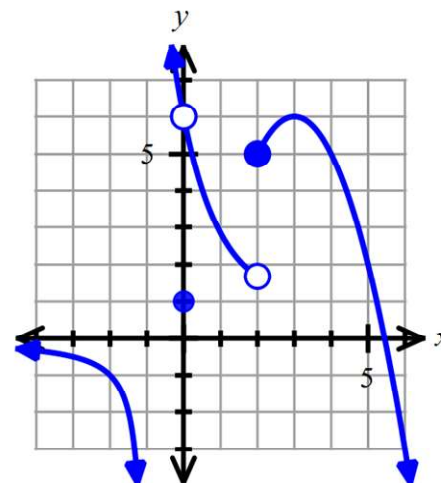


**11.1 Assignment: Do all work on your own paper.**

Calculator allowed.

For #1 – 12, use the graph of  $f(x)$  shown to find each value.

- 1)  $f(0)$       2)  $\lim_{x \rightarrow 0} f(x)$       3)  $f(5)$       4)  $\lim_{x \rightarrow 5} f(x)$   
 5)  $f(2)$       6)  $\lim_{x \rightarrow 2} f(x)$       7)  $f(-2)$       8)  $\lim_{x \rightarrow -2} f(x)$   
 9)  $\lim_{x \rightarrow -1} f(x)$       10)  $\lim_{x \rightarrow 4} f(x)$



11) Identify any values of  $x$ , if any, where  $f(x)$  has a discontinuity and the limit of  $f(x)$  also exists. In addition, identify the type of discontinuity for those  $x$ -values (if any).

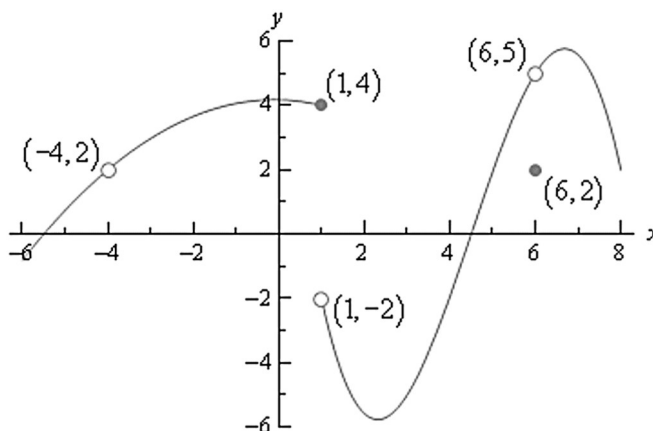
12) Identify any values of  $x$ , if any, where  $f(x)$  has a discontinuity and the limit of  $f(x)$  does **not** exist. In addition, identify the type of discontinuity for those  $x$ -values (if any).

For #13 – 15, create a table of values to find each limit.

- 13)  $\lim_{x \rightarrow -1} (3x^2 - 2x)$   
 14)  $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x}$       15)  $\lim_{x \rightarrow 5} \frac{x^2 - 25}{x - 5}$

For #16 – 22, use the graph of  $g(x)$  as shown to the right to find each value.

- 16)  $\lim_{x \rightarrow 1^-} g(x)$       17)  $\lim_{x \rightarrow 1^+} g(x)$       18)  $\lim_{x \rightarrow 1} g(x)$   
 19)  $\lim_{x \rightarrow 6^-} g(x)$       20)  $\lim_{x \rightarrow 6^+} g(x)$       21)  $\lim_{x \rightarrow 6} g(x)$



22)  $g(6)$

For #23 – 27, graph of  $h(x) = \begin{cases} -2x + 5 & \text{if } x \leq 3 \\ 6x - 7 & \text{if } x > 3 \end{cases}$  to find each value, if possible.

- 23) Find  $h(1)$ .      24) Find  $h(3)$ .      25) Find  $\lim_{x \rightarrow 3^+} h(x)$   
 26) Find  $\lim_{x \rightarrow 3^-} h(x)$       27) Find  $\lim_{x \rightarrow 3} h(x)$

For #28 – 31, decide if each statement is true or false.

- 28) If  $q(3) = 8$ , then  $\lim_{x \rightarrow 3} q(x) = 8$ .  
 29) If  $\lim_{x \rightarrow 3} q(x) = 8$ , then  $q(3) = 8$ .  
 30) If  $\lim_{x \rightarrow 3^+} q(x) = 8$  and  $\lim_{x \rightarrow 3^-} q(x) = 8$ , then  $\lim_{x \rightarrow 3} q(x) = 8$ .  
 31) If  $\lim_{x \rightarrow 3^+} q(x) = 8$  and  $\lim_{x \rightarrow 3^-} q(x) = 8$ , then  $q(3) = 8$ .

- Answers:**      1) 1      2) 6      3) 2      4) 2      5) 5      6) does not exist (DNE)  
 7) -1      8) -1      9) does not exist (DNE)      10) 5      11)  $x = 0$ ; hole  
 12)  $x = -1$  (vertical asymptote);  $x = 2$  (jump discontinuity)      13) 5      14) 0      15) 10  
 16) 4      17) -2      18) does not exist (DNE)      19) 5      20) 5      21) 5  
 22) 2      23) 3      24) -1      25) 11      26) -1      27) does not exist  
 28) False      29) False      30) True      31) False

**11.2 Assignment: Do all work on your own paper.**

Calculator allowed.

For #1 – 4, evaluate each limit given that  $\lim_{x \rightarrow 3} f(x) = 16$  and  $\lim_{x \rightarrow 3} g(x) = -4$ .

1)  $\lim_{x \rightarrow 3} (f(x) + 5g(x))$

2)  $\lim_{x \rightarrow 3} (f(x) \cdot g(x))$

3)  $\lim_{x \rightarrow 3} \frac{f(x)}{2g(x)}$

4)  $\lim_{x \rightarrow 3} (2f(x) - 3g(x))$

For #5 – 19, evaluate each limit. Simplify your final answer. Exact answers only (no decimals).

5)  $\lim_{x \rightarrow -5} (x^2 - 1)$

6)  $\lim_{x \rightarrow -8} (12 - \sqrt[3]{x})$

7)  $\lim_{x \rightarrow -4} (-6x + 3)$

8)  $\lim_{x \rightarrow -1} (-7x - 4x^3)$

9)  $\lim_{x \rightarrow 20} -6\sqrt{x}$

10)  $\lim_{x \rightarrow -2} (-x^3 + 7x^2 - 14)$

11)  $\lim_{x \rightarrow 4} (2e^{x-4} - 5)^3$

12)  $\lim_{x \rightarrow -2} \sqrt[3]{20x - 14}$

13)  $\lim_{x \rightarrow 3\pi} \frac{x}{3}$

14)  $\lim_{x \rightarrow 4} e^2$

15)  $\lim_{x \rightarrow -\pi} -3x^2$

16)  $\lim_{x \rightarrow 4} \frac{x^2 - 9x + 2}{x - 4}$

17)  $\lim_{x \rightarrow 0} \frac{\sqrt{25+x} - 5}{x}$

18)  $\lim_{x \rightarrow 0} \frac{-2x^2 + 5x}{x}$

19)  $\lim_{x \rightarrow 49} \frac{7 - \sqrt{x}}{49 - x}$

For #20 – 22: Given that  $f(x) = \begin{cases} -4x^2 + 5 & \text{if } x < 1 \\ 6x - 7 & \text{if } x \geq 1 \end{cases}$ , find each of the following limits, if possible.

20)  $\lim_{x \rightarrow 1^-} f(x)$

21)  $\lim_{x \rightarrow 1^+} f(x)$

22)  $\lim_{x \rightarrow 1} f(x)$

For #23 – 26: Given an expression for  $g(x)$ , find  $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ .

23)  $g(x) = -4x + 2$

24)  $g(x) = -5\sqrt{x}$

25)  $g(x) = 6x^2 - 3$

26)  $x^2 - 4x + 1$

**Answers:**

1)  $-4$

2)  $-64$

3)  $-2$

4)  $44$

5)  $24$

6)  $14$

7)  $27$

8)  $11$

9)  $-12\sqrt{5}$

10)  $22$

11)  $-27$

12)  $-3\sqrt[3]{2}$

13)  $\pi$

14)  $e^2$

15)  $-3\pi^2$

16)  $-1$

17)  $\frac{1}{10}$

18)  $5$

19)  $\frac{1}{14}$

20)  $1$

21)  $-1$

22) does not exist

23)  $-4$

24)  $\frac{-5\sqrt{x}}{2x}$

25)  $12x$

26)  $2x - 4$

**11.3 Assignment: Do all work on your own paper.****Calculator allowed.**

**For #1 – 4: Is the function continuous at the value of  $x$ ?** Use  $f(x) = \frac{3x-5}{3x^2+x-10}$  and  $g(x) = \frac{x^2-9}{x+3}$ .

- 1)  $f(x)$  at  $x = 4$     2)  $f(x)$  at  $x = -2$     3)  $g(x)$  at  $x = -3$     4)  $g(x)$  at  $x = 3$

**For #5 – 8:** For each piecewise function given, identify values of  $x$  that are discontinuities, if any.

$$5) f(x) = \begin{cases} -x + 3 & \text{if } x \leq 1 \\ 2 & \text{if } 1 < x \leq 3 \\ x^2 - 4 & \text{if } x > 3 \end{cases}$$

$$6) h(x) = \begin{cases} 4x^2 - 1 & \text{if } x \neq 5 \\ 2e^{x-5} + 97 & \text{if } x = 5 \end{cases}$$

$$7) g(x) = \begin{cases} -2x^3 - 4 & \text{if } x \leq -2 \\ 3x^2 - 1 & \text{if } -2 < x \leq 0 \\ 7 - x & \text{if } x > 0 \end{cases}$$

$$8) w(x) = \begin{cases} 3\sqrt[3]{x} + 5 & \text{if } x < -1 \\ 2e^{x^2-1} & \text{if } x > -1 \end{cases}$$

9) Consider the piecewise function from #6. Does the  $\lim_{x \rightarrow -1} w(x)$  exist? Explain your reasoning.

**For #10 – 14, use  $f(x) = \begin{cases} \frac{x^2-5x+6}{x^2-x-6} & \text{if } x \neq 3 \\ -2 & \text{if } x = 3 \end{cases}$**

10) Find  $f(3)$ .

11) Find  $\lim_{x \rightarrow 3^-} f(x)$ .

12) Find  $\lim_{x \rightarrow 3^+} f(x)$ .

13) Find  $\lim_{x \rightarrow 3} f(x)$ .

14) Is  $f(x)$  continuous at  $x = 3$ ? Explain.

**For #15 – 20, use  $g(x) = \begin{cases} \frac{x^2+8x+15}{x+5} & \text{if } x \neq -5 \\ -2 & \text{if } x = -5 \end{cases}$**

15) Find  $g(-5)$

16) Find  $\lim_{x \rightarrow -5^-} g(x)$ .

17) Find  $\lim_{x \rightarrow -5^+} g(x)$ .

18) Find  $\lim_{x \rightarrow -5} g(x)$ .

19) Is  $g(x)$  continuous at  $x = -5$ ? Explain.

20) Sketch  $g(x)$ .

**For #21 – 23: True or False?**

21) If  $h(x)$  is continuous at  $x = 14$ , then  $\lim_{x \rightarrow 14} h(x)$  exists.

22) If  $\lim_{x \rightarrow -2} d(x) = 7$ , then  $d(-2) = 7$ .

23) If  $g(x)$  is continuous everywhere and  $\lim_{x \rightarrow 0} g(x) = 14.2$ , then  $g(0) = 14.2$ .

**Answers:**

1) Yes    2) No    3) No    4) Yes    5)  $x = 3$     6) none

7)  $x = -2; x = 0$     8)  $x = -1$     9) yes,  $\lim_{x \rightarrow -1^-} w(x) = \lim_{x \rightarrow -1^+} w(x)$

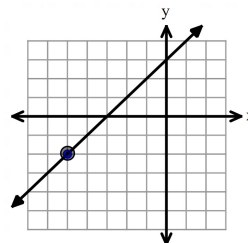
10)  $-2$     11)  $\frac{1}{5}$     12)  $\frac{1}{5}$     13)  $\frac{1}{5}$

14) no,  $\lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^+} f(x) \neq f(3)$     15)  $-2$

16)  $-2$     17)  $-2$     18)  $-2$

19) yes,  $\lim_{x \rightarrow -5^-} g(x) = \lim_{x \rightarrow -5^+} g(x) = g(-5)$     20)

21) True    22) False    23) True



**11.4 Worksheet. Do all work on your own paper.**

Calculator allowed.

**For #1 – 3, use the limit definition of a derivative to find  $g'(x)$ .**

1)  $g(x) = -6x + 13$

2)  $g(x) = 3x^2 - x$

3)  $g(x) = 2\sqrt{x}$

**For #4 – 7:** Use  $f(x) = -\frac{1}{4}x^2$ .4) Find  $f'(x)$  by using the limit definition of a derivative.5) Find the slope of the tangent line to  $f(x)$  at  $(2, -1)$ .6) Write the equation of the tangent line, in  $(h, k)$  form, to  $f(x)$  at  $(2, -1)$ .7) Sketch  $f(x)$  and the tangent line you found in #6.**For #8 – 11:** Given that  $g(x) = 5x^3 - 4x^2 + 2x$  and  $g'(x) = 15x^2 - 8x + 2$ .8) Find the slope of the tangent line to  $g(x)$  at  $(1, 3)$ .9) Write the equation of the tangent line, in  $(h, k)$  form, to  $g(x)$  at  $(1, 3)$ .10) Find the slope of the tangent line to  $g(x)$  at  $(-1, -11)$ .11) Write the equation of the tangent line, in  $(h, k)$  form, to  $g(x)$  at  $(-1, -11)$ .**For #12 – 16:** A tennis ball is tossed into the air from the roof of a building at an initial height of 37 feet and with an initial velocity of 24 feet per second. The height of the tennis ball can be modeled by  $h(t) = -16t^2 + 24t + 37$ , where the height  $h$  is measured in feet for  $t$  seconds after the ball was tossed. Also,  $h'(t) = -32t + 24$ . Assume that the ball did not hit the building once it begins to fall towards the ground.12) Find the instantaneous velocity of the ball at  $t = 1$  second.13) Find the time  $t$  where the ball hits the ground. If needed, round to two decimal places.

14) Find the instantaneous velocity of the ball at the instant that it hits the ground, to one decimal place.

15) Find the instantaneous velocity of the ball at  $t = 2$  seconds.

16) Explain why your answer for #15 is negative.

17) Multiple Choice: Which limit represents the derivative of  $y = -5x^2 + 2x + 1$ ?

A)  $\lim_{h \rightarrow 0} \frac{-5(x+h)^2 + 2(x+h) + 1 + 5x^2 + 2x + 1}{h}$

B)  $\lim_{h \rightarrow 0} \frac{-5(x+h)^2 + 2(x+h) + 1 + 5x^2 - 2x - 1}{h}$

C)  $\lim_{h \rightarrow 0} \frac{-5x^2 + 2x + 1 + h + 5x^2 + 2x + 1}{h}$

D)  $\lim_{h \rightarrow 0} \frac{-5x^2 + 2x + 1 + h + 5x^2 - 2x - 1}{h}$

18) After verifying your answer from #17, simplify the correct limit to find  $y'$ .**Answers:**

1)  $-6$

2)  $6x - 1$

3)  $\frac{\sqrt{x}}{x}$

4)  $-\frac{1}{2}x$

5)  $-1$

6)  $y = -1(x - 2) - 1$

7)

8)  $9$

9)  $y = 9(x - 1) + 3$

10)  $25$

11)  $y = 25(x + 1) - 11$

12)  $-8\frac{ft}{s}$

13)  $2.45$  seconds

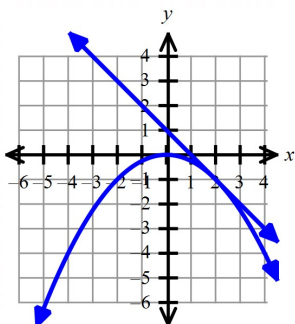
14)  $-54.4\frac{ft}{s}$

15)  $-40\frac{ft}{s}$

16) the ball is moving downward

17) B

18)  $-10x + 2$

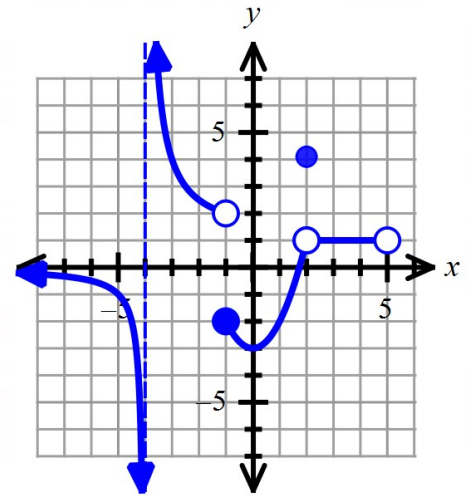


## Ch 11 Mid-Unit Review Worksheet: Do all work on your own paper.

Calculator allowed.

For #1 – 12: Use the graph of  $h(x)$  to the right to find the requested values, if possible.

- 1)  $h(2)$
- 2)  $\lim_{x \rightarrow 2} h(x)$
- 3)  $h(-1)$
- 4)  $\lim_{x \rightarrow -1} h(x)$
- 5)  $\lim_{x \rightarrow -1^+} h(x)$
- 6)  $\lim_{x \rightarrow -1^-} h(x)$
- 7)  $h(5)$
- 8)  $\lim_{x \rightarrow 5^-} h(x)$
- 9)  $\lim_{x \rightarrow 0} h(x)$
- 10) Identify any  $x$ -values where  $h(x)$  has a discontinuity but the limit exists.
- 11) Identify any  $x$ -values where  $h(x)$  has a discontinuity but the limit does not exist.
- 12) Explain why  $\lim_{x \rightarrow 5} h(x)$  does not exist.



For #13 – 14, use a table of values to evaluate each limit.

$$13) \lim_{x \rightarrow 0} \frac{\sin 3x}{4x} \qquad 14) \lim_{x \rightarrow -1} \frac{5x+5}{x^2-1}$$

For #15 – 23, evaluate each limit. Simplify your answer.

$$15) \lim_{x \rightarrow 3} \sqrt{5x+3} \qquad 16) \lim_{x \rightarrow 0} (7e^{x^2} - 4) \qquad 17) \lim_{x \rightarrow 4\pi} -11.3$$

$$18) \lim_{x \rightarrow -1} \frac{2x^2-3x+5}{x^2-8} \qquad 19) \lim_{x \rightarrow -8} \sqrt[3]{7x} \qquad 20) \lim_{x \rightarrow 25} \frac{\sqrt{x}-5}{x-25}$$

$$21) \lim_{x \rightarrow 4} \frac{3x^2-12x}{x^2-16} \qquad 22) \lim_{x \rightarrow 0} \frac{\sqrt{x+9}-3}{x} \qquad 23) \lim_{x \rightarrow -5} \frac{x+5}{x^2-x-30}$$

For #24 – 28, find each request value, if possible, given that  $f(x) = \begin{cases} -4x - 9 & \text{if } x \leq -2 \\ 2x + 3 & \text{if } -2 < x < 1 \\ 3e^{x-1} + 2 & \text{if } x > 1 \end{cases}$ 

$$24) \lim_{x \rightarrow -2^-} f(x) \qquad 25) \lim_{x \rightarrow -2^+} f(x) \qquad 26) \lim_{x \rightarrow -2} f(x) \qquad 27) f(-2)$$

$$28) \lim_{x \rightarrow 1^-} f(x) \qquad 29) \lim_{x \rightarrow 1^+} f(x) \qquad 30) \lim_{x \rightarrow 1} f(x) \qquad 31) f(1)$$

32) Identify any values of  $x$  for which  $f(x)$  is not continuous. Justify your conclusion with the definition of continuous.For #33 – 35, find  $f'(x)$  by using the limit definition of a derivative.

$$33) f(x) = 7x - 4 \qquad 34) f(x) = x^2 + 5x - 2 \qquad 35) f(x) = -2\sqrt{x}$$

For #36 – 37: Given that  $h(x) = 6x^3 - 2x^2 + 5$  and  $h'(x) = 18x^2 - 4x$ 

- 36) Find the slope of the tangent line to  $h(x)$  at  $(1, 9)$ .
- 37) Write the equation of the tangent line to  $h(x)$  in  $(h, k)$  form at  $(1, 9)$ .

## Ch 11 Mid-Unit Review Worksheet Answers:

- 1) 4
- 2) 1
- 3) -2
- 4) does not exist
- 5) -2
- 6) 2
- 7) does not exist
- 8) 1
- 9) -3
- 10)  $x = 2$
- 11)  $x = -4; x = -1; x = 5$
- 12) We do not have behavior approaching  $x = 5$  from the right side, thus the two-sided limit does not exist.
- 13)  $\frac{3}{4}$
- 14)  $-\frac{5}{2}$
- 15)  $3\sqrt{2}$
- 16) 3
- 17) -11.3
- 18)  $-\frac{10}{7}$
- 19)  $-2\sqrt[3]{7}$
- 20)  $\frac{1}{10}$
- 21)  $\frac{3}{2}$
- 22)  $\frac{1}{6}$
- 23)  $-\frac{1}{11}$
- 24) -1
- 25) -1
- 26) -1
- 27) -1
- 28) 5
- 29) 5
- 30) 5
- 31) does not exist (is not defined)
- 32)  $x = 1; \lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^+} f(x) \neq f(1)$
- 33) 7
- 34)  $2x + 5$
- 35)  $-\frac{\sqrt{x}}{x}$
- 36) 14
- 37)  $y = 14(x - 1) + 9$

## 11.5 Worksheet: Do all work on your own paper.

Calculator allowed.

For #1 – 12, find the derivative of each function. You do not need to rationalize answers.

1)  $f(x) = x^8 - 6x + 3$

2)  $y = \sqrt[3]{x} + x$

3)  $g(x) = x^{100} + 50x - 1$

4)  $h(x) = \frac{10}{x^3} + 7\sqrt{x}$

5)  $b(x) = x^2 + \frac{1}{8x^2}$

6)  $y = 10x^{-\frac{2}{5}} - \pi$

7)  $y = (3x + 5)(7 - 2x)$

8)  $f(x) = x - 3x^{-\frac{2}{3}}$

9)  $y = \frac{(x^2 + 4x + 3)}{x}$

10)  $y = \frac{5}{x^4} - \frac{7}{8x^2} + 3x^2 + 6$

11)  $h(x) = \frac{6x^{12} - 20x^9 + 5x^3}{2x^4}$

12)  $g(r) = 4\pi r^2$

For #13 – 15, write the equation of the tangent line, in  $(h, k)$  form, to  $f(x)$  at the given point.

13)  $f(x) = -x + 7; (8, -1)$

14)  $f(x) = 7x^2 + 3x; (-1, 4)$

15)  $f(x) = 2\sqrt{x} - 4x; (9, -30)$

16) Find the ordered pairs on  $y = x^3 - x^2 - x + 1$  where the tangent line to  $y$  is horizontal.

For #17 – 21: True or False?

17) Given that the limit of  $f(x)$  exists at  $x = 3$ , then  $f(x)$  is continuous at  $x = 3$ .18) If  $g(-2) = 4$  and  $g(x)$  is continuous everywhere, then  $\lim_{x \rightarrow -2} g(x) = 4$ .19) If  $h(6) = 2$  and  $h'(6) = \frac{1}{4}$ , then the equation of the tangent line to  $h(x)$  at  $(6, 2)$  is  $y = \frac{1}{4}(x - 6) + 2$ .20) If  $\lim_{x \rightarrow a} f(x)$  exists and  $f(a)$  exists, then  $f(x)$  is continuous at  $x = a$ .21) If  $\lim_{x \rightarrow -2^-} d(x) = 9$  and  $\lim_{x \rightarrow -2^+} d(x) = 9$  but  $d(-2)$  is not defined, then  $\lim_{x \rightarrow -2} d(x)$  does not exist.22) Multiple Choice. Given that  $f(x) = -3x^3 - 2x + 4$ , then  $f'(x) = ?$ 

A)  $\lim_{x \rightarrow 0} \frac{-3(x+h)^3 - 2(x+h) + 4 + 3x^2 + 2x - 4}{h}$

B)  $\lim_{x \rightarrow 0} \frac{-3(x+h)^3 - 2(x+h) + 4 - 3x^2 - 2x + 4}{h}$

C)  $\lim_{h \rightarrow 0} \frac{-3(x+h)^3 - 2(x+h) + 4 + 3x^2 + 2x - 4}{h}$

D)  $\lim_{h \rightarrow 0} \frac{-3x^3 - 2x + 4 + h + 3x^2 + 2x - 4}{h}$

## 11.5 Worksheet Answers:

1)  $f'(x) = 8x^7 - 6$

2)  $y' = \frac{1}{3}x^{-\frac{2}{3}} + 1$

3)  $g'(x) = 100x^{99} + 50$

4)  $h'(x) = -30x^{-4} + \frac{7}{2}x^{-\frac{1}{2}}$

5)  $b'(x) = 2x - \frac{1}{4}x^{-3}$

6)  $y' = -4x^{-\frac{7}{5}}$

7)  $y' = -12x + 11$

8)  $f'(x) = 1 + 2x^{-\frac{5}{3}}$

9)  $y' = 1 - 3x^{-2}$

10)  $y' = -20x^{-5} + \frac{7}{4}x^{-3} + 6x$

11)  $h'(x) = 24x^7 - 50x^4 - \frac{5}{2}x^{-2}$

12)  $g'(r) = 8\pi r$

13)  $y = -(x - 8) - 1$

14)  $y = -11(x + 1) + 4$

15)  $y = -\frac{11}{3}(x - 9) - 30$

16)  $(1, 0)$  and  $(-\frac{1}{3}, \frac{32}{27})$

17) False

18) True

19) True

20) False

21) False

22) C

**11.6 Worksheet: Do all work on your own paper.**

Calculator allowed.

**For #1 – 8, find the derivative of each function by using the method of your choice. You do not need to rationalize answers.**

- 1)  $y = -6x^5(7x^2 + 5x - 2)$       2)  $g(x) = 11x^4(x^{-3} - 2x + 6x^{-4})$       3)  $y = (4 - 5x)(3x + 1)$   
 4)  $f(x) = (2x^2 + 4x)(3x - 5)$       5)  $y = (9x^{-2} + 4)(2x^3 - 5x)$       6)  $y = 2\sqrt{x}(6x^2 + 5x)$   
 7)  $h(x) = 5\sqrt[3]{x}(4 - 3x^4 + 12x)$       8)  $y = (3x^{-1} - 6x^5)(5 + 2x^{-3})$

**For #9 – 14, use the provided table to find the requested values if  $h(x) = f(x) \cdot g(x)$ .**

$x$	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
6	-5	3	-2	4
-4	$\frac{1}{4}$	0	6	1
5	8	$-\frac{2}{5}$	7	-1

- 9)  $h'(6)$       10)  $h'(-4)$       11)  $h'(5)$   
 12) Write the equation of the tangent line, in  $(h, k)$  form, to  $h(x)$  at the point  $(5, 56)$ .  
 13) Write the equation of the tangent line, in  $(h, k)$  form, to  $h(x)$  at the point  $(6, 10)$ .  
 14) Write the equation of the tangent line, in  $(h, k)$  form, to  $h(x)$  at the point  $(-4, \frac{3}{2})$ .  
 15) **Multiple Choice.** Given that  $\lim_{h \rightarrow 0} \frac{g(x+h) - g(x)}{h} = -8x^3 + 4x$ . Which of the following could equal  $g(x)$ ?  
 A)  $-4x(2x^2 - 1)$   
 B)  $-2x^2(x^2 - 1)$   
 C)  $-24x^2 + 4$   
 D) none of these

**For #16 – 18:** A football is tossed into the air from the roof of a building at an initial height of 40 feet and with an initial velocity of 35 feet per second. The height of the tennis ball can be modeled by  $h(t) = -16t^2 + 35t + 40$ , where the height  $h$  is measured in feet for  $t$  seconds after the football was tossed. Also,  $h'(t) = -32t + 35$ . Assume that the football did not hit the building as it fall towards the ground.

- 16) Find the instantaneous velocity of the ball at  $t = 2$  second.  
 17) Find the time  $t$  when the ball hits the ground. If needed, round to two decimal places.  
 18) Find the instantaneous velocity of the ball at the instant that it hits the ground, to one decimal place.

**Bonus #1: Factor your answer from #5 by using the GCF.**

**Bonus #2: Factor your answer from #6 by using the GCF.**

**Bonus #3: Factor your answer from #7 by using the GCF.**

**11.6 Worksheet Answers:**

- 1)  $y' = -294x^6 - 180x^5 + 60x^4$       2)  $g'(x) = 11 - 110x^4$       3)  $y' = -30x + 7$   
 4)  $f'(x) = 18x^2 + 4x - 20$       5)  $y' = 24x^2 - 2 + 45x^{-2}$       6)  $y' = 30x^{\frac{3}{2}} + 15x^{\frac{1}{2}}$   
 7)  $h'(x) = \frac{20}{3}x^{-\frac{2}{3}} - 65x^{\frac{10}{3}} + 80x^{\frac{1}{3}}$       8)  $y' = -15x^{-2} - 24x^{-5} - 150x^4 - 24x$   
 9)  $-26$       10)  $\frac{1}{4}$       11)  $-\frac{54}{5}$       12)  $y = -\frac{54}{5}(x - 5) + 56$   
 13)  $y = -26(x - 6) + 10$       14)  $y = \frac{1}{4}(x + 4) + \frac{3}{2}$       15) B  
 16)  $-29\frac{ft}{s}$       17) 3.02 seconds      18)  $-61.5\frac{ft}{s}$

## 11.7 Worksheet: Do all work on your own paper.

Calculator allowed.

For #1 – 9, find the derivative of each expression. Keep the denominator in factored form (do not expand.) Use any method of your choice.

1)  $y = \frac{3}{2x-1}$

2)  $f(x) = \frac{5x+2}{4x+3}$

3)  $h(x) = \frac{15x^4+3x^2+6x}{3x}$

4)  $g(x) = \frac{x^2-9}{x-3}$

5)  $y = \frac{-6x+3}{x^2+1}$

6)  $y = \frac{2\sqrt{x}-4x}{3x+2}$

7)  $y = \frac{5x^3+4x^4-2x}{x+6}$

8)  $f(x) = \frac{-4}{5x+3}$

9)  $h(x) = \frac{x^5-30x}{6x^2}$

For #10 – 14, use the table as shown to find each value, if  $h(x) = \frac{f(x)}{g(x)}$ .

$x$	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
6	-5	3	-2	4
-4	$\frac{1}{4}$	0	6	1
5	8	-10	7	-1

10)  $h'(6)$

11)  $h'(-4)$

12)  $h'(5)$

13) Write the equation of the tangent line, in  $(h, k)$  form, to  $h(x)$  at the point  $(6, \frac{5}{2})$ .

14) Write the equation of the tangent line, in  $(h, k)$  form, to  $h(x)$  at the point  $(5, \frac{8}{7})$ .

For #15 – 17, use  $g(x) = \begin{cases} \frac{x^2-7x+6}{x-6} & \text{if } x \neq 6 \\ 2 & \text{if } x = 6 \end{cases}$

15) Is  $g(x)$  continuous at  $x = 6$ ? Justify your conclusion with the definition of continuous.

16) Find  $\lim_{x \rightarrow 6} g(x)$ , if possible.

17) Sketch the graph of  $g(x)$ .

## 11.7 Worksheet Answers:

1)  $y' = \frac{-6}{(2x-1)^2}$

2)  $f'(x) = \frac{7}{(4x+3)^2}$

3)  $h'(x) = 15x^2 + 1$

4)  $y' = 1$

5)  $y' = \frac{6x^2-6x-6}{(x^2+1)^2}$

6)  $y' = \frac{-3\sqrt{x} + \frac{2}{\sqrt{x}} - 8}{(3x+2)^2}$  or  $\frac{-3x+2-8\sqrt{x}}{\sqrt{x}(3x+2)^2}$

7)  $y' = \frac{12x^4+106x^3+90x^2-12}{(x+6)^2}$

8)  $f'(x) = \frac{20}{(5x+3)^2}$

9)  $h'(x) = \frac{1}{2}x^2 + 5x^{-2}$  or  $\frac{18x^6+180x^2}{36x^4}$  or  $\frac{x^4+10}{2x^2}$

10)  $h'(6) = \frac{7}{2}$

11)  $h'(-4) = -\frac{1}{144}$

12)  $h'(5) = -\frac{62}{49}$

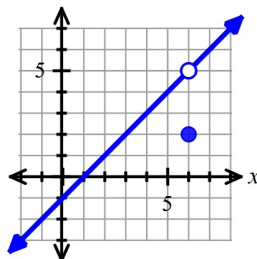
13)  $y = \frac{7}{2}(x-6) + \frac{5}{2}$

14)  $y = -\frac{62}{49}(x-5) + \frac{8}{7}$

15) no;  $\lim_{x \rightarrow 6^-} g(x) = \lim_{x \rightarrow 6^+} g(x) \neq g(6)$

16) 5

17)





## 11.8 Worksheet: Do all work on your own paper.

Calculator allowed.

For #1 – 8, find the derivative of each expression. Use any method of your choice.

1)  $y = 2(4x - 1)^3$

2)  $f(x) = -7(6 - x)^5$

3)  $g(x) = 6(3x^2 - 4)^2$

4)  $h(x) = -(5x^3 - 2x)^4$

5)  $y = \sqrt{8 - 2x^3}$

6)  $y = -2(4\sqrt{x} + 6)^3$

7)  $f(x) = 2(7x - x^6)^{-3}$

8)  $g(x) = \sqrt[3]{9x^2 - 5}$

For #9 – 12, find the requested value by using the table shown.

9)  $\frac{d}{dx} [f(g(-1))]$

10)  $\frac{d}{dx} [g(f(4))]$

11)  $\frac{d}{dx} [f(g(6))]$

12)  $\frac{d}{dx} [g(f(11))]$

$x$	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
4	6	3	5	-3
-1	8	4	13	6
6	1	11	14	-9
11	-1	7	10	-2

13) Given that  $y = \sqrt{(x - 3)(2x + 7)}$ , write the equation of the tangent line, in  $(h, k)$  form, to  $y$  at  $(5, \sqrt{34})$ .14) Given that  $f(x) = -2x + 5$ , select all of the following options that correctly represent  $f'(x)$ .

A)  $\lim_{h \rightarrow 0} \frac{-2(x+h)+5-2x+5}{h}$

B)  $\lim_{h \rightarrow 0} \frac{-2(x+h)+5+2x-5}{h}$

C)  $\lim_{h \rightarrow 0} \frac{-2x+h+5+2x-5}{h}$

D)  $-2$

For #15 – 23, use the graph of  $f(x)$  shown to find the requested values.

15)  $f(2)$

16)  $\lim_{x \rightarrow -3} f(x)$

17)  $f(-1)$

18)  $\lim_{x \rightarrow -1} f(x)$

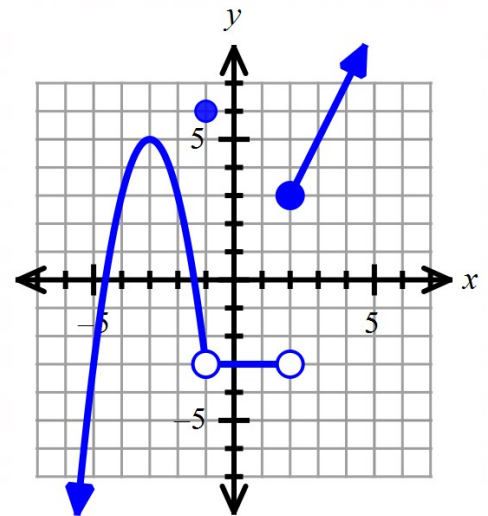
19)  $\lim_{x \rightarrow 2^-} f(x)$

20)  $\lim_{x \rightarrow 2^+} f(x)$

21)  $\lim_{x \rightarrow 2} f(x)$

22)  $f(-4)$

23)  $\lim_{x \rightarrow -4} f(x)$



## 11.8 Worksheet Answers

1)  $y' = 24(4x - 1)^2$

2)  $f'(x) = 35(6 - x)^4$

3)  $g'(x) = 72x(3x^2 - 4)$

4)  $h'(x) = -4(5x^3 - 2x)^3(15x^2 - 2)$

5)  $y' = -3x^2(8 - 2x^3)^{-\frac{1}{2}}$  or  $y' = -\frac{3x^2}{\sqrt{8-2x^3}}$

6)  $y' = -12x^{-\frac{1}{2}}(4\sqrt{x} + 6)^2$

7)  $f'(x) = -6(7x - x^6)^{-4}(7 - 6x^5)$  or  $\frac{-6(7-6x^5)}{(7x-x^6)^4}$

8)  $g'(x) = 6x(9x^2 - 5)^{-\frac{2}{3}}$

9) 30

10) -45

11) -90

12) 60

13)  $y = \frac{21}{2\sqrt{34}}(x - 5) + \sqrt{34}$  or  $y = \frac{21\sqrt{34}}{68}(x - 5) + \sqrt{34}$

14) B, D

15) 3

16) 5

17) 6

18) -3

19) -3

20) 3

21) does not exist

22) 3

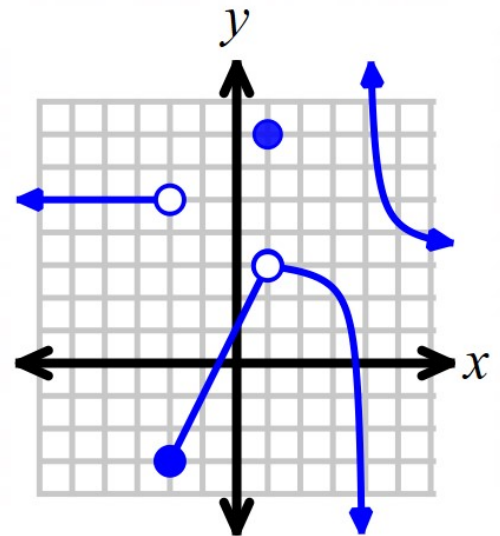
23) 3

## Ch 11 Review Worksheet: Do all work on your own paper.

Calculator allowed.

For #1 – 10: Use the graph of  $h(x)$  below to find the requested values, if possible. #1 – 9: Multiple Choice.

- 1)  $h(-2)$   
A) 5      B) -3      C) -2      D) DNE
- 2)  $\lim_{x \rightarrow -2} h(x)$   
A) 5      B) -3      C) -2      D) DNE
- 3)  $h(1)$   
A) 7      B) 1      C) 3      D) DNE
- 4)  $\lim_{x \rightarrow 1} h(x)$   
A) 7      B) 1      C) 3      D) DNE
- 5)  $\lim_{x \rightarrow 1^+} h(x)$   
A) 7      B) 1      C) 3      D) DNE
- 6)  $\lim_{x \rightarrow 1^-} h(x)$   
A) 7      B) 1      C) 3      D) DNE
- 7)  $h(4)$   
A) 8      B) 0      C) -1      D) DNE
- 8)  $\lim_{x \rightarrow 4} h(x)$   
A) 8      B) 0      C) -1      D) DNE
- 9)  $\lim_{x \rightarrow -3} h(x)$   
A) 5      B) 0      C) -5      D) DNE

10) Identify any  $x$ -values where  $h(x)$  has a discontinuity but the limit exists.

For #11 – 17, evaluate each limit. Simplify your answer.

- 11)  $\lim_{x \rightarrow 10} \sqrt{5x - 2}$
- 12)  $\lim_{x \rightarrow 2} (-3e^{x-2} - 4)$
- 13)  $\lim_{x \rightarrow 16} \frac{\sqrt{x}-4}{x-16}$
- 14)  $\lim_{x \rightarrow -3} \frac{2x^2+6x}{x^2-9}$
- 15)  $\lim_{x \rightarrow 0} \frac{\sqrt{x+64}-8}{x}$
- 16)  $\lim_{x \rightarrow -1} \sqrt[3]{x+2}$

For #17 – 21, use  $f(x) = \begin{cases} -4e^{x^2-9} + 5 & \text{if } x < 3 \\ 8x - 20 & \text{if } x = 3 \\ 4 - x & \text{if } x > 3 \end{cases}$

- 17) Find  $\lim_{x \rightarrow 3^-} f(x)$ .
- 18) Find  $\lim_{x \rightarrow 3^+} f(x)$ .
- 19) Find  $\lim_{x \rightarrow 3} f(x)$ .
- 20) Find  $f(-3)$ .
- 21) Is  $f(x)$  continuous at  $x = 3$ ? Use the definition of continuity.

For #22 – 24, find  $f'(x)$  by using the **limit definition** of a derivative. Show your work (do not use power rule except to verify your result.)

22)  $f(x) = 5x - 1$

23)  $f(x) = x^2 + 2x - 1$

24)  $f(x) = -3\sqrt{x}$

For #25 – 26: Given that  $h(x) = -2x^3 + 3x^2 + 4$

25) Find the slope of the tangent line to  $h(x)$  at  $(1, 5)$ .

26) Write the equation of the tangent line to  $h(x)$  in  $(h, k)$  form at  $(1, 5)$ .

For #27 – 37, find the derivative of each function.

27)  $g(x) = -5x^2 + 10x - 3$

28)  $y = 5\sqrt{x} - \frac{1}{x^3} + \pi$

29)  $f(x) = \frac{8x^3 - 4x^2 + 5x}{2x}$

30)  $y = (2x - 3)(5x + 4)$

31)  $h(x) = -3\sqrt{x}(4x^2 + 4x)$

32)  $y = (2x^{-3} + 5x)(x^2 - 4x^{-2})$

33)  $f(x) = \frac{7x-3}{x+4}$

34)  $y = -\frac{5}{x^2+3}$

35)  $g(x) = \frac{x^2+5x}{3x-1}$

36)  $h(x) = -2(3x + 11)^4$

37)  $m(x) = -5\sqrt{6x - 8}$

38)  $y = -5(1 - 8x)^{-3}$

For #38 – 40: Multiple Choice. Use the table shown to find the requested derivative.

39) Find  $h'(-1)$  if  $h(x) = f(x) \cdot g(x)$ .

- A) -12      B) -8      C) 46      D) 50

40) Find  $w'(4)$  if  $w(x) = \frac{f(x)}{g(x)}$ .

- A)  $-\frac{4}{3}$       B) -4      C) -1      D)  $-\frac{5}{3}$

41) Find  $k'(6)$  if  $k(x) = f(g(x))$ .

- A) -12      B) -20      C) -80      D) -160

$x$	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
4	-9	3	5	-3
-1	8	-1	-2	6
6	1	-5	4	-4
-5	-10	7	20	-8

**Answers:**

- 1) B      2) D      3) A      4) C      5) C      6) C      7) D  
 8) D      9) A      10)  $x = 1$  (hole)      11)  $4\sqrt{3}$       12) -7      13)  $\frac{1}{8}$   
 14) 1      15)  $\frac{1}{16}$       16) -2      17) 1      18) 1      19) 1      20) -44  
 21) no;  $\lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^+} f(x) \neq f(3)$       22) 5      23)  $2x + 2$       24)  $\frac{-3\sqrt{x}}{2x}$  or  $\frac{-3}{2\sqrt{x}}$   
 25) -36      26)  $y = -36(x + 2) + 32$       27)  $g'(x) = -10x + 10$   
 28)  $y' = \frac{5}{2}x^{-1/2} + 3x^{-4}$       29)  $f'(x) = 8x - 2$       30)  $y' = 20x - 7$   
 31)  $h'(x) = -30x^{3/2} - 18x^{1/2}$       32)  $y' = 15x^2 + 18x^{-2} + 40x^{-6}$       33)  $f'(x) = \frac{31}{(x+4)^2}$   
 34)  $y' = 10x(x^2 + 3)^{-2}$       35)  $g'(x) = \frac{3x^2 - 2x - 5}{(3x-1)^2}$       36)  $h'(x) = -24(3x + 11)^3$   
 37)  $m'(x) = -15(6x - 8)^{-1/2}$       38)  $y' = -120(1 - 8x)^{-4}$       39) D  
 40) A      41) C