possible Hamilton circuits.
- Determine the __________ of the weights of the edges for each of these Hamilton circuits.
- The Hamilton circuit with the __________________ sum of weights is the optimal solution.

Example: A businessman lives in City D and needs to travel to cities A, B, and C before returning home. The weighted graph shows the cost (in dollars) of gasoline used to drive to each city. Use the Brute Force Method to find his optimal circuit.



Hamilton Circuit	Weights	Sum of Weights

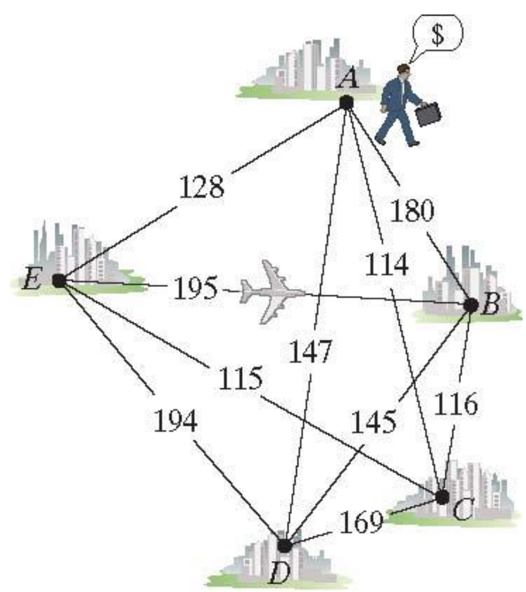
The optimal circuit is __________________________________, with a weight of ______________.

Although the Bruce Force Method always provides the optimal solution, it is time-consuming and can be difficult for more difficult problems.
Reminder: for a complete graph, the number of possibilities can be found by using (n – 1)!
How many circuits would need to be considered in order to find the optimal solution to a TSP if there are 5 vertices? 6 vertices?

The Nearest Neighbor Method: Used to ______________________ solutions to a TSP.

1. Model the problem with a complete, weighted graph.
2. Identify the _______________ that serves as the starting point.
3. From the starting point, choose an edge with the ______________ weight. **Darken** this edge to the second vertex.
4. From the 2nd vertex, choose the edge with the smallest weight that does __________ lead to a vertex already visited. **Darken** this edge to the third vertex.
5. Continue building the circuit, one vertex at a time, by moving along the edge with the smallest weight until all vertices are visited.
6. From the last vertex, return to the _________________ point.

Example: A sales director who lives in city *A* is required to fly to regional offices in cities *B*, *C*, *D*, and *E*. The weighted graph shown gives the one-way airfares. Use the Nearest Neighbor Method to find an approximate solution. What is the total cost?



Is this always going to be the optimal solution?

How could the optimal solution be found by using the Nearest Neighbor Algorithm?

Why would the Nearest Neighbor Algorithm be used instead of the Brute Force method?

12.4 Notes: Trees

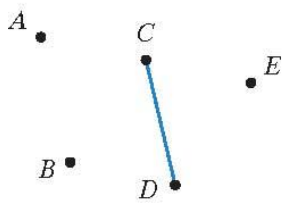
Objectives

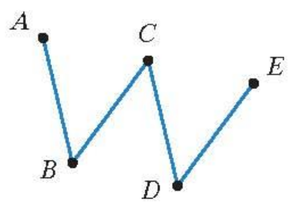
1. **Can you decide if a graph is a tree and justify your reasoning?**
2. **Can you use the properties of a tree?**
3. **Can you find a spanning tree for a connected graph?**
4. **Can you discover the minimum spanning tree for a weighted graph?**

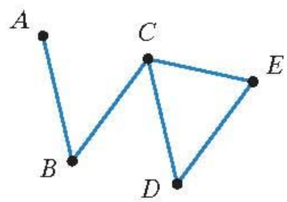
Definition: A ***tree*** is a graph that is ______________________ and has _________ __________________. Some other ways to say this are:

- A tree is a connected graph in which every edge is a bridge.
- A tree is a connected graph with n vertices and $n - 1$ edges.

Example: Which of the following graphs are trees? Why or why not?

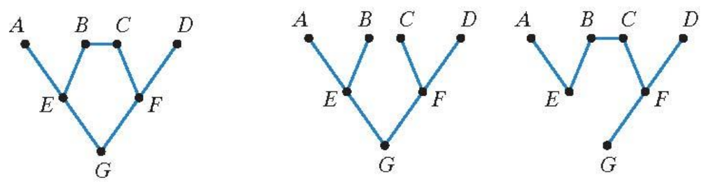
1) 

2) 

3) 

Vocabulary:

- A ______________________ is a set of vertices and edges chosen from among those of the original graph.

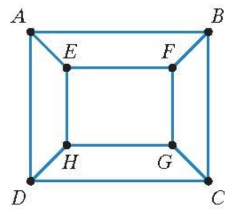


Original graph   Two possible subgraphs

- A subgraph that contains ________ of a connected graph's vertices, is connected, and contains no circuits is called a _____________________ __________________.

Are the two subgraphs shown above spanning trees? Explain.

Example: Find a spanning tree for the graph shown.



Vocabulary: The ____________________ _________________ ________________ for a weighted graph is a spanning tree with the smallest possible total weight.

Kruskal's Algorithm to find a minimum spanning tree from a weighted graph.

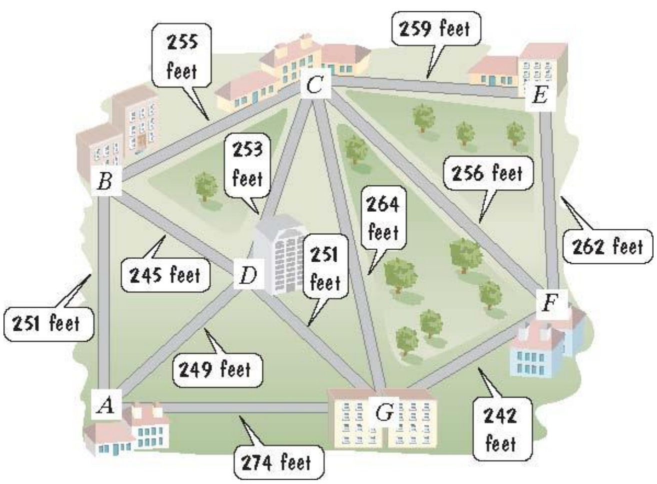
1. Find the edge with the _______________ weight in the graph. If there is more than one, pick one at random. Darken (or highlight) that edge.

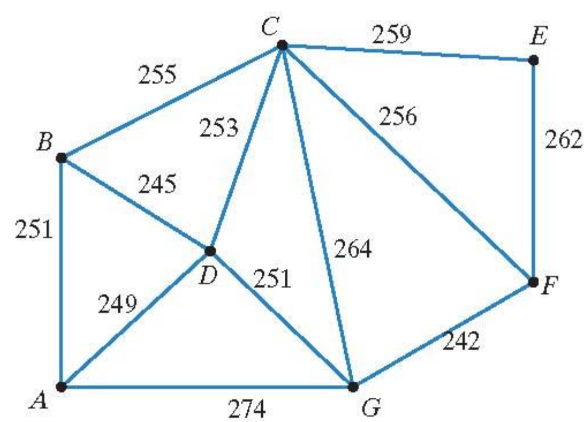
2. Find the ___________-smallest edge in the graph. If there is more than one, pick one at random. Darken (or highlight) that edge.

3. Find the next-smallest unmarked edge in the graph that does not create a darkened _____________. If there is more than one, pick one at random. Darken (or highlight) that edge.

4. Repeat step 3 until all ____________________ have been included ($n - 1$ edges). The darkened edges are the desired minimum spanning tree.

Example: Seven buildings on a college are connected by the sidewalks shown in the figure. The weighted graph represents buildings as vertices, sidewalks as edges, and sidewalk lengths as weights. A heavy snow has fallen and the sidewalks need to be cleared quickly. Campus decides to clear as little as possible and still ensure that students walking from building to building will be able to do so along cleared paths. Determine the shortest series of sidewalks to clear. What is the total length of the sidewalks that need to be cleared?





The total minimum length of sidewalks to be cleared is _________________.

12.5 Notes: Map Coloring

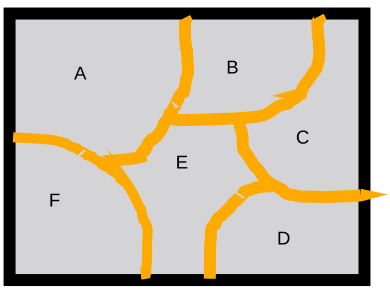
Note: This section is not from your textbook.

Objectives:

- **Can you create a diagram to model a coloring situation?**
- **Can you find the fewest possible number of items needed for a coloring situation?**

For centuries, people have wondered what minimum number of ___________________ are required to shade countries on a map so that countries with a common border will not be shaded the same color.

Consider the map shown below. Using a vertex for each region and an edge to represent a shared border, draw a diagram representing the map.



To find the minimum number of colors needed for this graph, start by finding the degree of each vertex.

Degree of a vertex: the number of __________________ using that vertex.

1. Starting with the vertex with the highest degree, assign a ___________________ to that vertex.
2. Are there any vertices that are NOT adjacent to the vertex from #1? If so, assign the same _______________ to those vertices, making sure that they are not adjacent to each other either.
    a. Note: If you have multiple options, start with the vertices with the highest _________________.
3. Continue steps 1 and 2, starting with the vertex with the highest degree that has not already been assigned a color.
4. When all vertices have been assigned a color, ______________ the number of colors used. This value is your final answer.

Map Coloring: Avoiding Scheduling Conflicts

The table shows various sports at a school, along with some female students who participate in them.

The coaches want to organize end-of-the-year banquets for each sport. What is the fewest number of nights that will be needed if they are scheduled so that each student shown does not have a scheduling conflict?

Student	Cross-County	Softball	Basketball	Volleyball
Amy	X		X	X
Heather	X	X		
Sara			X	X
Amber	X	X	X	

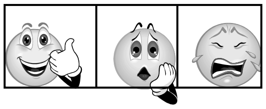
Step 1: Create a graph. Use a vertex for each sport. An edge indicates a conflict (a girl in both sports).

Step 2: Start with the vertex with the highest degree. Assign a day to that vertex and any that do not share an edge with it.

Step 3: Go to a vertex with the next highest degree. Assign a new day to it and any vertices that do not share an edge with it (or each other).

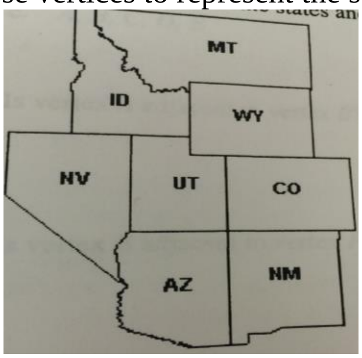
Step 4: Continue until all sports are assigned a day.

Prob/Stat/Discrete **Name**____
Unit 12 Objectives

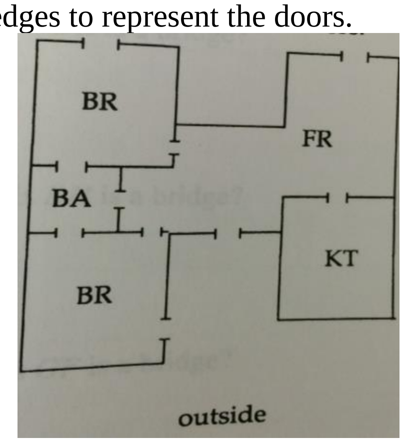


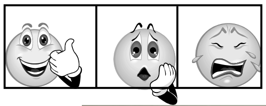
Objective #1: Can you model relationships using graphs?

1) Create a graph that models the bordering relationship among the sates shown in the map. Use vertices to represent the states and edges to represent common borders.

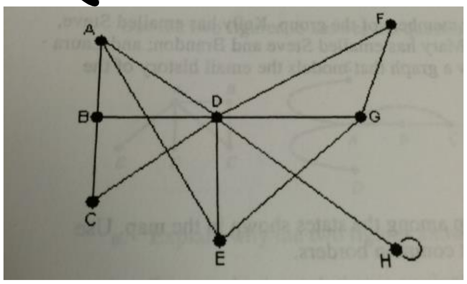


2) Draw a graph that models the floor plan. Use vertices to represent the rooms and the outside, and edges to represent the doors.

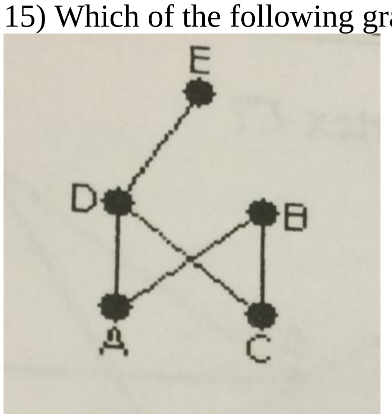


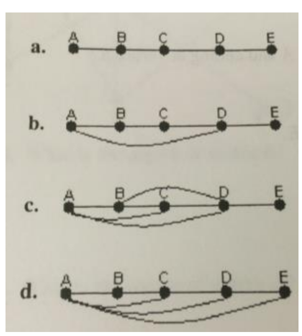


Objective #2: Can you use the vocabulary of graph theory?

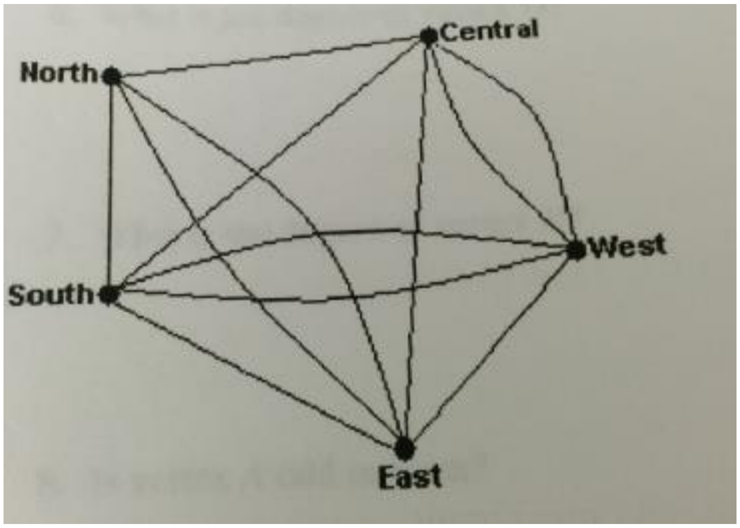


1) What is the degree of vertex B?
2) What is the degree of vertex C?
3) What is the degree of vertex D?
4) What is the degree of vertex H?
5) Is vertex A odd or even?
6) Is vertex f odd or even?
7) Which of the following is NOT a path starting at vertex A and ending at vertex E?
   a. A, B, C, D, H, E
   b. A, E
   c. A, B, D, G, E
   d. A, D, F, G, E
8) Which of the following is NOT a path starting at vertex A and ending at vertex E?
   a. A, C, D, E
   b. A, D, E
   c. A, B, C, D, E
   d. A, D, G, E
9) Is vertex A adjacent to vertex B?
10) Is vertex E adjacent to vertex F?
11) Is vertex H adjacent to vertex A?
12) Is DE is a bridge?
13) Is DH is a bridge?
14) Is GF is a bridge?
15) Which of the following graphs is equivalent to the graph shown?





For Exercises 16 and 17, the graph models the football schedule for 5 area high schools. The vertices represent the teams and each game played is represented as an edge between two teams.

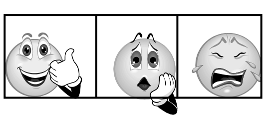


16) How many times does East play each of the other teams?

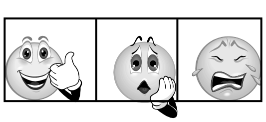
Number of times that East plays each of these other teams			
North	South	West	Central

17) How many times does West play each of the other teams?

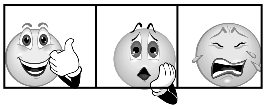
Number of times that West plays each of these other teams			
North	South	East	Central



Objective #3: Can you compare and contrast an Euler path and an Euler circuit?



Objective #4: Can you summarize Euler's Theorem?

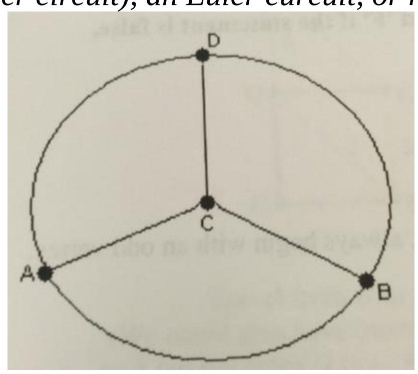


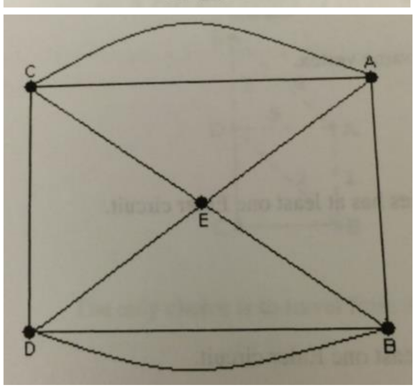
Objective #5: Can you solve problems using Euler's Theorem?

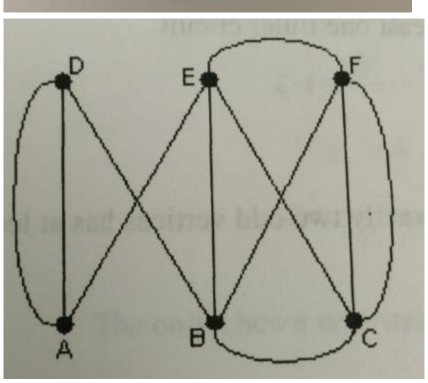
For Exercises 1-8, write 'T' if the statement is true and 'F' if the statement is false

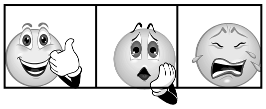
1) A Euler path can start and end at the same vertex.
2) When using Fleury's Algorithm to find an Euler path, always begin with an odd vertex.
3) When using Fleury's Algorithm to find an Euler circuit, always being with an even vertex.
4) An Euler circuit always starts and ends at the same vertex.
5) A connected graph with exactly two odd vertices has at least one Euler circuit.
6) A connected graph with no odd vertex has at least one Euler circuit.
7) A connected graph with exactly one odd vertex has at least one Euler circuit.
8) A connected graph with exactly two even vertices and exactly two odd vertices has at least one Euler path.

For Exercises 9-11, use Euler's theorem to determine whether the graph has an Euler path (but no and Euler circuit), an Euler curcuit, or neither. Explain your reasoning.

9) 

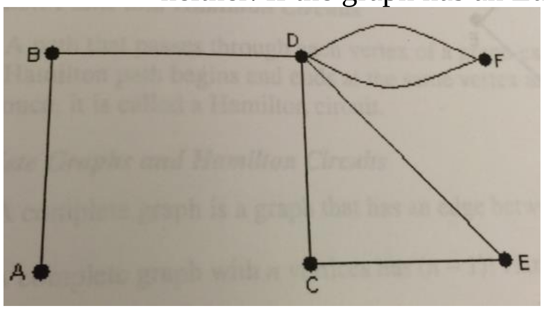
10) 

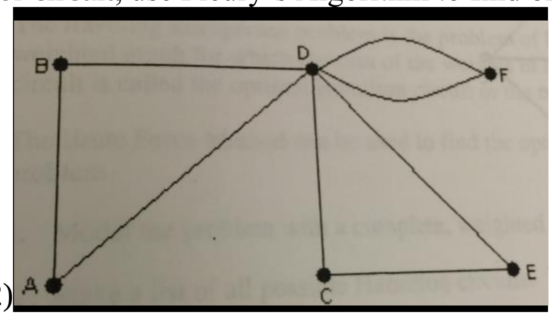
11) 

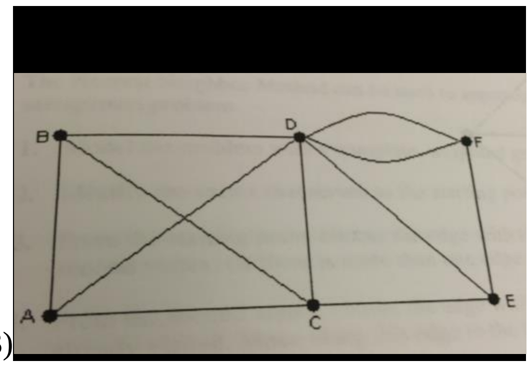


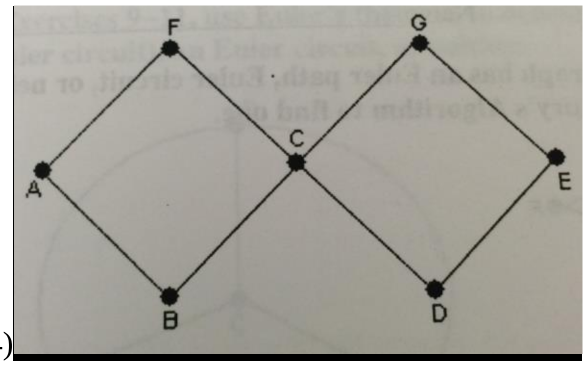
Objective #6: Can you use Fleury's Algorithm to find possible Euler Paths and Euler Circuits?

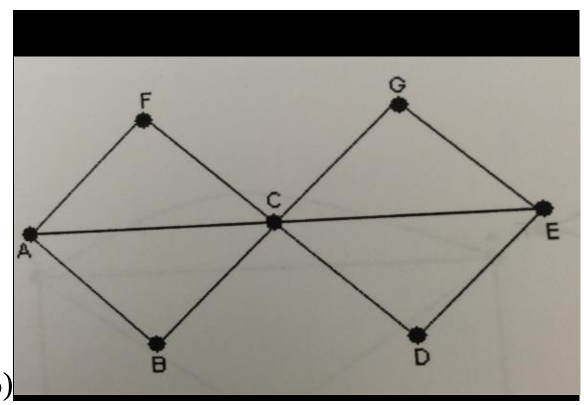
For Exercises 1-6, determine whether the graph has an Euler path, Euler circuit, or neither. If the graph has an Euler path or circuit, use Fleury's Algorithm to find one.

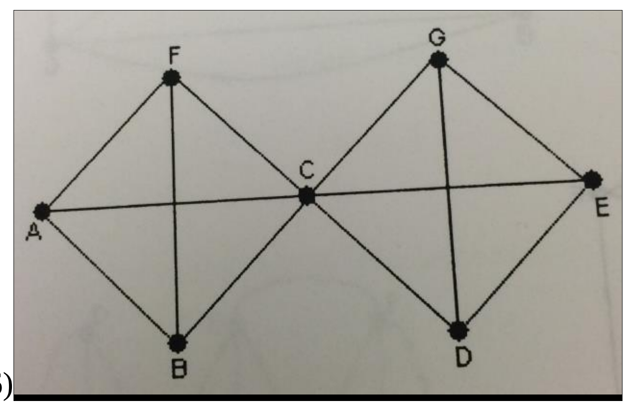
1) 

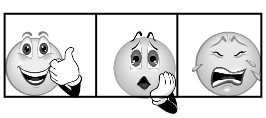
2) 

3) 

4) 

5) 

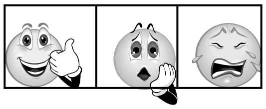
6) 



Objective #7: Can you determine whether a situation is asking you to find an Euler Path or an Euler Circuit?

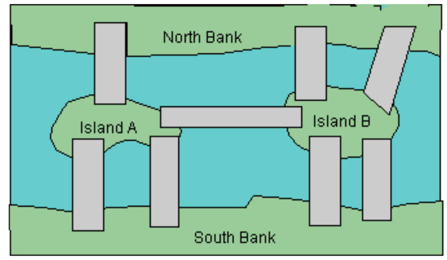
a) A snow removal truck enters a neighborhood, cleans each street (traveling on each street exactly one time), and exists the neighborhood at the same location that it entered it. Does this situation describe an Euler Path, an Euler Circuit, or neither? Explain your reasoning.

b) Angela would like to plan a roadtrip that starts in Massachusetts, visits every state in the continental US exactly once (crossing each common border once), and ending in California. Does this situation describe an Euler Path, an Euler Circuit, or neither? Explain your reasoning.



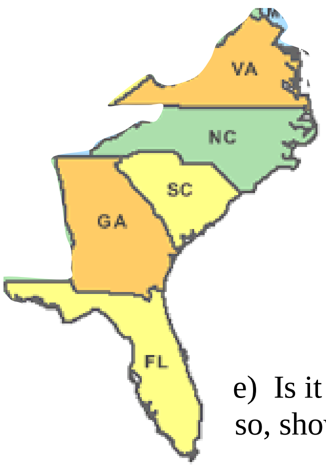
Objective #8: Can you use Fleury's Algorithm in application problems?

a) Draw a graph that models the layout of the city shown.



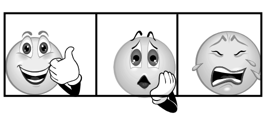
b) Use the graph to determine if city residents would be able to walk across all of the bridges without crossing the same bridge twice. If such a walk is possible, show the path on your graph from part a).

c) Trace this route on the city map in a manner that is clear to the city's residents.



d) Draw a map that models the map of the southern coast of the United States as shown. Use vertices to represent states and edges for common borders.

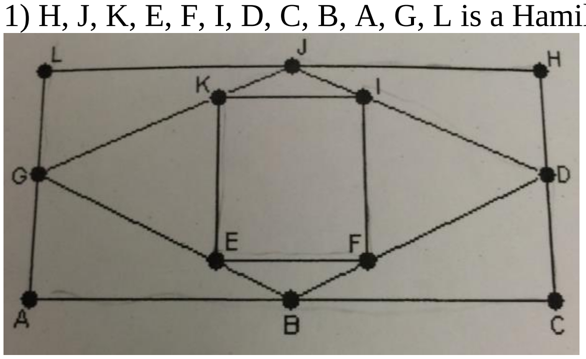
e) Is it possible to find a path that crosses each common state border exactly once? If so, show the path on your graph for part d).



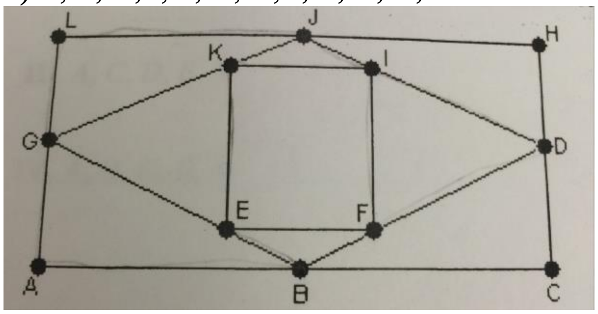
Objective #9: Can you answer questions about Hamilton Paths and circuits?

For Exercises 1-6, write 'T' if the statement is true and 'F' if the statement is false.

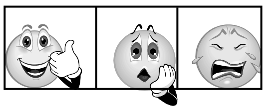
1) H, J, K, E, F, I, D, C, B, A, G, L is a Hamilton path of the given graph.



2) B, E, K, I, F, D, H, J, L, G, A, B is a Hamilton circuit of the given graph.



3) A Hamilton path must contain every edge in the graph exactly once.
4) Hamilton path must contain every edge in the graph exactly once.
5) A Hamilton circuit must begin and end at the same edge.
6) A Hamilton circuit must begin and end at the same vertex.

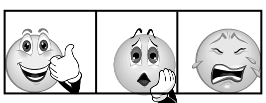


Objective #10: Can you find the number of Hamilton circuits in a complete graph?

For Exercises 1 and 2, determine the number of Hamilton circuits in a complete graph with the given number of vertices.

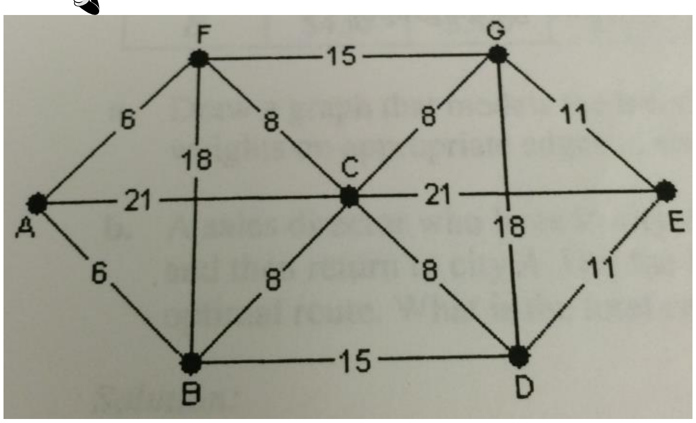
1) 4 Vertices

2) 8 Vertices

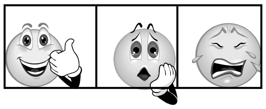


Objective #11: Can you use weighted graphs to solve problems?

For Exercises 1-3, use the weighted graph shown to answer the question.

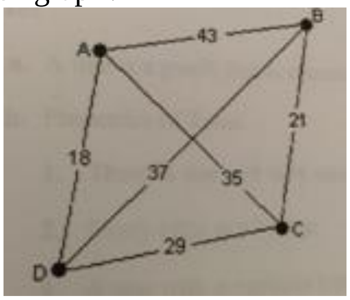


1) Find the total weight of the Hamilton circuit C, F, A, B, D, E, G, C.
2) Find the total weight of the Hamilton circuit A, B, F, G, D, E, C, A.
3) Find the total weight of the Hamilton circuit B, C, A, F, G, E, D, B.



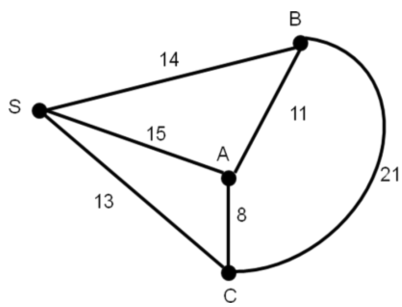
Objective #12: Can you use the Brute Force Method to solve traveling salesperson problems?

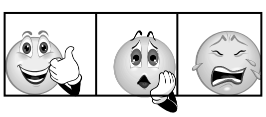
a) Using the Brute Force Method, which of the following is not an optimal solution for the given graph?



I. A, B, C, D, A
II. A, C, B, D, A
III. A, C, D, B, A
IV. A, D, C, B, A

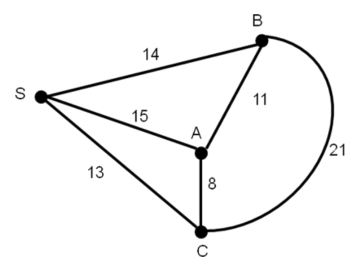
b) Use the Brute Force method to find the optimal solution for the TSP shown.



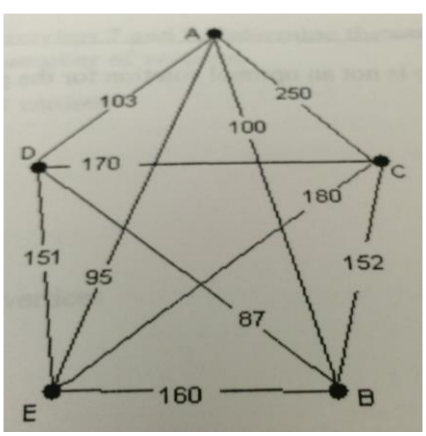


Objective #13: Can you use the Nearest Neighbor Method to approximate solutions to traveling salesperson problems?

a) Starting at point S on the graph shown.



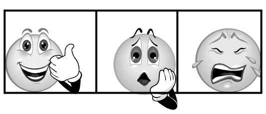
b) Is this solution from a) optimal? How do you know?

c) 

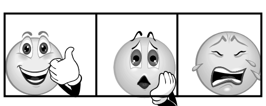
Jon is a traveling salesman for a pharmaceutical company. His territory includes 5 cities and he needs to find the least expensive route to the cities and home.
Repeat the Nearest Neighbor Method beginning for each city (Cities A and B are completed) to find the optimal route.

City	Edge #1, Weight	Edge #2, Weight	Edge #3, Weight	Edge #4, Weight	Edge #5, Weight	Full Route, Total Weight
A	AE	ED	DB	BC	CA	A, E, D, B, C, A
	95	151	87	152	250	735
B	BD	DA	AE	EC	CB	B, D, A, E, C, B
	87	103	95	180	152	617
C						
D						
E						

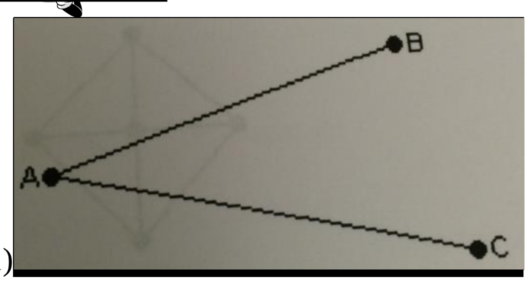
Jon lives in City A. Rewrite the optimal route such that it begins and ends in city A.

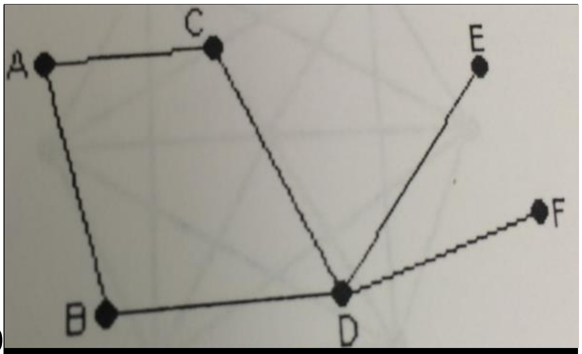


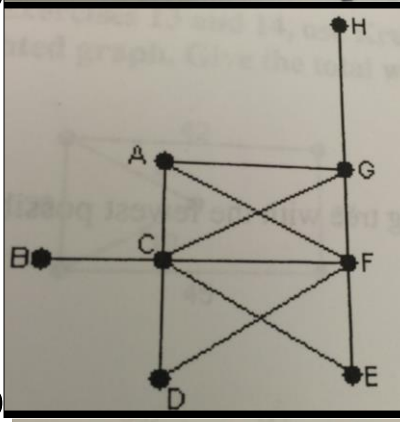
Objective #14: Can you describe the benefits and weaknesses of the Nearest Neighbor Method?

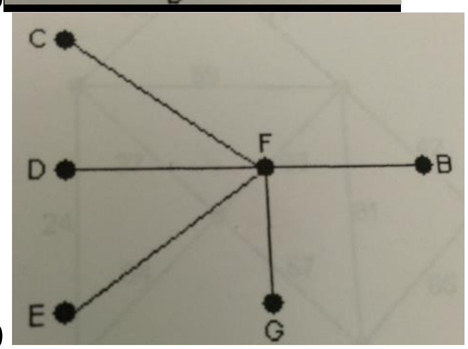


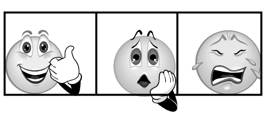
Objective #15: Can you decide if a graph is a tree and justify your reasoning?
For Exercises 1-4, determine whether the graph is a tree, explain your answer if "no".

1) 

2) 

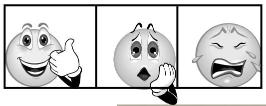
3) 

4) 

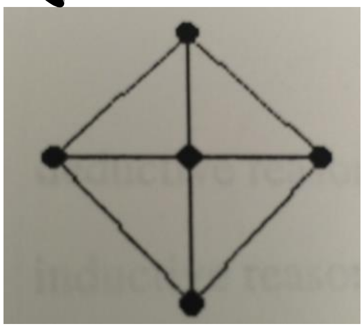


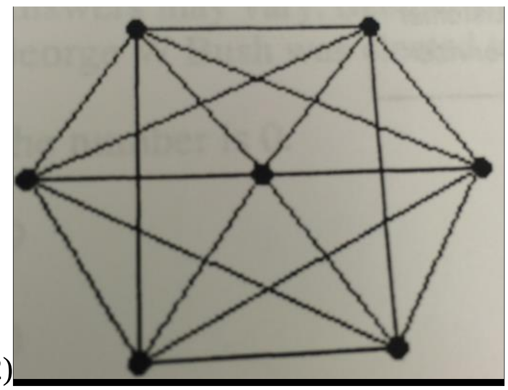
Objective #16: Can you use the properties of a tree?
For Exercises 1-6, write 'T' if the statement is true and 'F' if the statement is false.

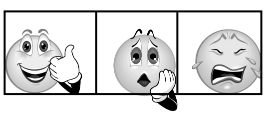
1) In the graph of a tree, the number of edges is one fewer that the number of vertices.
2) A spanning tree contains one and only one circuit.
3) A spanning tree contains all of a graph's vertices.
4) A spanning tree must be a connected graph.
5) A minimum spanning tree for a weighted graph is a spanning tree with the fewest possible edges.
6) A minimum spanning tree for a weighted graph is a spanning tree with the smallest possible total weight.



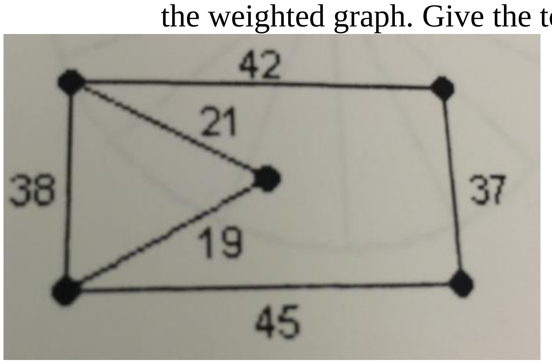
Objective #17: Can you find a spanning tree for a connected graph?
For Exercises 11 and 12, draw a spanning tree for the connected graph.

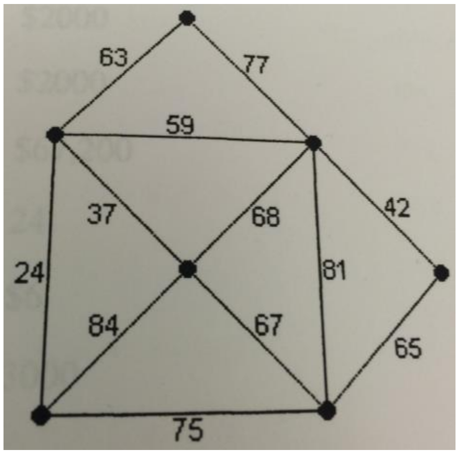
1) 

2) 

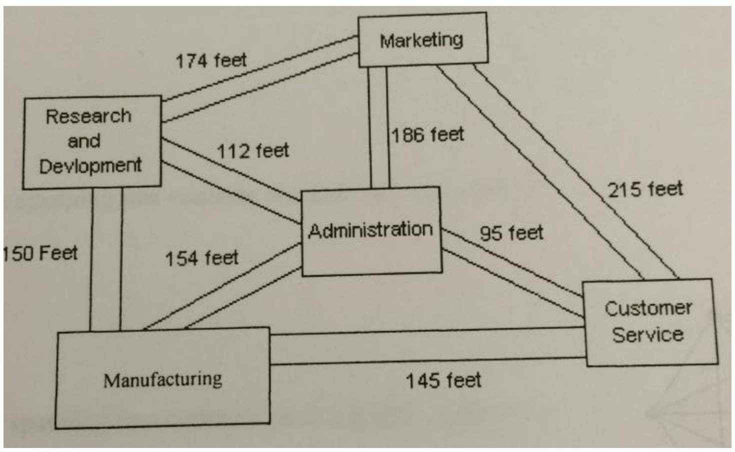


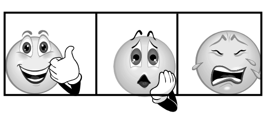
Objective #18: Can you discover the minimum spanning tree for a weighted graph?
For Exercises 1 and 2, use Kruskal’s Algorithm to find the minimum spanning tree for the weighted graph. Give the total weight of the minimum spanning tree.

1) 

2) 

3) A corporate campus plans to run computer network cable between buildings. Use Kruskal’s Algorithm to find the minimum spanning tree that allows the network to connect all the buildings. How much cable is needed?



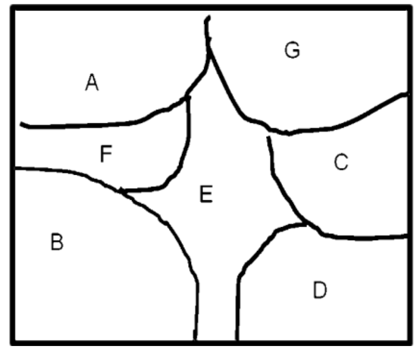


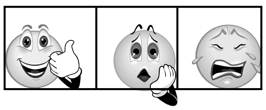
Objective #19: Can you create a diagram to model a coloring situation?

a) Create a graph to show the conflicts that may occur from scheduling meeting for the following committees.

	Fund-raising	Activities	Commun-ication	Service	Account-ing
Amy	X		X		X
Heather	X	X		X	
Sara			X	X	X
Amber	X	X	X		

b) Create a graph showing the common borders for each region.





Objective #20: Can you find the fewest possible number of items needed for a coloring situation?

Use the graphs created in Objective 19.

a) How many different meetings must be used as a minimum to avoid conflicts from Objective 19a?

b) What is the minimum number of colors that can be used to color the graph from Objective 19b?