Section 10.1: Applications of Linear Equations

Objective
1. Use linear equations to solve problems.

Strategy for Solving Word Problems

Step 1. Read the problem carefully at least twice. Attempt to state the problem in your own words and state what the problem is looking for. Let x (or any variable) represent one of the quantities in the problem.

Step 2. If necessary, write expressions for any other unknown quantities in the problem in terms of X.
Use one var!

Step 3. Write an equation in x that describes the verbal conditions of the problem.

Step 4. Solve the equation and answer the problem's question.

Step 5. Check the solution *in the original wording* of the problem, not in the equation obtained from the words.

Algebraic Translations of English Phrases

Addition	Subtraction	Multiplication	Division
sum	minus	times	divided by
more than	decreased by	product of	quotient
increased by	subtracted from	percent of a number	reciprocal
exceeded by	difference between	multiplied by	
	less than	twice	
	fewer than		

Example 1: Solving a Word Problem

Nine subtracted from eight times a number is 39. Find the number. Step1: Read twice

Step 2 $\boxed{\text{Let } n = \text{the number}}$

Step 3 ___ -9 is 39

$8n - 9 = 39$

$\quad +9 \quad +9$

Step 4 $8n = 48$

$\boxed{n = 6}$

Step 5: 9 subtr. from 8×6 is 39

9 subtr from 48 is 39

$48 - 9 = 39$ ✓

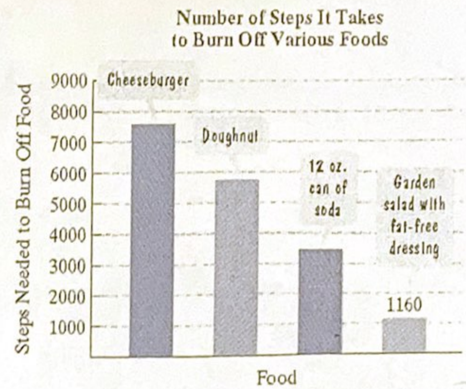
Ex3

min (x)	Plan A (y)	Plan B
0	20	5
1	20.05	5.10
2	20.10	5.20
3	20.15	5.30
4	20.20	5.40

Better? Plan B starts lower BUT grows faster

$\boxed{A}$: $y = 20 + .05x$

$\boxed{B}$: $y = 5 + .10x$



Example 2: Walk it Off

This graph shows the number of steps needed to burn off various foods.

The number of steps needed to burn off a cheeseburger exceeds (more) the number needed to burn off a 12-ounce soda (1) by 4140. The number needed to burn off a doughnut exceeds the number needed to burn off a 12-ounce soda by 2300. (1)

- Determine the number of steps it takes to burn off each of a cheeseburger, doughnut and 12-ounce soda if a 16,790 step walk is needed to burn off all three combined. TOTAL

Read twice

Notice all dep on soda

cheeseburger : 4140 more than soda

doughnut : 2300 more than soda

Solution:

Let x = number of steps needed to burn off a 12-ounce soda. (1)

Let $x + 4140$ = number of steps needed to burn off a cheeseburger.

Let $x + 2300$ = number of steps needed to burn off a doughnut.

Write an equation that models the conditions:

Steps needed to burn off cheeseburger	plus	steps needed to burn off doughnut	plus	steps needed to burn off soda	equals	16,790.
$x + 4140$	$+$	$x + 2300$	$+$	x		$= 16790$

$\boxed{10.1 \text{ WS } \#1, 4}$

$3x + 6440 = 16790$

$3x = 10350$

$x = 3450$ soda

Example 3: You are choosing between two long-distance telephone plans. Plan A has a monthly fee of $20 with a charge of $0.05 per minute for all long-distance calls. Plan B has a monthly fee of $5 with a charge of $0.10 per minute for all long-distance calls. For how many minutes of long-distance calls will the costs for the two-plans be the same?

$\boxed{10.1 \text{ WS } \#2, 3}$

$20 + .05x = 5 + .10x$

$-.05x \qquad -.05x$

$20 = 5 + .05x$

$-5 \qquad -5$

$15 = .05x$

$\boxed{x = 300}$

✓ Cost A: $20 + .05(300) = \$35$

Cost B: $5 + .10(300) = \$35$

same

Section 10.2: Ratio, Proportion, and Variation

Objectives

1. Solve proportions.
2. Solve problems using proportions.
3. Solve direct variation problems.
4. Solve inverse variation problems.

Proportions:

- A ratio compares quantities by division.

Example 1: a group contains 60 women and 30 men. The ratio of women to men is: $\frac{60 \text{ women}}{30 \text{ men}}$

- A proportion is a statement that says two ratios are equal: $\frac{a}{b} = \frac{c}{d}$

Example 1

The Cross-Products Principle for Proportions

$\frac{a}{b} = \frac{c}{d} \Rightarrow a \cdot d = b \cdot c$

If $\frac{a}{b} = \frac{c}{d}$ then $ad = bc$. ($b \neq 0$ and $d \neq 0$)

The cross products *ad* and *bc* are equal.

Solve this proportion for x:

$\frac{63}{x} = \frac{7}{5}$

$7x = 63 \cdot 5$

$7x = 315$

$\boxed{x = 45}$

Check the solution:

$\frac{63 \div 9}{45 \div 9} = \frac{7}{5}$ ✓

Applications of Proportions

Solving Applied Problems Using Proportions

1. Read the problem and represent the unknown quantity by x (or any letter).
2. Set up a proportion by listing the given info on one side and the ratio with the unknown quantity on the other side. Each respective quantity should occupy the same corresponding position on each side of the proportion.
3. Drop units and apply the cross - products principle.
4. Solve for x and answer the question.

Example 2
Calculating Taxes

- The property tax on a house whose assessed value is $65,000 is $825. Determine the property tax on a house with an assessed value of $180,000, assuming the same tax rate.

comparing 2 quantites
→ tax
→ value of house

- Solution:

Step 1. Let x = tax on a $180,000 house.

Step 2. Set up the proportion:

given info: ratio w/ x

$$\frac{\text{value}}{\text{tax}} \quad \frac{\$65{,}000}{\$825} = \frac{\$180{,}000}{x} \begin{matrix}\leftarrow \text{value} \\ \leftarrow \text{tax}\end{matrix}$$

Step 3 Drop the units and apply the cross products principle

$$\frac{65{,}000}{825} = \frac{180{,}000}{x}$$

$$65000x = 148500000$$

$$\boxed{x = 2284.62}$$

Step 4 Solve for x and answer the question.

The tax on a 180,000 house is $2284.62

The property tax on the $180,000 house is approximately $2285

Pay / hours   Cost / # of candy bars

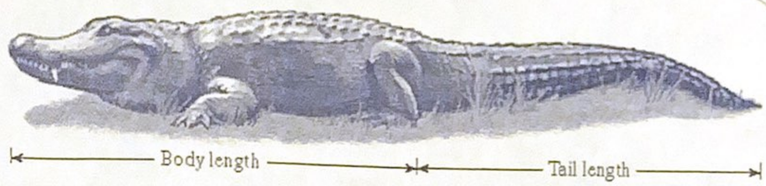
Example 3
Direct Variation

As one quantity increases, the other quantity increases and vice versa.

- An alligator's tail length varies directly as its body length.

  An alligator with a body length of 4 feet has a tail length of 3.6 feet. What is the tail length of an alligator whose body length is 6 ft?

tail length   4   6   body length



- Solution

  Step 1. Let x = tail length of an alligator whose body length is 6 feet.

  Step 2. Set up the proportion:

given info

$$\frac{\text{body}}{\text{tail}} \quad \frac{4\text{ ft}}{3.6\text{ ft}} = \frac{6\text{ ft}}{x} \quad \frac{\text{body}}{\text{tail}}$$

Step 3. Apply the cross-products principle, solve and answer the question.

$$\frac{4}{3.6} = \frac{6}{x}$$

$$4x = 6(3.6)$$

$$4x = 21.6$$

$$\boxed{x = 5.4}$$

An alligator whose body length is 6 feet has a tail length measuring 5.4 feet.

Alt. method: rate $\rightarrow$ $\boxed{\frac{\text{tail}}{\text{body}}} \times \frac{\text{body}}{1} = \text{tail}$ $\boxed{\frac{3.6}{4} = .9} \Rightarrow .9(6) = \boxed{5.4}$

rate, scale factor

Inverse Variation

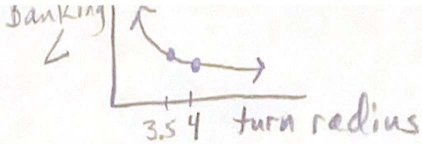
- As one quantity increases, the other quantity decreases and vice versa.

Time to finish / # workers   Cost each / # of friends sharing

- Setting up a Proportion when y varies inversely as x

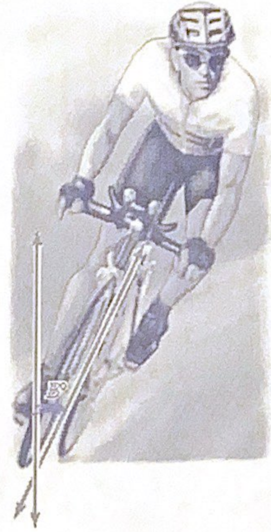
$$\frac{\textit{The first value for y}}{\textit{The value for x corresponding to the second value for y}} = \frac{\textit{The second value for y}}{\textit{The value for x corresponding to the first value for y}}$$

All flipped around! Values that go together are diagonal!



Example 4
Inverse Variation

A bicyclist tips the cycle when making a turn. The angle B, formed by the vertical direction and the bicycle is called the **banking angle**. The banking angle varies inversely as the cycle's turning radius. When the turning radius is 4 feet the banking angle is 28°.



↓ less

- What is the banking angle when the turning radius is 3.5 feet?

So banking ∠ will be more

- Solution

Step 1. Represent the unknown x

a = banking angle when turning radius is 3.5 feet.

Step 2. Set up the proportion

$$\frac{\angle}{\text{radius}} \qquad \frac{28^\circ}{3.5\text{ ft}} = \frac{a}{4\text{ ft}} \qquad \frac{\angle}{\text{radius}}$$

← info finding

given info

Step 3 and 4. Apply the cross products principle, solve, and answer the question.

$$3.5a = 112$$
$$\boxed{a = 32}$$

When the turning radius is 3.5 feet, the banking angle is 32°.

OLD WAY:

$$y = \frac{k}{x} \Rightarrow y = \frac{112}{x}$$

$$28 = \frac{k}{4} \qquad y = \frac{112}{3.5}$$

$$k = 112 \qquad \boxed{y = 32}$$

Section 10.3: Linear Inequalities in One Variable

Objectives
1. Graph subsets of real numbers on a number line.
2. Solve linear inequalities.
3. Solve applied problems using linear inequalities.

- Linear Inequalities

A linear inequality : ax + b [inequality symbol] c, where the inequality symbol can be $<, \leq, >, \geq$

- **Solving an inequality** is the process of finding the set of numbers that make an inequality a true statement.
- **A solution set** is the set of all numbers that solve the inequality.

Graphing Subsets of Real Numbers on a Number Line

Open dots – indicate a number is not included in a set.

Closed dots- indicate a number is included in a set.

Let a and b be real numbers such that $a < b$.		
Set-Builder Notation		**Graph**
$\{x \mid x < a\}$	x is a real number less than a.	a b
$\{x \mid x \leq a\}$	x is a real number less than or equal to a.	a b
$\{x \mid x > b\}$	x is a real number greater than b.	a b
$\{x \mid x \geq b\}$	x is a real number greater than or equal to b.	a b
$\{x \mid a < x < b\}$	x is a real number greater than a and less than b.	a b
$\{x \mid a \leq x \leq b\}$	x is a real number greater than or equal to a and less than or equal to b.	a b
$\{x \mid a \leq x < b\}$	x is a real number greater than or equal to a and less than b.	a b
$\{x \mid a < x \leq b\}$	x is a real number greater than a and less than equal to b.	a b

Example 1: Graphing Subsets of Real Numbers

Less

a) $\{x : x < 3\}$ 2 3 4

b) $\{x : x \geq -1\}$ -2 -1 0

c) $\{x : -1 < x \leq 3\}$ -1 3

between

Solving Linear Inequalities in One Variable

- The procedure for solving linear inequalities is the same as the procedure for solving linear equations, with one important exception:

Remember!! **When multiplying or dividing both sides of the inequality by a negative number, reverse the direction of the inequality symbol, changing the sense of the inequality.

$\frac{3n}{3} > \frac{12}{3}$

$n > 4$

Check:

3 4 5

Try 5: $3 \cdot 5 > 12$

$15 > 12$ ✓

$\frac{-3n}{-3} > \frac{12}{-3}$

$n < -4$

-5 -4 -3

Try -5;

$-3(-5) > 12$

$15 > 12$ ✓

Per. 1: Q $\boxed{10.2}$ → Collect
Q $\boxed{10.3}$, Ex. 3 & 4 below $\boxed{10.3}$

Example 2: Solving a Linear Inequality

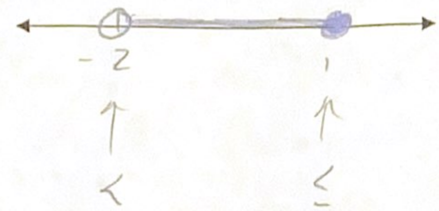
Solve and graph the solution set: $6x - 12 > 8x + 2$

$-8x \quad -8x$
$-2x - 12 > 2$
$-2x > 14$
$\boxed{x < -7}$

other way: $6x - 12 > 8x + 2$
$-6x \quad -6x$
$-12 > 2x + 2$
$-14 > 2x$
$-7 > x$ SAME!

Example 3: Solving a Linear Inequality

Solve the three part inequality: $-3 < 2x + 1 \leq 3.$



2x+1 is betw. -3 & 3

$2x + 1$ is greater than -3 and less than or equal to 3.

$-3 < 2x + 1 \leq 3$
$-1 \quad -1 \quad -1$
$-4 < 2x \leq 2$
$\div 2 \quad \div 2 \quad \div 2$

x is betw. -2 & 1

$\boxed{-2 < x \leq 1}$

Example 4: To earn an A in a course, you must have a final average of at least 90%. On the first four examinations, you have grades of 86%, 88%, 92%, and 84%. If the final examination counts as two grades, what must you get on the final to earn an A in the course?

avg $\geq$ 90%

avg: add, $\div 6$

4 grades

+ 2 more grades = 6 total

$$\frac{86 + 88 + 92 + 84 + x + x}{6} \geq 90$$

$$\frac{\cancel{6}}{1} \cdot \frac{350 + 2x}{\cancel{6}} \geq 90 \cdot 6$$

$$350 + 2x \geq 540$$

$$2x \geq 190$$

$$\boxed{x \geq 95}$$