Section 10.5: Linear Inequalities in Two Variables

Objectives
1. Graph a linear inequality in two variables.
2. Use mathematical models involving linear inequalities.
3. Graph a system of linear inequalities.

Linear Inequalities in Two Variables and Their Solutions

- The *graph of an inequality in two variables* is the set of all points whose coordinates satisfy the inequality.

Graphing a linear inequality in two variables:

1. Replace the inequality with an equal sign and graph the linear equation.
   - Draw a solid line if the original inequality has a $\le$ or $\ge$.
   - Draw a dotted line if the original inequality has a $<$ or $>$.
2. Choose a test point in one of the half-planes that is not on the line. Substitute the coordinates of the test point into the inequality.
3. If a true statement results, shade the half-plane containing this test point.

   If a false statement results, shade the half-plane not containing this test point.

solid line

Example 1: Graph: $3x - 5y \ge 15$.

$3x - 5y = 15$

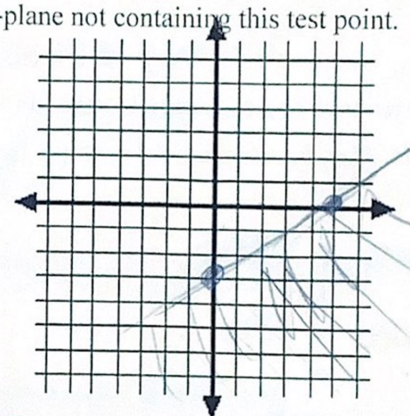
$(0, \ )$ $(\ ,0)$

$(0,-3)$ $(5,0)$

$3\cdot 0 - 5y = 15$ $3x - 5\cdot 0 = 15$

$-5y = 15$ $3x = 15$

$y = -3$ $x = 5$



$3x - 5y = 15$

$-5y = -3x + 15$

$y = \frac{-3x + 15}{-5}$

$y = \frac{3}{5}x - 3$

slope y-int

Test $(0,0)$:

$3\cdot 0 - 5\cdot 0 \ge 15$

$0 \ge 15$ No - shade other side

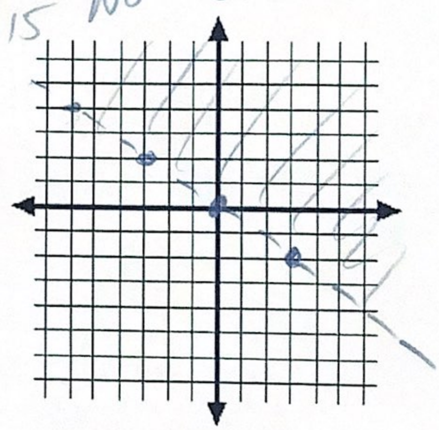
The Graph of a Linear Inequality in Two Variables

Example 2: Graph: $y > -\frac{2}{3}x$. dotted line

$y > -\frac{2}{3}x$

$y = -\frac{2}{3}x + 0$

slope y-int.



Test a point: $(1,1)$

$1 > -\frac{2}{3}\cdot 1$

$1 > -\frac{2}{3}$ True - shade that side

Graphing Linear Inequalities without Using Test Points

For the vertical line $x = a$:

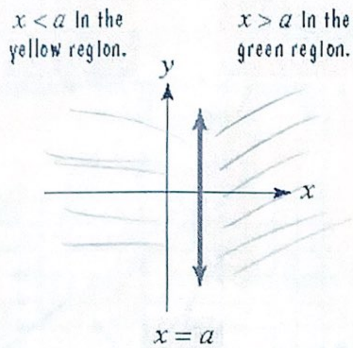
- If $x > a$, shade the half-plane to the right of $x = a$.
- If $x < a$, shade the half-plane to the left of $x = a$.

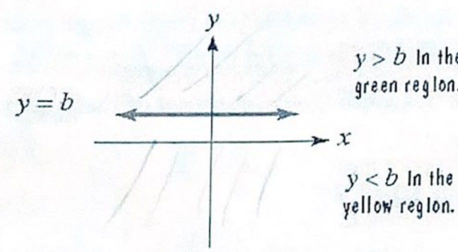
x: lf. or rt.

For the horizontal line $y = b$:

- If $y > b$, shade the half-plane above $y = b$.
- If $y < b$, shade the half-plane below $y = b$.

y: up or down





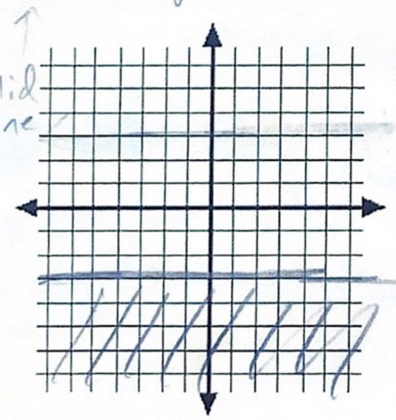
Graphing Linear Inequalities without Using Test Points

Example 3:

Graph each inequality in a rectangular coordinate system:

a. $y \leq -3$ y = -3 horiz. line

Solid line

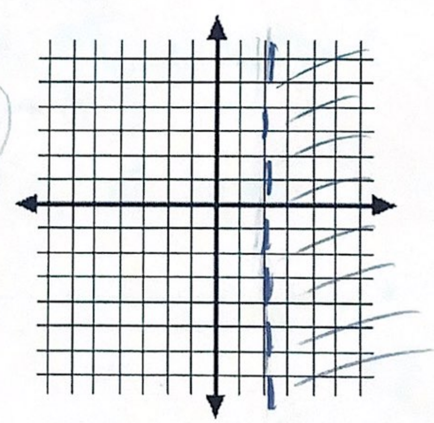


y < -3

y is below -3

b. $x > 2$ dotted line

x = 2 vert. line

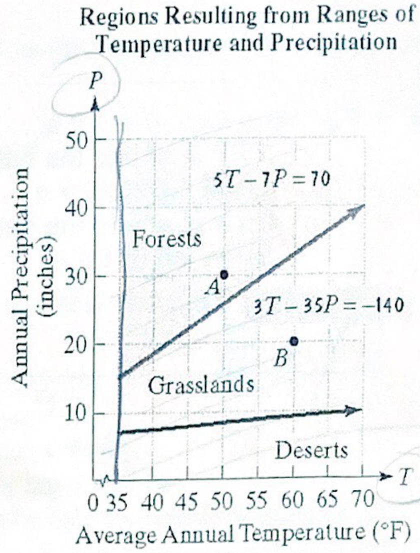


x > 2

x is more, so rt

Modeling with Systems of Linear Inequalities

Example 4: The graph displays three kinds of regions- deserts, grasslands, and forests- temperatures, $T \geq 35$, and precipitation, P. Show that point A is a solution of the system of inequalities that describes where forests occur.



↳ (50, 30)
T P

40 < 70 ✓

250 − 210 < 70

50 ≥ 35 ✓ 5·50 − 7·30 < 70

A forest occurs if: $T \geq 35$ and $5T - 7P < 70$.

- Graphing Systems of Linear Inequalities: The **solution set of a system of linear inequalities in two variables** is the set of all points that satisfy all inequalities in the system.

Example 5: Graph the solution set of the system:

$x - y < 1$

$2x + 3y \geq 12$.

$3y = 12 - 2x$

$y = \frac{12-2x}{3}$

$y = 4 - \frac{2}{3}x$

$y = -\frac{2}{3}x + 4$

solid line

$x - y = 1$ dotted line

$-x \quad -x$

$-y = 1 - x$

$\div -1 \quad \div -1$

$y = -1 + x$

$y = x - 1$

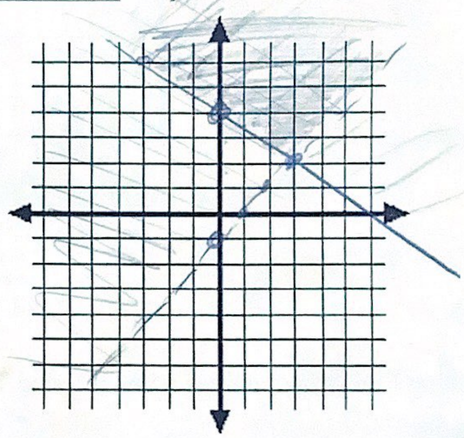
Test (0,0)

$x - y$

$0 - 0 < 1$

$0 < 1$

True



Test (0,0) : $2x + 3y \geq 12$

$0 + 0 \geq 12$

False − shade other side

$3y \geq 12 - 2x$

$y \geq 4 - \frac{2}{3}x$

shade above

Section 10.6: Linear Programming

Objectives

1. Write an objective function describing a quantity that must be maximized or minimized.
2. Use inequalities to describe limitations in a situation.
3. Use linear programming to solve problems.

Objective Functions in Linear Programming

- A method for solving problems in which a particular quantity that must be maximized or minimized is limited by other factors is called ***linear programming.***
- An objective function is an algebraic expression in two or more variables describing a quantity that must be maximized or minimized.

Example 1: Bottled water and medical supplies are to be shipped to victims of an earthquake by plane. Each container of bottled water will serve 10 people and each medical kit will aid 6 people. Let x represent the number of bottles of water to be shipped and y the number of medical kits. Write the objective function that describes the number of people that can be helped.

Solution: Because each water serves 10 people and each medical kit aids 6 people, we have

$X = \#$ bottles

$Y = \#$ of medical kits

Ex. of objective fnt

The number of people helped	is	10 times the number of bottles of water	plus	6 times the number of medical kits.
	$=$	$10x$	$+$	$6y.$

Want to maximize →

OBJECTIVE: help max. no. of people

Using z to represent the number of people helped, the objective function is

$$\boxed{z = 10x + 6y.}$$

For a value of x and a value for y, there is only one value of z. Thus, z is a function of x and y.

Constraints on Linear Programming

- A constraint is expressed as an inequality.
- The list of constraints forms a system of linear inequalities.

constraint (limitations)

Example 2: Each plane can carry no more than 80,000 pounds. The bottled water weighs 20 pounds per container and each medical kit weighs 10 pounds. Let x represent the number of bottles of water to be shipped and y the number of medical kits. Write an inequality that describes this constraint.

Solution: Because each plane carries no more than 80,000 pounds, we have

The total weight of the water bottles	plus	the total weight of the medical kits	must be less than or equal to	80,000 pounds.

Ex. of one constraint: weight

$$\boxed{20x + 10y \leq 80000}$$

Full problem!

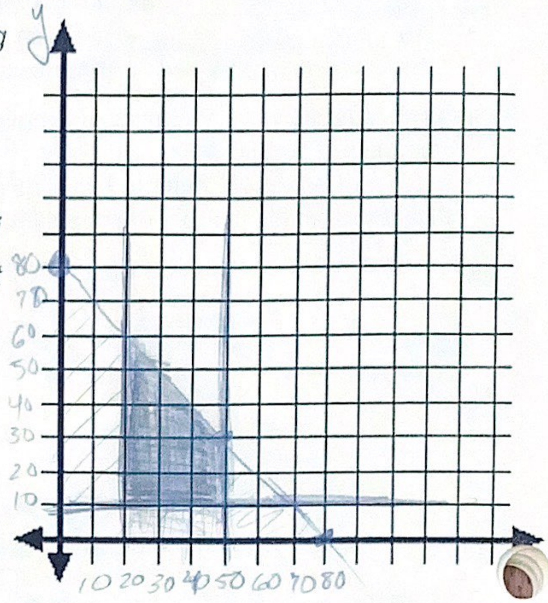
Solving Problems with Linear Programming

Let $z = ax + by$ be an objective function that depends on x and y. Furthermore, z is subject to a number of constraints on x and y. If a maximum or minimum value exists, it can be determined as follows:

1. Graph the system of inequalities representing the constraints.
2. Find the value of the objective function at each corner, or ***vertex***, of the graphed region. The maximum and minimum of the objective function occur at one or more of the corner points.

The Japanese Club at Damonte Ranch specializes in creating origami (paper making) for their fundraiser each school year. They produce paper cranes and paper flowers. Each paper crane sold yields 50 cents and each paper flower sold yields 75 cents. They can produce up to 80 pieces of origami per week. Based on the popularity of paper cranes, they must produce at least 20 per week, but no more than 50 pieces. They know that the minimum number of paper flowers sold per week is 10 pieces. How many of each type of origami should they produce to maximize their profits?

profit



(1) Let x = no. of paper cranes
y = no. of paper flowers

(2) OBJECTIVE: make max \$ (profit)
obj. fnt: $\boxed{z = .50x + .75y}$

(3) Contstraints:

can produce: up to 80 — $x + y \le 80$ — $y = 80 - x$ (y-int, slope) — test $(0,0)$ ✓

must make cranes — $20 \le x \le 50$

know min. flowers — $y \ge 10$ (no upper limit — always sell out)

(4) Test each vertex for max profit w/ obj fnt

$z = .50x + .75y$

$(20, 10)$ — $z = .50(20) + .75(10) = 17.50$

$(20, 60)$ — $z = .50(20) + .75(60) = \$55.00$

make 20 cranes, 60 flowers — Best choice ↑

~~$(50, 10)$~~ X — better

$(50, 30)$: $z = .50(50) + .75(30) = \$47.50$