

## 5.1 The Product Rule and Power Rules for Exponents

| Base | Exponent/Power |
|------|----------------|
| $x$  | $x$            |

$x = x \cdot x \cdot x$

$2^4 = 2 \cdot 2 \cdot 2 \cdot 2 = 16$

**EXAMPLE 1** Using ExponentsWrite  $3 \cdot 3 \cdot 3 \cdot 3$  in exponential form and evaluate.

$$3^4 = 81$$

$$2^4 = 16$$

**EXAMPLE 2** Evaluating Exponential Expressions

Evaluate. Name the base and the exponent.

(a)  $5^4 = 5 \cdot 5 \cdot 5 \cdot 5 = 625$   
 $b = 5$   
 $e = 4$

(b)  $-5^4 = -625$   
 $b = 5$   
 $e = 4$

(c)  $(-5)^4 = 625$   
 $b = -5$   
 $e = 4$

(d)  $(-5)^3 = -125$   
 $b = -5$   
 $e = 3$

(e)  $-(-5)^3 = 125$   
 $b = -5$   
 $e = 3$

**Product rule for exponents**

Same base raised to a power multiplied,  
 then add the exponents:

$x^3 \cdot x^4 = x^{3+4} = x^7$

**EXAMPLE 3** Using the Product Rule

Use the product rule for exponents to find each product if possible.

(a)  $6^3 \cdot 6^5 = 6^8 = 1,679,616$

(b)  $(-4)^2(-4)^3 = (-4)^5 = -262,144$

(c)  $x^2 \cdot x^1 = x^3$

(d)  $m^4 m^3 m^5 = m^{12}$

(e)  $2^3 \cdot 3^2 = 8 \cdot 9 = 72$

(f)  $2^3 + 2^4 = 8 + 16 = 24$

(g)  $(2x^3)(3x^7) = 6x^{10}$

(h)  $(m+n)^2(m+n)^3 = (m+n)^5$

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**Power rules for exponents**

(a)  $(n^4)^3 = n^{12}$

(b)  $(mn)^a = m^a n^a$

(c)  $\left(\frac{m}{n}\right)^a = \frac{m^a}{n^a}$

$(n^a)^b = n^{a \cdot b}$

$\left(\frac{m}{n}\right)^3 = \left(\frac{m}{n}\right)\left(\frac{m}{n}\right)\left(\frac{m}{n}\right) = \frac{m^3}{n^3}$

**Example 4:** Use the Power Rules for exponents to simplify each expression.

a)  $(2^5)^3 = 2^{15}$

b)  $(5^7)^2 = 5^{14}$

c)  $(x^2)^5 = x^{10}$

d)  $(3xy)^2 = 9x^2y^2$

e)  $5(4pq)^2 = 80p^2q^2$

f)  $3(2m^2p^3)^4 = 48m^8p^{12}$

g)  $(-5^6)^3 = -5^{18}$

h)  $\left(\frac{2}{3}\right)^5 = \frac{2^5}{3^5}$

i)  $\left(\frac{m}{n}\right)^3 = \frac{m^3}{n^3}$

$$-5^{10}$$

$$\frac{-}{3^5}$$

$$\frac{m^3}{h^3}$$

i)  $\left(\frac{1}{5}\right)^4$

$$\frac{1^4}{5^4} = \frac{1}{625}$$

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**Example 5: Simplify by using a combination of rules.**

(a)  $\left(\frac{2}{3}\right)^2 \cdot 2^3$

$$\frac{2^2}{3^2} = \frac{4}{9} \rightarrow \frac{8}{9} = \frac{32}{9}$$

(b)  $(5x^7)^2 \cdot (5x^7)^3$

$$5^2 \cdot 5^3 = 5^5$$

$$x^{14} = 78125 x^{14}$$

(c)  $(2x^2y^3)^4 \cdot (3xy^2)^6$

$$2^4 \cdot x^8 \cdot y^{12} \cdot 3^6 \cdot x^6 \cdot y^{12}$$

$$2^4 \cdot 3^6 \cdot x^{14} \cdot y^{24} = 16 \cdot 27 \cdot x^{14} \cdot y^{24}$$

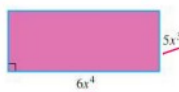
(d)  $(-4x^2y^3)^2(-4x^2y^3)^3$

$$(-1)^2 \cdot 4^2 \cdot x^4 \cdot y^6 \cdot (-1)^3 \cdot 4^3 \cdot x^6 \cdot y^9$$

$$+ \cdot x^4 \cdot y^6 \cdot (-1) \cdot 4^3 \cdot x^6 \cdot y^9 = -x^{10} y^{15}$$

**Example 6: Using Area Formulas.** Find an expression that represents the area in each figure.

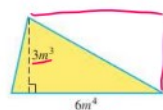
a)



$$A = lw$$

$$A = (6x^4)(5x^3) = 30x^7$$

b)



$$A = \frac{1}{2}bh = \frac{1}{2}(6m^4)(3m^3) = 9m^7$$

## 5.2 Integer Exponents and the Quotient Rules

### OBJECTIVES

- 1 Use 0 as an exponent.
- 2 Use negative numbers as exponents.
- 3 Use the quotient rule for exponents.
- 4 Use combinations of rules.

Zero exponent

Anything raised to 0 power = 1  
 $(3x^2y^4)^0 = 1$

### EXAMPLE 1

Using Zero Exponents

Evaluate.

(a)  $60^0 = 1$

(c)  $-60^0 = -1$

(e)  $6y^0 = 6 \cdot 1 = 6$

(g)  $8^0 + 11^0 = 1 + 1 = 2$

(b)  $(-60)^0 = 1$

(d)  $y^0 = 1$

(f)  $(6y)^0 = 1$

(h)  $-8^0 - 11^0 = -1 - 1 = -2$

**Negative exponents**

They cause the base to move to the

$$1 + 1 = 2$$

$$-1 - 1 = -2$$

### Negative exponents

They cause the base to move to the opposite side of a fraction, keep the exponent, but make it positive

$$5^{-1} = \frac{1}{5}$$

$$\frac{1}{5^{-2}} = 5^2$$

#### EXAMPLE 2 Using Negative Exponents

Simplify by writing with positive exponents. Assume that all variables represent nonzero real numbers.

(a)  $3^{-2} = \frac{1}{3^2}$  (b)  $5^{-3} = \frac{1}{5^3}$  (c)  $\left(\frac{1}{2}\right)^{-3} = \left(\frac{2}{1}\right)^3 = 2^3$  (d)  $\left(\frac{2}{5}\right)^{-4} = \left(\frac{5}{2}\right)^4 = \frac{5^4}{2^4}$

(e)  $\left(\frac{4}{3}\right)^{-5} = \left(\frac{3}{4}\right)^5 = \frac{3^5}{4^5}$  (f)  $\frac{1}{4} - 2^{-1} = \frac{1}{4} - \frac{1}{2}$  (g)  $p^{-2} = \frac{1}{p^2}$  (h)  $\frac{1}{x^{-4}} = \frac{x^4}{1} = x^4$  (i)  $x^3 y^{-4} = \frac{x^3}{y^4}$

#### Changing from Negative to Positive Exponents

For any nonzero numbers  $a$  and  $b$  and any integers  $m$  and  $n$ , the following are true.

$$\frac{a^{-m} b^n}{b^{-n} a^m} = \frac{b^n}{a^m} \quad \text{and} \quad \left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m$$

Examples:  $\frac{3^{-5}}{2^{-4}} = \frac{2^4}{3^5}$  and  $\left(\frac{4}{5}\right)^{-3} = \left(\frac{5}{4}\right)^3$

#### EXAMPLE 3 Changing from Negative to Positive Exponents

Simplify by writing with positive exponents. Assume that all variables represent nonzero real numbers.

(a)  $\frac{4^{-5}}{5^{-3}} = \frac{5^3}{4^5}$  (b)  $\frac{m^{-5}}{p^{-1}} = \frac{p}{m^5}$

(c)  $\frac{a^{-2} b^3}{3a^2} = \frac{bd^3}{3a^2}$  (d)  $\left(\frac{x}{2y}\right)^{-4} = \left(\frac{2y}{x}\right)^4 = \frac{2^4 y^4}{x^4} = \frac{16y^4}{x^4}$

### Quotient rule for exponents

If there's the same base, you subtract the exponent in denominator from the exponent in the numerator top-bottom

$$\frac{x^5}{x^3} = x^{5-3} = x^2$$

$$\frac{x^3}{x^3} = x^{3-3} = x^0 = 1$$

#### EXAMPLE 4 Using the Quotient Rule

Simplify by writing with positive exponents. Assume that all variables represent nonzero real numbers.

(a)  $\frac{5^8}{5^6} = \frac{5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5}{5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5} = 5^2 = 25$  (b)  $\frac{4^3}{4^9} = 4^{-6} = \frac{1}{4^6}$

$5^8 \div 5^6 = 5^{8-6} = 5^2 = 25$

$4^3 \div 4^9 = 4^{3-9} = 4^{-6} = \frac{1}{4^6}$

$$5^{8-6} = 5^2 = 25$$

$$(c) \frac{5^{-3}}{5^{-7}} = 5^{-3-(-7)} = 5^4$$

$$(d) \frac{q^5}{q^{-3}} = q^{5-(-3)} = q^8$$

$$(e) \frac{3^2 \cdot 3^4}{3^4 \cdot 3^2} = 3^{2+4-4-2} = 3^0 = 1$$

$$(f) \frac{(m+n)^{-2}}{(m+n)^{-4}} = (m+n)^{-2-(-4)} = (m+n)^2$$

$$(g) \frac{2^7 y^2 z^5}{x^3 y^2 z^2} = \frac{2^7 y^2 z^5}{x^3 y^2 z^2} = \frac{14y^7}{x^5}$$

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**EXAMPLE 5** Using Combinations of Rules

Simplify. Assume that all variables represent nonzero real numbers.

$$(a) \frac{(4^2)^3}{4^5} = \frac{4^6}{4^5} = 4^{6-5} = 4^1 = 4$$

$$(b) (2x)^3(2x)^2 = (2x)^{3+2} = (2x)^5 = 2^5 x^5 = 32x^5$$

$$(c) \left(\frac{2x^3}{5}\right)^4 = \frac{(2x^3)^4}{5^4} = \frac{2^4 x^{12}}{5^4} = \frac{16x^{12}}{625}$$

$$(d) \frac{(3x^{-2})^{-3}}{(4^{-1}y^3)^2} = \frac{3^9 x^6}{4^2 y^6} = \frac{y^9 x^6}{4^3 \cdot 3^3} = \frac{x^6 y^9}{1728}$$

$$(e) \frac{(4m)^{-3}}{(3m)^{-4}} = \frac{4^3 m^3}{3^4 m^3} = \frac{3 \cdot 3 \cdot 3 \cdot m^3}{4 \cdot 4 \cdot 4 \cdot m^3} = \frac{3 \cdot 3 \cdot 3 \cdot m}{4 \cdot 4 \cdot 4} = \frac{81m}{64}$$

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$$6^{-1} + 4^{-1}$$

$$\frac{4 \cdot 1}{4 \cdot 6} + \frac{1 \cdot 6}{4 \cdot 6}$$

$$\frac{4}{24} + \frac{6}{24} = \frac{10 \div 2}{24 \div 2} = \frac{5}{12}$$

## 5.3 An Application of Exponents: Scientific Notation

## OBJECTIVES

- 1 Express numbers in scientific notation.
- 2 Convert numbers in scientific notation to numbers without exponents.
- 3 Use scientific notation in calculations.

Scientific Notation

$5 \times 10^8$  ← how many times to move the decimal  
 $5 \times 10 = 50$   
 $5 \times 10 \times 10 = 500$   
 $5 \times 10 \times 10 \times 10 = 5000$   
 $5.2 \times 10^6$   
 ↑ number between 0-10  
 6 decimal between 1<sup>st</sup> & 2<sup>nd</sup> digit  
 $500000000$   
 $52000000$

## EXAMPLE 1 Using Scientific Notation

Write each number in scientific notation.

(a)  $93,000,000$   $9.3 \times 10^7$

(b)  $63,200,000,000$   $6.32 \times 10^9$

(c)  $0.00462$   $4.62 \times 10^{-3}$

(d)  $-0.0000762$   $-7.62 \times 10^{-5}$

## EXAMPLE 2 Writing Numbers without Exponents

Write each number without exponents.

(a)  $6.2 \times 10^3$   $6200 = \boxed{6,200}$

(b)  $4.283 \times 10^6$   $4.283 \times 10^6 = \boxed{4283000}$

(c)  $7.04 \times 10^{-3}$   $0.00704 = \boxed{.00704}$

**EXAMPLE 3** Multiplying and Dividing with Scientific Notation

Perform each calculation.

(a)  $(7 \times 10^3)(5 \times 10^4)$

$$35 \times 10^7 \quad 35,000,000$$

$$3.5 \times 10^8$$

(b)  $\frac{4 \times 10^{-5}}{2 \times 10^3}$

$$= 2 \times 10^{-8}$$

(c)  $(3 \times 10^4)(8 \times 10^7)$

$$= 24 \times 10^{11}$$

$$2.4 \times 10^{12}$$

(d)  $\frac{6 \times 10^{-2}}{2 \times 10^8}$

$$3 \times 10^{-10}$$

$$(3.937) \times 10^{-8}$$

$$700,000 = (7) \times 10^5$$

$$0.27559 \times 10^{-3}$$

$$0.27559 \times 10^{-2}$$

$$2.7559 \times 10^{-2}$$

**EXAMPLE 4** Using Scientific Notation to Solve an Application

A *nanometer* is a very small unit of measure that is equivalent to about 0.00000003937 in. About how much would 700,000 nanometers measure in inches? (Source: *World Almanac and Book of Facts*.)

**EXAMPLE 5** Using Scientific Notation to Solve an Application

In 2008, the national debt was  $\$1.0025 \times 10^{13}$  (which is more than \$10 trillion). The population of the United States was approximately 304 million that year. About how much would each person have had to contribute in order to pay off the national debt? (Source: Bureau of Public Land; U.S. Census Bureau.)

$$\frac{1.0025 \times 10^{13}}{3.04 \times 10^8}$$

$$329 \times 10^5$$

$$3.04,000,000$$

$$3.04 \times 10^8$$

$$.32900$$

$$329 \times 10^5 \quad .32900$$

$$\boxed{3.29 \times 10^4}$$

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### 5.4 Adding and Subtracting Polynomials; Graphing Simple Polynomials

#### OBJECTIVES

- 1 Identify terms and coefficients.
- 2 Add like terms.
- 3 Know the vocabulary for polynomials.
- 4 Evaluate polynomials.
- 5 Add and subtract polynomials.
- 6 Graph equations defined by polynomials of degree 2.

Terms pieces of an expression - separated by a + or -

Numerical coefficient

$$4x^2y$$

all the things that are multiplied

number in front

$$4x^2y +$$

#### EXAMPLE 1

##### Identifying Coefficients

Name the coefficient of each term in these expressions.

(a)  $x - 6x^4$

$$\uparrow \quad \uparrow$$

$$1 \quad -6$$

(b)  $5 - y^3$

$$\uparrow \quad \uparrow$$

$$5 \quad -1$$

#### EXAMPLE 2

##### Adding Like Terms

Simplify by adding like terms.

(a)  $4x^3 + 6x^3$

$$2x^3$$

(b)  $9x^6 - 14x^6 + x^6$

$$-4x^6$$

(c)  $12m^2 + 5m + 4m^2$

$$16m^2 + 5m$$

(d)  $3x^2y + 4x^2y - x^2y$

$$6x^2y$$

combine = add

Like terms- same variable raised to same power

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#### Standard form

write the biggest exponent first

$$x^3 + 2x^2 - 3x + 2$$

#### Degree of a term

$x^2$  - exponent of the term

#### Degree of a polynomial

The biggest exponent

3rd degree -  $x^3$

#### Monomial

1 term

#### Binomial

2 terms

$$x - 2$$

#### Trinomial

3 terms

$$x^2 + 3x - 2$$

| Term                   | Degree | Polynomial         | Degree |
|------------------------|--------|--------------------|--------|
| $3x^4$                 | 4      | $3x^4 - 5x^2 + 6$  | 4      |
| $5x$ , or $5x^1$       | 1      | $5x + 7$           | 1      |
| $-7$ , or $-7x^0$      | 0      | $x^2y + xy - 5y^2$ | 3      |
| $2x^3y$ , or $2x^3y^1$ | 3      | $x^5 + 3x^6$       | 6      |

#### EXAMPLE 3

##### Classifying Polynomials

For each polynomial, first simplify, if possible. Then give the degree and tell whether the polynomial is a *monomial*, a *binomial*, a *trinomial*, or *none of these*.

(a)  $2x^3 + 5$

binomial

(b)  $4xy - 5xy + 2xy$

1 x y

the polynomial is a *monomial*, a *binomial*, a *trinomial*, or *none of these*.

(a)  $2x^3 + 5$

binomial  
degree 3

(b)  $4xy - 5xy + 2xy$

$1xy$   
 $xy$  degree 2

#### EXAMPLE 4 Evaluating a Polynomial

Find the value of  $3x^4 + 5x^3 - 4x - 4$  for (a)  $x = -2$  and (b)  $x = 3$ .

$$\begin{aligned} 3(-2)^4 + 5(-2)^3 - 4(-2) - 4 \\ 3(16) + 5(-8) + 8 - 4 \\ 48 - 40 + 8 - 4 \end{aligned}$$

$$\begin{aligned} 8 + 8 \\ 16 - 4 \end{aligned} \quad \boxed{12}$$

$$\begin{aligned} 3(3)^4 + 5(3)^3 - 4(3) - 4 \\ 3(81) + 5(27) - 12 - 4 \\ 243 + 135 - 12 - 4 \\ 365 - 12 - 4 \\ 365 - 16 \\ 349 \end{aligned}$$

#### Adding polynomials

Add any like terms  
arrange in standard form

#### EXAMPLE 5 Adding Polynomials Vertically

(a) Add:  $(6x^3 - 4x^2 + 3) + (-2x^3 + 7x^2 - 5)$ .

$$-2x^3 + 7x^2 - 5$$

$$\boxed{4x^3 + 3x^2 - 2}$$

#### EXAMPLE 6 Adding Polynomials Horizontally

(a) Add:  $(6x^3 - 4x^2 + 3) + (-2x^3 + 7x^2 - 5)$

$$\boxed{4x^3 + 3x^2 - 2}$$

#### Subtracting polynomials

Turn it into addition  
by distributing the negative (change all the signs after the subtract)

#### EXAMPLE 7 Subtracting Polynomials Horizontally

(a) Perform the subtraction  $(5x - 2) - (3x + 8)$ .

$$-3x + 8$$

$$2x + 6$$

(b) Subtract:  $(6x^3 - 4x^2 + 2) - (11x^3 + 2x^2 + 8)$ .

$$-5x^3 - 6x^2 + 10$$

#### EXAMPLE 8 Subtracting Polynomials Vertically

Subtract by columns to find

$$(14y^3 - 6y^2 + 2y - 5) - (2y^3 - 7y^2 - 4y + 6)$$

$$+ (-2y^3 + 7y^2 + 4y + 6)$$

$$\boxed{12y^3 + y^2 + 6y - 11}$$

#### EXAMPLE 9 Adding and Subtracting Polynomials with More Than One Variable

Add or subtract as indicated.

(a)  $(4a + 2ab - b) + (3a - ab + b)$

**EXAMPLE 2** Adding and Subtracting Polynomials with More Than One Variable

Add or subtract as indicated.

(a)  $(4a + 2ab - b) + (3a - ab + b)$

$7a + ab$

(b)  $(2x^2y + 3xy + y^2) + (3x^2y - xy + 2y^2)$

$5x^2y + 4xy + 3y^2$

(c)  $(8a^3 - 2a^2 + 3) + (-2a^3 + 6a - 2)$

$6a^3 - 2a^2 + 6a + 1$

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**5.5 Multiplying Polynomials**

**OBJECTIVES**

- 1 Multiply a monomial and a polynomial.
- 2 Multiply two polynomials.
- 3 Multiply binomials by the FOIL method.

**EXAMPLE 1** Multiplying Monomials and Polynomials

Find each product.

(a)  $4x^2(3x + 5)$

$12x^3 + 20x^2$

(b)  $-8m^3(4m^3 + 3m^2 + 2m - 1)$

$-32m^6 - 24m^5 - 16m^4 + 8m^3$

$3x^3(2x^2 + 4x + 1)$   
 $6x^5 + 12x^4 + 3x^3$

**EXAMPLE 2** Multiplying Two Polynomials

a) Multiply  $(m^2 + 5)(4m^3 - 2m^2 + 4m)$ .

|       |          |          |        |
|-------|----------|----------|--------|
|       | $4m^3$   | $-2m^2$  | $4m$   |
| $m^2$ | $4m^5$   | $-2m^4$  | $4m^3$ |
| $+5$  | $+20m^3$ | $-10m^2$ | $20m$  |

$4m^5 - 2m^4 + 24m^3 - 10m^2 + 20m$

b) Multiply  $(x^3 + 2x^2 + 4x + 1)(3x + 5)$ .

$(3x+5)(x^3+2x^2+4x+1)$   
 $3x^4 + 6x^3 + 12x^2 + 3x + 5x^3 + 10x^2 + 20x + 5$   
 $3x^4 + 11x^3 + 22x^2 + 23x + 5$

$4x^2 - 10x$   
 $-(2x^2 - 6x)$   
 $(2x-3)(4x^2+2x-5)$   
 $8x^3 - 8x^2 - 16x + 15$

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**EXAMPLE 4** Multiplying Polynomials with Fractional Coefficients

Find the product of  $4m^3 - 2m^2 + 4m$  and  $\frac{1}{2}m^2 + \frac{5}{2}$ .

$(\frac{1}{2}m^2 + \frac{5}{2})(4m^3 - 2m^2 + 4m)$

**EXAMPLE 4** Multiplying Polynomials with Fractional CoefficientsFind the product of  $4m^2 - 2m^2 + 4m$  and  $\frac{1}{2}m^2 + \frac{5}{2}$ .

$$\left(\frac{1}{2}m^2 + \frac{5}{2}\right)(4m^3 - 2m^2 + 4m)$$

$$2m^5 - 1m^4 + 2m^3 + 10m^3 - 5m^2 + 10m$$

$$2m^5 - m^4 + 12m^3 - 5m^2 + 10m$$

FOIL

When multiplying 2 binomials

F - first terms

O - outer

I - inner

L - last

$$(x+2)(x-3)$$

**EXAMPLE 5** Using the FOIL MethodUse the FOIL method to find the product  $(x+8)(x-6)$ .

$$x^2 - 6x + 8x - 48$$

$$x^2 + 2x - 48$$

**EXAMPLE 6** Using the FOIL MethodMultiply  $(9x-2)(3y+1)$ .

$$27xy + 9x - 6y - 2$$

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**EXAMPLE 7** Using the FOIL Method

Find each product.

(a)  $(2k+5y)(k+3y)$ 

$$2k^2 + 11ky + 15y^2$$

(c)  $2x^2(x-3)(3x+4)$ 

$$2x^2(3x^2 + 4x - 9x - 12)$$

$$2x^2(3x^2 - 5x - 12)$$

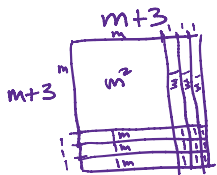
$$6x^4 - 10x^3 - 24x^2$$

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## 5.6 Special Products

## OBJECTIVES

- 1 Square binomials.
- 2 Find the product of the sum and difference of two terms.
- 3 Find greater powers of binomials.



## EXAMPLE 1 Squaring a Binomial

Find  $(m+3)(m+3)$ 

$$\begin{aligned} (m+3)(m+3) &= m^2 + 3m + 3m + 9 \\ &= m^2 + 6m + 9 \end{aligned}$$

CAUTION:

$$(m+3)^2 \neq m^2 + 9$$

$$(3m)^2 = 3^2 m^2 = 9m^2$$

## EXAMPLE 2 Squaring Binomials

Find each binomial square and simplify.

(a)  $(5z-1)(5z-1)$

$$\begin{aligned} (5z-1)(5z-1) &= 25z^2 - 5z - 5z + 1 \\ &= 25z^2 - 10z + 1 \end{aligned}$$

(b)  $(3b+5r)(3b+5r)$

$$\begin{aligned} (3b+5r)(3b+5r) &= 9b^2 + 15br + 15br + 25r^2 \\ &= 9b^2 + 30br + 25r^2 \end{aligned}$$

(c)  $(2a-9x)(2a-9x)$

$$\begin{aligned} (2a-9x)(2a-9x) &= 4a^2 - 18ax - 18ax + 81x^2 \\ &= 4a^2 - 36ax + 81x^2 \end{aligned}$$

(d)  $(4m + \frac{1}{2})(4m + \frac{1}{2})$

$$\begin{aligned} (4m + \frac{1}{2})(4m + \frac{1}{2}) &= 16m^2 + 2m + 2m + \frac{1}{4} \\ &= 16m^2 + 4m + \frac{1}{4} \end{aligned}$$

$\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

(e)  $x(4x-3)(4x-3)$

$$\begin{aligned} x(4x-3)(4x-3) &= x(16x^2 - 12x - 12x + 9) \\ &= x(16x^2 - 24x + 9) \\ &= 16x^3 - 24x^2 + 9x \end{aligned}$$

Conjugates: Binomials with same terms but different signs  
 $(x+5)(x-5)$   $(6x+7)(6x-7)$

Example 3: Find the product of each set of conjugates.

(a)  $(x+4)(x-4)$

$$\begin{aligned} (x+4)(x-4) &= x^2 - 4x + 4x - 16 \\ &= x^2 - 16 \end{aligned}$$

(b)  $(x+10)(x-10)$

$$\begin{aligned} (x+10)(x-10) &= x^2 - 10x + 10x - 100 \\ &= x^2 - 100 \end{aligned}$$

(c)  $(x-5)(x+5)$

$$\begin{aligned} (x-5)(x+5) &= x^2 - 5x + 5x - 25 \\ &= x^2 - 25 \end{aligned}$$

(d)  $(\frac{2}{3} - w)(\frac{2}{3} + w)$

$$\begin{aligned} (\frac{2}{3} - w)(\frac{2}{3} + w) &= \frac{4}{9} - w^2 \end{aligned}$$

$\frac{2}{3} \cdot \frac{2}{3} = \frac{4}{9}$

(e)  $(5m-3)(5m+3)$

$$\begin{aligned} (5m-3)(5m+3) &= 25m^2 - 9 \end{aligned}$$

(f)  $(4x-6y)(4x+6y)$

$$\begin{aligned} (4x-6y)(4x+6y) &= 16x^2 + 24xy - 24xy - 36y^2 \\ &= 16x^2 - 36y^2 \end{aligned}$$

(g)  $(z - \frac{1}{4})(z + \frac{1}{4})$

$$\begin{aligned} (z - \frac{1}{4})(z + \frac{1}{4}) &= z^2 - \frac{1}{16} \end{aligned}$$

$$z^2 - \frac{1}{16}$$

**EXAMPLE 5** Finding Greater Powers of Binomials

Find each product.

(a)  $(x + 5)(x + 5)(x + 5)$

$$(x + 5)(x^2 + 5x + 5x + 25)$$

$$(x + 5)(x^2 + 10x + 25)$$

$$x^3 + 10x^2 + 25x$$

$$+ 5x^2 + 50x + 125$$

$$x^3 + 15x^2 + 75x + 125$$

(c)  $-2r(r + 2)(r + 2)(r + 2)$

$$-2r(r + 2)(r^2 + 4r + 4)$$

$$-2r(r^3 + 4r^2 + 4r + 2r^2 + 8r + 8)$$

$$-2r(r^3 + 6r^2 + 12r + 8)$$

$$-2r^4 - 12r^3 - 24r^2 - 16r$$

**5.7 Dividing Polynomials Day 1****OBJECTIVES**

- 1** Divide a polynomial by a monomial.
- 2** Divide a polynomial by a polynomial.

**EXAMPLE 1** Dividing a Polynomial by a Monomial

Divide  $5m^5 - 10m^3$  by  $5m^2$ .

**EXAMPLE 2** Dividing a Polynomial by a Monomial

Divide.

$$\begin{array}{r} 16a^5 - 12a^4 + 8a^2 \\ \hline 4a^3 \end{array}$$

**EXAMPLE 3** Dividing a Polynomial by a Monomial with a Negative Coefficient

Divide  $-7x^3 + 12x^4 - 4x$  by  $-4x$ .

**EXAMPLE 4** Dividing a Polynomial by a Monomial

Divide  $180x^4y^{10} - 150x^3y^8 + 120x^2y^6 - 90xy^4 + 100y$  by  $30xy^2$ .

Stop here and do Ch 6 next. Do long and synthetic division after Ch 6?

**5.7 Day 2: Dividing Polynomials****Synthetic Division:**

Recall: Use long division to find  $853 \div 6$ .

**Long Division with Polynomials:****EXAMPLE 6** Dividing a Polynomial by a Polynomial

(a) Divide by using long division:  $\frac{3x^2 - 5x - 28}{x - 4}$

b) Divide by using long division:  $\frac{6k^3 - 20k - k^2 + 1}{2k - 3}$

c) Divide by using long division:  $\frac{4a^2 - 22a + 32}{2a + 3}$

**EXAMPLE 5** Dividing a Polynomial by a Polynomial

Divide.

a)  $\frac{3x^2 - 5x - 28}{x - 4}$

b)  $\frac{4x^2 + x - 18}{x - 2}$

c)  $\frac{2y^2 + 9y - 35}{y + 7}$

**EXAMPLE 7** Dividing into a Polynomial with Missing Terms

a) Divide  $x^3 - 1$  by  $x - 1$ .

b) Divide  $m^3 - 1000$  by  $m - 10$ .

**EXAMPLE 8** Dividing by a Polynomial with Missing Terms

a) Divide  $x^4 + 2x^3 + 2x^2 - x - 1$  by  $x^2 + 1$ .

b) Divide  $y^4 - 5y^3 + 6y^2 + y - 4$  by  $y^2 + 2$