Objectives

1. Use the compound interest formulas.
2. Calculate present value.
3. Understand and compute effective annual yield.

▪Compound Interest

Compound interest is interest computed on the accumulated interest as well as on the principle.
To calculate a total amount accumulated with compound interest paid ~~once~~ per year we use: annually

A = total amount (future value)
P = principal: starting $ (amount borrowed OR invested)

$$\boxed{A = P(1 + r)^t}$$

r = rate
t = time (in years)

Example 1: You deposit $2000 in a savings account at Hometown Bank, which has a rate of 6%. ⇒ r = .06

a. Find the amount, A, of money in the account after 3 years subject to compound interest, compounded annually

P = 2000, t = 3

$A = 2000(1+.06)^3$

$A = 2000(1.06)^3$

$A = \$2382.03$ Nice! Your $ has grown!

b. Find the interest.

How much did it grow? SUBTRACT

$2382.03 - 2000 = \boxed{\$382.03}$

Example 2: Review of using natural logarithms to solve exponential equations. Solve each equation for t. → LN

a) $50 = 4^t$

$LN(50) = LN 4^t$ ← take LN of both sides

$LN(50) = t \cdot LN(4)$ ← log property: exponent moves in front of LN (multiplier)

$\frac{LN(50)}{LN(4)} = \frac{t \cdot LN(4)}{LN(4)}$ Divide to SOLVE

$\boxed{2.82 = t}$

b) $9 = 3(8.2)^t$ First: isolate $(\text{base})^{\text{exp}}$

$\frac{9}{3} = \frac{3(8.2)^t}{3}$

$LN\,3 = LN(8.2)^t$ ← take LN of each side

$LN(3) = t\,LN(8.2)$ ← move exp. in front of LN

$\frac{LN(3)}{LN(8.2)} = \frac{t\,LN(8.2)}{LN(8.2)}$ Divide to solve

$\boxed{0.52 = t}$

Example 3: How many years would it take to have a total of $15,000, if $5000 deposited into an account with an interest rate of 7%, compounded annually?

A = 15,000, P = 5000, r = .07

Find $\boxed{t = ?}$

$A = P(1+r)^t$

$\frac{15000}{5000} = \frac{5000(1+.07)^t}{5000}$

$3 = (1.07)^t$

$LN(3) = LN(1.07)^t$

$LN(3) = t \cdot LN(1.07)$

$\frac{ln(3)}{ln(1.07)} = \frac{t \cdot ln(1.07)}{ln(1.07)}$

$\boxed{16.24 = t}$ → $\boxed{\text{over 16 years}}$

LN or ln

■**Compound Interest:** To calculate the compound interest paid **more** than once a year we use

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

$\boxed{n = \text{\# of times per year}}$

Note: We will assume 360 days per year and round answers to the nearest cent (2 decimal places.)

compounded daily : $\boxed{n=360}$

Ex. compounded monthly: $\boxed{n=12}$

Example 4: You deposit $7500 in a savings account that has a rate of 6%. The interest is compounded monthly.

$P=$ $r=.06$

a. How much money will you have after five years? $t=5$

→ Find A (future value)

$$A = 7500\left(1+\frac{.06}{12}\right)^{12\cdot 5}$$

$$\boxed{A = \$10,116.38}$$

b. Find the interest after five years.

How much has it grown? $10116.38 - 7500 = \boxed{2616.38}$

Example 5: Find the number of years it would take for an initial investment of $3000 to grow to $30,000, if it is in an account earning 4.2% interest, compounded quarterly.

$P=$ $A=$

Find t

$r=.042$ $n=4$

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

$$\frac{30{,}000}{3000} = \frac{3000}{3000}\left(1+\frac{.042}{4}\right)^{4t}$$

$$\ln 10 = \ln\left(1+\frac{.042}{4}\right)^{4t}$$

$$\frac{\ln(10)}{(4\cdot\ln(1.015))} = \frac{4t\cdot\ln(1.0105)}{4\cdot\ln(1.0105)} \Rightarrow \boxed{t = 55.1} \Rightarrow \boxed{\text{over 55 years}}$$

Note: need to use ( ) to divide by whole expression $(4\cdot\ln(1.0105))$

Example 6: An investment is made into a fund that earns 4.2%, compounded daily. If $8,000 is initially invested, how much money will be in the account after 5 years? How much interest will have accrued?

Find A $r=.042$ $t=5$ $n=360$ $P=8000$

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

$$A = 8000\left(1+\frac{.042}{360}\right)^{360\cdot 5}$$

$$A = \boxed{9869.30}$$

Future value
accumulated value

How much interest?
SUBTRACT!

$$\begin{array}{r} 9869.30 \\ -8000.00 \\ \hline \end{array}$$

\$1869.30 interest

Compound Interest: Continuous Compounding

- Some banks use ***continuous compounding***, where the compounding periods increase infinitely.

After t years, the balance, A, in an account with principal P and annual interest rate r (in decimal form) is given by the following formulas:

1. For n compounding periods per year: $A = P\left(1+\frac{r}{n}\right)^{nt}$.
2. For continuous compounding: $\boxed{A = Pe^{rt}}$

where $e \approx 2.718\ldots$

Big idea!
- $n=4$ (quarterly) grows more than $n=1$ (annually)
- $n=12$ (monthly) also grows some more
- $n=52$ (weekly) grows a bit more
- $n=360$ (daily) grows a little more
- $n \Rightarrow \infty$ starts to plateau $\Rightarrow$ use e

Example 7: You decide to invest \$8000 for 6 years and you have a choice between two accounts. The first pays 7% per year, compounded monthly. The second pays 6.85% per year, compounded continuously. Which is the better investment?

$P = 8000$
$t = 6$

First option

$r = .07$
$n = 12$

$A = P\left(1+\frac{r}{n}\right)^{nt}$

$A = 8000\left(1+\frac{.07}{12}\right)^{12\cdot 6}$

$\boxed{A = \$12160.84}$

Better option

Second option

$r = .0685$
~~$n=$~~
Use $A = P\cdot e^{rt}$

$A = P\cdot e^{rt}$

$A = 8000\cdot e^{.0685\times 6}$

$\boxed{A = 12066.60}$

Use $\boxed{\times}$ key, not ( ) to multiply in calculator!

Example 8: Charlie invests \$3000 in an account that earns 5% interest, compounded continuously. Around how many years would it take for this amount to grow to \$12,000?

$P=3000$, $r=.05$, $A=12{,}000$

Use $A = Pe^{rt}$

Find t

$A = P\cdot e^{rt}$

$\frac{12000}{3000} = \frac{3000\cdot e^{.05t}}{3000}$

$\ln 4 = \ln e^{.05t}$

$\ln(e) = 1$

$\ln(4) = .05t\cdot \underbrace{\ln(e)}$

$\frac{\ln(4)}{.05} = \frac{.05t\cdot 1}{.05}$

$\boxed{27.7 = t} \Rightarrow \boxed{\text{Almost 28 years}}$

Recall: A = future value
P = present value (\$ invested)

Calculating Present Value

If A dollars are to be accumulated in t years in an account that pays rate r compounded n times per year, then the present value P that needs to be invested now is given by $\boxed{P = \dfrac{A}{\left(1+\dfrac{r}{n}\right)^{nt}}}$.

Example 7: How much money should be deposited in an account today that earns 8% compounded monthly so that it will accumulate to \$20,000 in five years?

$A = 20{,}000$ $t = 5$ $r = .08$ $n = 12$

$$P = \frac{20{,}000}{\left(1+\frac{.08}{12}\right)^{12\cdot 5}}$$

$$\boxed{P = \$13{,}424.21}$$

Could you use $A = P\left(1+\frac{r}{n}\right)^{nt}$ and solve for P?
Yes, but it loses accuracy if you round off $\left(1+\frac{r}{n}\right)^{nt}$ to divide by it.

$$\frac{A}{\left(1+\frac{r}{n}\right)^{nt}} = \frac{P\cancel{\left(1+\frac{r}{n}\right)^{nt}}}{\cancel{\left(1+\frac{r}{n}\right)^{nt}}}$$

Instead, divide both sides of the formula

$$\boxed{\frac{A}{\left(1+\frac{r}{n}\right)^{nt}} = P}$$

Then plug in the values in this new version of it ☺