Chapter 3 Calendar **Name:** Key

Day	Date	Assignment (Due the next class meeting)
		3.1 Worksheet Adding/subtracting & Multiplying polynomial functions
		3.2 Worksheet Factoring polynomials
		3.3 Worksheet Polynomial division, remainder and factor theorems
		~~3.4 Worksheet Rational Roots Theorem~~ Names changed
		~~3.5 Worksheet Graphing polynomial functions~~ ~~day 1~~
		Chapter 3 Practice Test
		Ch 3 TEST 1st Semester Review Packet

- Be prepared for daily quizzes.
- Every student is expected to do every assignment for the entire unit.
- Students who complete *every assignment* for this semester are eligible for a 2% semester grade bonus and a pizza lunch paid by the math department.
- Try www.khanacademy.org or www.mathguy.us (Earl's website) if you need help.

3.1 Notes: Adding, Subtracting, & Multiplying Polynomials

Work with a partner to perform the indicated operation. Be prepared to share with the class.

1) Add: $2x^3 - 5x^2 + 3x - 9$ and $x^3 + 6x^2 + 11$

$3x^3 + x^2 + 3x + 2$

2) $(5x^5 - 3x^4 + 2x) + (-x^5 + 3x^4 - x)$

$4x^5 + 2x - x$

3) Subtract: $5z^2 - z + 3$ from $4z^2 + 9z - 12$

$4z^2 + 9z - 12 - (5z^2 - z + 3)$

$4z^2 + 9z - 12 - 5z^2 + z - 3$

$-z^2 + 10z - 15$

4) $-(t^2-6t+2)-(5t^2-t-8)$

$-t^2+6t-2-5t^2+t+8$

$-6t^2+7t+6$

5) $-2+3(t^2-6t+2)+4(5t^2-t-8)-(6t+1)$

$-2+3t^2-18t+6+20t^2-4t-32-6t-1$

$23t^2-28t-29$

6) $20-\frac{1}{2}(-6t^2+8t-4)-3(t^2-6t+5)+(t-3)$

$20+3t^2-4t+2-3t^2+18t-15+t-3$

$15t+4$

7) According to data from the U.S. Census Bureau for the period 2000-2007, the number of male students enrolled in high school in the United States can be approximated by the function $M(x)=-0.004x^3+0.037x^2+.049x+8.11$ where x is the number of years since 2000 and $M(x)$ is the number of male students in the millions. The number of female students enrolled in high school in the United States can be approximated by the function $F(x)=-0.006x^3+0.029x^2+0.165x+7.67$ where x is the number of years since 2000 and $F(x)$ is the number of female students in millions. Estimate the total number of students enrolled in high school in the United States in 2007.

$x=7$

$T(x)=M(x)+F(x)$

$T(x)=-0.01x^3+0.066x^2+0.214x+15.78$

$=-0.01(7)^3+0.066(7)^2+0.214(7)+15.78$

$T(7)=-3.43+3.234+1.498+15.78=\boxed{17.08 \text{ million}}$

Multiplying Polynomials

8) If $f(x)=2x+1$ and $g(x)=x-6$, find $f(x)\cdot g(x)$.

$(2x+1)(x-6)$

$2x^2-12x+x-6$

$\boxed{2x^2-11x-6}$

9) If $h(x) = x^2 - 5$ and $g(x) = x - 1$, find $h(x) \cdot g(x)$.

$(x^2-5)(x-1)$

$\boxed{x^3 - x^2 - 5x + 5}$

10) Multiply: $-2y^2 + 3y - 6$ and $y - 2$

$(-2y^2+3y-6)(y-2)$

$-2y^3+4y^2+3y^2-6y-6y+12$

$\boxed{-2y^3+7y^2-12y+12}$

11) $-(2x - 3y)(x^2 + 3y - 2)$

$-(2x^3+6xy-4x-3x^2y-9y^2+6y)$

$\boxed{-2x^3-6xy+4x+3x^2y+9y^2-6y}$

12) $(x^2 + 3x)(3x^2 - 2x + 4)$

$3x^4-2x^3+4x^2+9x^3-6x^2+12x$

$3x^4+7x^3-2x^2+12x$

13) $-(x - 5)(x + 2)(3x - 1)$

$-(x^2-3x-10)(3x-1)$

$-(3x^3-x^2-9x^2+3x-30x+10)$

$-(3x^3-10x^2-27x+10)$

$\boxed{-3x^3+10x^2+27x-10}$

14) What is the degree of the function, $f(x) = (x^2 + 4x - 3)(3x^5 + 6x^3)$?

$3x^7$

$\boxed{7^{th} \text{ degree}}$

15) Write the area of a triangle if its height is $2x - 3$ and its base is $5x^2 + 1$.

$A = \frac{1}{2}(5x^2+1)(2x-3)$

$= \frac{1}{2}(10x^3-15x^2+2x-3)$

$\boxed{A = 5x^3 - \frac{15}{2}x^2 + x - \frac{3}{2}}$

16) You want to build a raised rectangular garden bed with a certain height h. You want the width to be the height plus 10 feet, and the length to be 5 times the height. Write a polynomial to describe the volume of the garden bed, in feet.

$h = h$

$w = h+10$

$L = 5h$

$V = 5h(h+10)(h)$

$V = 5h^2(h+10)$

$\boxed{V = 5h^3 + 50h^2}$

b.) From -2 to 1

$x=-2$ $\quad x=1$

$y=4$ $\quad y=1$

$m = \frac{1-4}{1-(-2)} = \frac{-3}{3} = -1$

Average Rate of Change: slope, plug in x to find y.

17.) Find the average rate of change for $f(x)=x^2$ a.) from 0 to 2

$x=0$ $\quad x=2$

$y=0$ $\quad y=4$

$m = \frac{4-0}{2-0} = \frac{4}{2} = 2$

3.2 Notes: Factoring Polynomials

Last term mult. ↗ + has +,+ or -,- ↘ - has +,-

In boxes 1 – 4, Factor:

1) Greatest Common Factor (GCF) $3x + 6$	2) Difference of Perfect Squares $x^2 - 9$	3) Trinomials $x^2 - x - 2$
$3(x+2)$	$(x+3)(x-3)$	$(x-2)(x+1)$
4) Trinomials with coefficient $5x^2 - 7x - 6$	**5) Grouping** $(x^3 + 6x^2)(-3x - 18)$	**6) Sum/Difference of Cubes** $x^3 + 27$
$(5x+3)(x-2)$	$x^2(x+6) - 3(x+6)$ $(x^2-3)(x+6)$	$x^3 + 3^3$ $(x+3)(x^2-3x+9)$

Not a Perfect square

Sum of two cubes

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

Difference of two cubes

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Look at these two formulas and describe the similarities between them.

- a/b in the same locations
- Differen + - + vs. - + +

What are the differences?

Factor the polynomials completely: ****Remember to find the GCF first*****

1) $x^3 + 64$

$x^3 + 4^3$

$(x+4)(x^2-4x+16)$

2) $x^3 - 8$

$x^3 - 2^3$

$(x-2)(x^2+2x+4)$

3) $27x^3 - 125$

$(3x)^3 - 5^3$ $(3x)^2$

$(3x-5)(9x^2+15x+25)$

4) $-2d^5 - 250d^2$

$-2d^2(d^3+125)$

$-2d^3(d^3+5^3)$

$-2d^3(d+5)(d^2-5d+25)$

5) $16b^6 - 686b^3$

$2b^3(8b^3-343)$

$2b^3((2b)^3-7^3)$

$2b^3(2b-7)(4b^2+14b+49)$

For some polynomials, you can factor by grouping pairs of terms that have a common monomial factor. * minamum GCF is +1

$$ra + rb + sa + sb = r(a + b) + s(a + b) \qquad \text{*or use the box method}$$
$$= (r+s)(a+b)$$

Factor the polynomials completely:

6) $(x^3 - 3x^2)(-16x + 48)$

$x^2(x-3) - 16(x-3)$

$(x^2-16)(x-3)$

$(x+4)(x-4)(x-3)$

7) $(8t^2 + 28ts)(-6ts - 21s^2)$

$4t(2t+7s) - 3s(2t+7s)$

$(2t+7s)(4t-3s)$

8) $(2x^3 - 6x^2)(+x - 3)$

$2x^2(x-3) + 1(x-3)$

$(2x^2+1)(x-3)$

*Go back to the chart and complete the examples in boxes 5 and 6.

3.3 Notes: Composition of Functions

Work with a partner to perform the indicated operations given $f(x) = 2x^2 - x + 11$, $g(x) = 7x - 9$, and $h(x) = -x^2 + 6$.

a) $f(x) + h(x)$

$2x^2 - x + 11 + -x^2 + 6$

$\boxed{x^2 - x + 17}$

c) $g(x) - f(x)$

$7x - 9 - (2x^2 - x + 11)$

$7x - 9 - 2x^2 + x - 11$

$-2x^2 + 8x - 20$

b) $g(x) \cdot h(x)$

$(7x-9)(-x^2+6)$

$-7x^3 + 9x^2 + 42x - 54$

Functional Notation:

$f(x) = \text{rule}$

$f(-2)$: means "What is y when $x = -2$?"

$f(-2) = 6$: means $y = 6$ when $x = -2$.

What does the word "composition" mean?

"Put together"

Use the following worked-out examples as a model to help you with the following examples:

Example: Find $f(3)$ if $f(x) = 8x - 1$.

$= 8(3) - 1$

$= 24 - 1$

so $f(3) = 23$

Try the following problems with a partner:

1) Find $g(-2)$ if $g(x) = -7x + 9$.

$-7(-2)+9$

$14+9$

23

2) Find $h(5)$ if $h(x) = -2x^2 + 23$.

$-2(5)^2+23$

$-2(25)+23$

$-50+23$

-27

3) Find $d(-8)$ if $d(x) = \frac{3}{4}x + 6$.

$\frac{3}{4}(-8)+6$

$-6+6$

0

Example: Solve for x if $f(x) = 5x - 3$ and $f(x) = 17$.

$17 = 5x - 3$

$20 = 5x$

so $4 = x$

Try the following problems with a partner:

1) Solve for x if $g(x) = 2x + 9$ and $g(x) = -33$

$2x+9 = -33$

$2x = -42$

$x = -21$

2) Solve for x if $h(x) = -2x - 3$ and $h(x) = 6$

$-2x-3 = 6$

$-2x = 9$

$x = -9/2$

3) Solve for x if $d(x) = x^2 + 7$ and $d(x) = 32$

$x^2+7 = 32$

$x^2 = 25$

$x = \pm 5$

More examples: If $f(x) = 6x + 18$, $g(x) = 9x - 1$, and $h(x) = -3x + 7$, then find the following compositions.

4) Find $f(g(x))$.

$6(9x-1)+18$

$54x-6+18$

$54x+12$

5) Find $g(f(x))$.

$9(6x+18)-1$

$54x+162-1$

$54x+161$

6) Find $h(g(x))$.

$-3(9x-1)+7$

$-27x+3+7$

$-27x+10$

7) Find $h(h(x))$.

$-3(-3x+7)+7$

$9x-21+7$

$9x-14$

8) Find $f(g(-6))$.

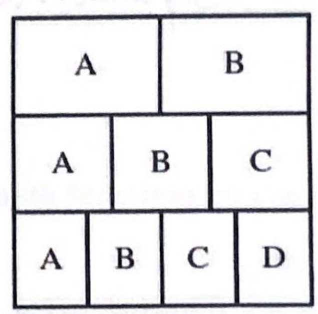
Your notes on composition of functions:

oops

$g(-6) = 9(-6)-1 = -54-1 = -55$

$f(g(-6)) = 6(-55)+18 = -330+18 = \boxed{-312}$

Example 9: The square below is divided into 3 rows of equal area. In the top row, the region labeled A has the same area as the region labeled B. In the middle row, the 3 regions have equal areas. In the bottom row, the 4 regions have equal areas. What fraction of the square's area is in a region labeled A?



row 1 $\frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}$

row 2 $\frac{1}{3} \cdot \frac{1}{3} = \frac{1}{9}$

row 3 $\frac{1}{4} \cdot \frac{1}{3} = \frac{1}{12}$

$\frac{1}{6} + \frac{1}{9} + \frac{1}{12}$

$\frac{6}{36} + \frac{4}{36} + \frac{3}{36}$

$\boxed{= \frac{13}{36}}$

3.4 Notes: Inverses of Linear and Quadratic Functions

What do you think the word "inverse" means in math? Hint: What is the inverse of adding? What is the inverse of dividing?

·Opposite or undo

Exploration: Complete the following input/output tables for the given linear functions. What do you notice?

Function A: $f(x) = 3x - 6$

x	$f(x)$	
3	3	$3(3)-6$
2	0	$3(2)-6$
1	-3	$3(1)-6$
0	-6	$3(0)-6$

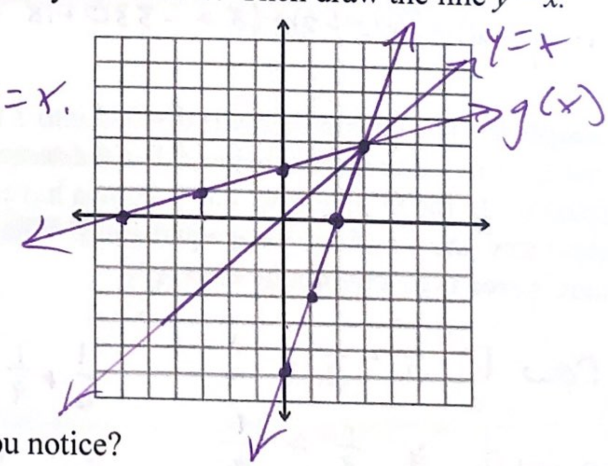
Function B: $g(x) = \frac{1}{3}x + 2$

x	$g(x)$	
3	3	$\frac{1}{3}(3)+2$
0	2	$\frac{1}{3}(0)+2$
-3	1	$\frac{1}{3}(-3)+2$
-6	0	$\frac{1}{3}(-6)+2$

(x, y) coordinates switch

Now graph each function on the coordinate system below. Then draw the line $y = x$. What do you notice?

Symmetrical about $y=x$.



Now find $f(g(x))$ and $g(f(x))$. What do you notice?

$$f(g(x)) = 3\left(\tfrac{1}{3}x+2\right)-6$$
$$= x+6-6$$
$$= x$$

$$g(f(x)) = \tfrac{1}{3}(3x-6)+2$$
$$= x-2+2$$
$$= x$$

Both $= x$.

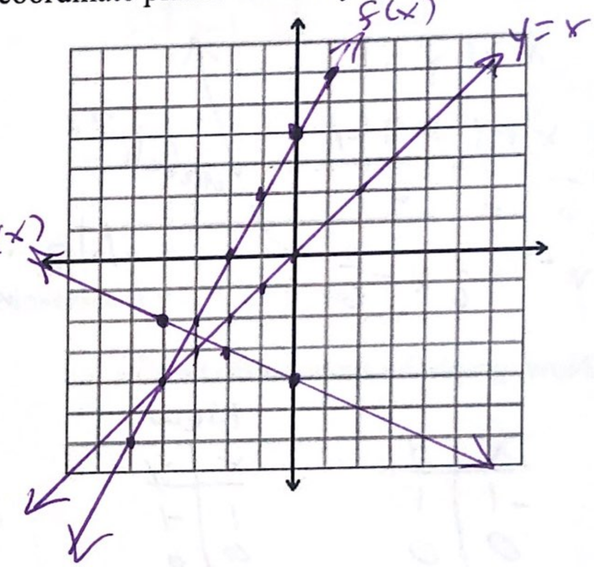
Properties of Inverse Functions:

1.) (x, y) coordinates switch

2.) Both lines are symmetrical about y = x

3.) $f(g(x)) = g(f(x)) = x$

Example 1: Graph the following lines on the same coordinate plane. Are they inverse functions? How do you know?



$y = 2x + 4$ and $y = -\frac{1}{2}x - 4$

f(x) g(x)

No, Not sym. about y=x

How to find the inverse (y^{-1}) of a function:

1.) switch the x & y

2.) solve for y.

3.) put y^{-1} to indicate inverse

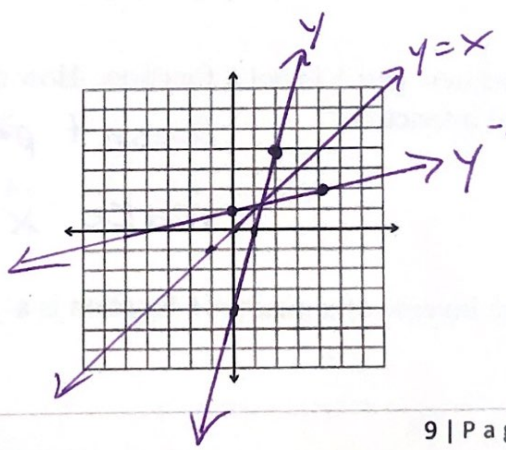
Examples: Find the inverse of each linear function. Then graph both functions on the same coordinate plane.

2) $y = 4x - 4$

$x = 4y - 4$

$\frac{x+4}{4} = \frac{4y}{4}$

$y^{-1} = \frac{1}{4}x + 1$

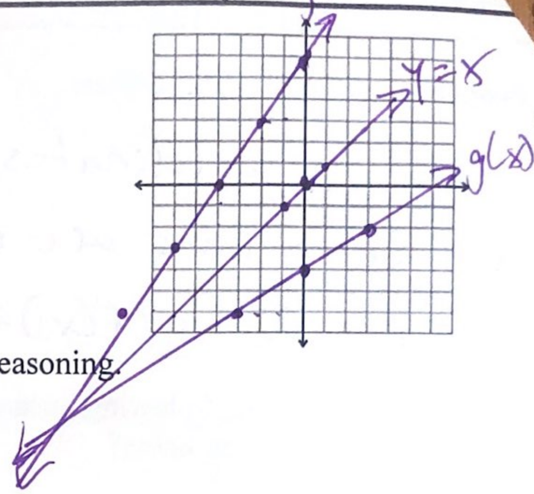


3) $g(x) = \frac{2}{3}x - 4$

$y^{-1} = \frac{3}{2}x + 6$

$x = \frac{2}{3}y - 4$

$(x + 4 = \frac{2}{3}y) \cdot \frac{3}{2}$



4) Are the following functions inverses? Explain your reasoning.
$h(x) = 6x - 1$ and $k(x) = 6x + 1$

$x = 6y - 1$

$\frac{x+1}{6} = \frac{6y}{6}$

$y^{-1} = \frac{1}{6}x + \frac{1}{6}$

match?

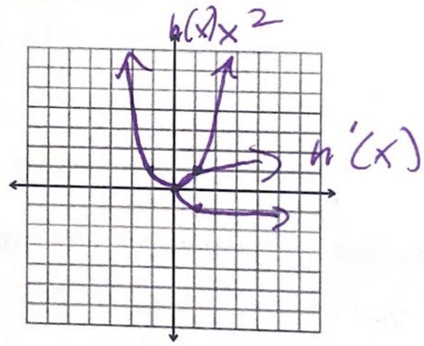
No, $\therefore$ h(x) & k(x) are not inverses

Now, graph the function $h(x) = x^2$

x	y
-1	1
0	0
1	1

New

x	y
1	-1
0	0
1	1



Switch the input and output values from the graph above and graph the new ordered pairs on the same grid. What did you just graph?

The inverse

This new graph is not a function. How can you limit the domain of *h(x)* so that the inverse is also a function?

Doesn't pass vertical line test.

$\therefore$ Make $x \geq 0$

The inverse of a quadratic function is a square root function.

5) Find the inverse of $f(x) = 4x^2$ if $x \geq 0$. Then graph both functions on the same grid.

$f(x)$

x	y
0	0
1	4

$f^{-1}(x)$

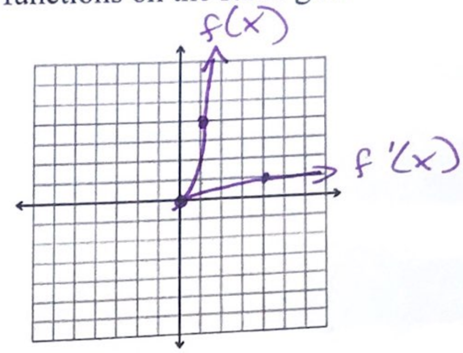
x	y
0	0
4	1

$x = 4y^2$

$\frac{x}{4} = \frac{4y^2}{4}$

$\sqrt{\frac{x}{4}} = \sqrt{y^2}$

$y^{-1} = \frac{\sqrt{x}}{2}$



6) Find the inverse of $y = x^2 - 8$ if $x \geq 0$.

$x = y^2 - 8$

$x + 8 = y^2$

$y^{-1} = \sqrt{x+8}$

7) Determine if $f(x) = x^2 + 3$ and $g(x) = \sqrt{x-3}$ are inverses if $x \geq 0$. Explain.

Yes, $\therefore$ f(x) & g(x) are inverses. matches?

$x = \sqrt{y-3}$

$x^2 = y - 3$

$y^{-1} = x^2 + 3$

8) Write a function.

$y = x + 7$

Now, find its inverse.

$x = y + 7$

$y^{-1} = x - 7$

Finally, is the inverse a function? Explain your reasoning.

Yes, it passes the vertical line test.