Section 10.4: Systems of Linear Equations in Two Variables

Objectives

1. Decide whether an ordered pair is a solution of a linear system.
2. Solve linear systems by graphing.
3. Solve linear systems by substitution.
4. Solve linear systems by addition.
5. Identify systems that do not have exactly one ordered-pair solution.
6. Solve problems using systems of linear equations.

Systems of Linear Equations & Their Solutions

- Two linear equations are called a ***system of linear equations*** or a ***linear system***.
- A ***solution to a system of linear equations in two variables*** is an ordered pr that satisfies both equations in the system.

(x, y)

Example 1: Determine whether (1,2) is a solution of the system:

$x \; y$

$2x - 3y = -4 \rightarrow 2\cdot 1 - 3\cdot 2 = -4 \rightarrow 2 - 6 = -4$ ✓

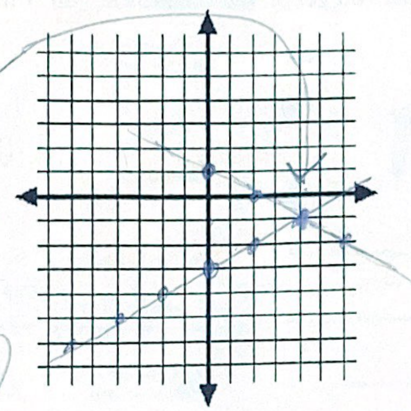
$2x + y = 4 \rightarrow 2\cdot 1 + 2 = 4 \rightarrow 2 + 2 = 4$ ✓

$\boxed{\text{Yes, it is a sol.}}$

Solving Linear Systems by **Graphing**

- For a system with one solution, *the pair of coordinates of the point of intersection of the lines is the system's solution*.

$\boxed{\text{Sol: } (4, -1)}$



Example 2: Solve by graphing:

$x + 2y = 2 \rightarrow$

$x + 2y = 2$
$-x \qquad -x$
$2y = 2 - x$
$\frac{2y}{2} = \frac{2 - x}{2}$
$y = \frac{2}{2} - \frac{1x}{2}$
$y = 1 - \frac{1}{2}x$
$\boxed{y = -\frac{1}{2}x + 1}$

$x - 2y = 6.$
$-x \qquad -x$
$-2y = 6 - x$
$\frac{-2y}{-2} = \frac{6 - x}{-2}$
$y = -3 + \frac{1}{2}x$
$y = \frac{1}{2}x - 3$

Solving Linear Systems by the **Substitution** Method

- This method involves converting the system to one equation in one variable by an appropriate substitution.

Example 3: Solve by the substitution method:

$y = -x - 1$ plug in for y

$4x - 3y = 24$

$4x - 3(\quad) = 24$
$4x - 3(-x - 1) = 24$
$4x + 3x + 3 = 24$
$7x + 3 = 24$
$7x = 21$
$\boxed{x = 3}$
$\downarrow$
$(3, \_)$

$y = -x - 1$
$y = -(3) - 1$
$\boxed{y = -4}$

$\boxed{\text{Sol: } (3, -4)}$

Solving Linear Systems by the Addition Method ___AKA Elimination Method___

Example 4: Solve by the addition (elimination) method:

$x + 2y = 48$ → $3x + 2y = 48 \xrightarrow{\times 4} 12x + 8y = 192$

$9x - 8y = -24.$ → $9x - 8y = -24$ → $9x - 8y = -24$

$21x = 168$

$\boxed{x = 8}$ → $3(8) + 2y = 48$ (1st eqt)

$24 + 2y = 48$

$2y = 24$

$\boxed{y = 12}$

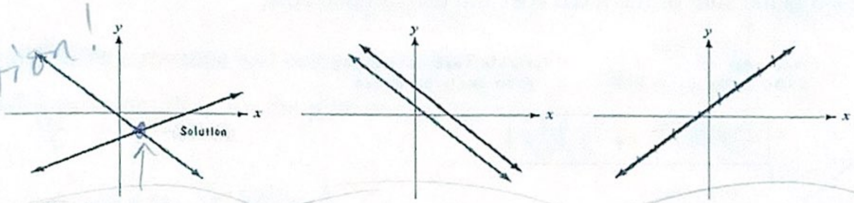
$x = 8$
$y = 12$

Linear Systems Having No Solution or Infinitely Many Solutions

The number of solutions to a system of two linear equations in two variables is given by one of the following:

Number of Solutions	What This Means Graphically
Exactly one ordered-pair solution	The two lines intersect at one point.
No Solution	The two lines are parallel, i.e., have the same slope.
Infinitely many solutions	The two lines are identical.

Sol. is pt. of intersection!



Exactly one solution   No solution (parallel lines)   Infinitely many solutions (lines coincide)

Example 5: Solve the system:

$4x + 6y = 12$ $\xrightarrow{\times(3)}$ $12x + 18y = 36$ No sol.

$6x + 9y = 12.$ $\xrightarrow{\times(-2)}$ $-12x - 18y = -24$

$0 \neq 12$ False

→ No comb. of x & y can make this true

alt. method...
$2x + 3y = 12$
$2x + 3y = 4$
Solve for y : same slope

Example 6: Solve the system:

$y = 3x - 2$

$15x - 5y = 10.$ → $15x - 5(\quad) = 10$

$15x - 5(3x - 2) = 10$

$15x - 15x + 10 = 10$

$10 = 10$ True!

Always true — many comb. of x & y → Inf Many Sol.

alt. method : prep to graph it :
$y = 3x - 2$
$15x - 5y = 10 \xrightarrow{\div 5} 3x - y = 2$
$\boxed{3x - 2 = y}$
same eqt.

Modeling with Systems of Equations: Making Money

Revenue and Cost Functions

A company produces and sells x units of a product.

- Revenue Function: $R(x)$ = (price per unit sold)x
- Cost Function: $C(x)$ = fixed cost + (cost per unit produced)x

The point of intersection of the graphs of the revenue and cost functions is called the break-even pt.

Modeling with Systems of Equations: Making Money
Finding a Break-even Point

Example 7: A company is planning to manufacture radically different wheelchairs. Fixed cost will be $500,000 and it will cost $400 to produce each wheelchair. Each wheelchair will be sold for $600.

a. Write the cost function, C, of producing x wheelchairs.
b. Write the revenue function, R, from the sale of x wheelchairs.
c. Determine the break-even point. Describe what this means.

Solution:

- The cost function is the sum of the fixed cost and the variable cost.

Fixed cost of $500,000 | plus | Variable cost: $400 for each chair produced

$$C(x) = \boxed{500000 + 400x}$$

money spent

- The revenue function is the money generated from the sale of x wheelchairs.

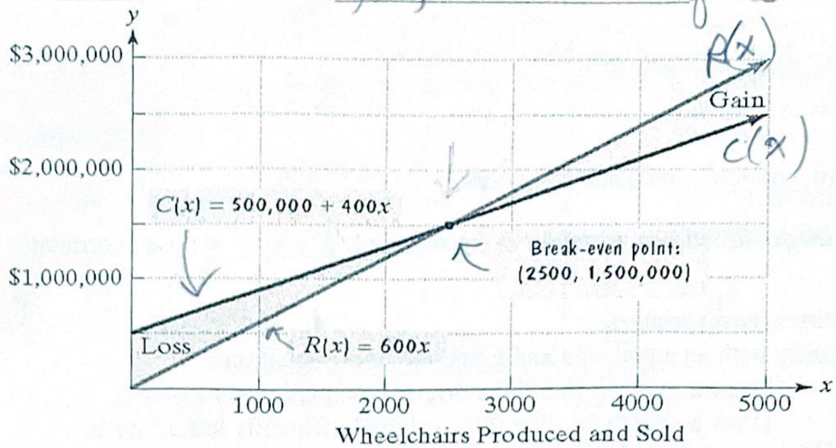
Revenue per chair, $600, times | the number of chairs sold

$$R(x) = \boxed{600x}$$

money gained

The break-even point is (2500, 1500000). This means that the company will break even if it produces and sells 2500 wheelchairs for $ 1,500,000 → which equals cost

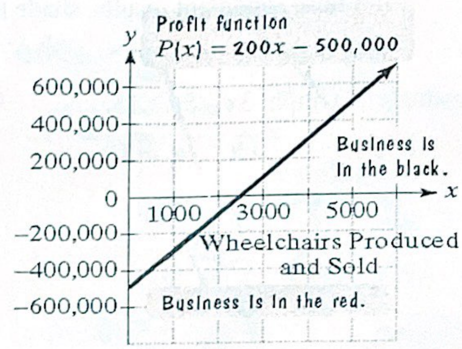
(so profit here is $0)



- The profit, $P(x)$, generated after producing and selling x units of a product is given by the **profit function**

$$P(x) = R(x) - C(x),$$

where R and C are the revenue and cost, respectively.



Example: The profit function, $P(x)$, for the previous example is

$P(x) = R(x) - C(x)$

$= 600x - (500{,}000 + 400x)$

$= 600x - 500{,}000 - 400x$

spend spend

$P(x) = 200x - 500{,}000$