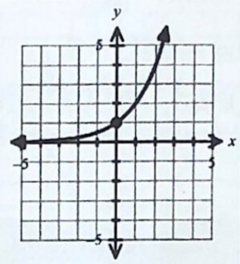
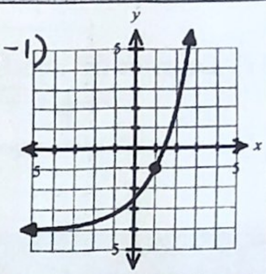
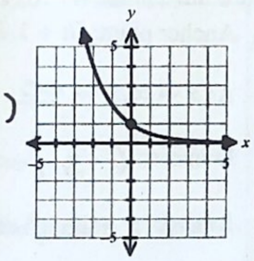
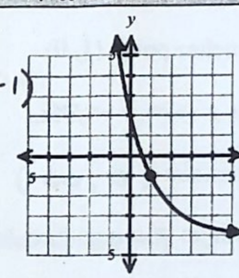
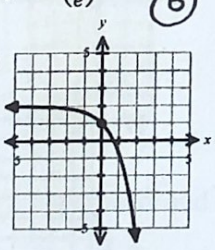
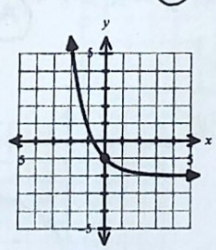
3.1 Notes: Graphs of Exponential and Logarithmic Functions

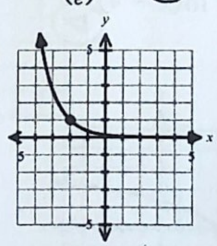
Parent Functions for Exponential Functions		
	$y = b^x$ where $b > 1$	$y = a \cdot b^{x-h} + k$
Exponential Growth Functions	HA: $y = 0$ Anchor point: $(0, 1)$ As $x \to \infty, y \to$ ∞ Domain: $(-\infty, \infty)$ Range: $(0, \infty)$ *A table of values can also be used.  Sample: $y = 2^x$	HA: $y = k$ $y = -4$ Anchor point: $(h, a + k)$ $(1, -1)$ As $x \to \infty, y \to$ ∞ Domain: $(-\infty, \infty)$ Range: $(-4, \infty)$  Sample: $y = 3 \cdot 2^{x-1} - 4$
	$y = b^x$ where $0 < b < 1$	$y = a \cdot b^{x-h} + k$
Exponential Decay Functions	HA: $y = 0$ Anchor point: $(0, 1)$ As $x \to \infty, y \to$ 0 Domain: $(-\infty, \infty)$ Range: $(0, \infty)$ *A table of values can also be used.  Sample: $y = \left(\frac{1}{2}\right)^x$	HA: $y = k$ $y = -4$ Anchor point: $(h, a + k)$ $(1, -1)$ As $x \to \infty, y \to$ -4 Domain: $(-\infty, \infty)$ Range: $(-4, \infty)$  Sample: $y = 3 \cdot \left(\frac{1}{2}\right)^{x-1} - 4$

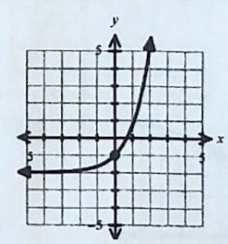
Examples 1 – 8: Match each equation with its graph below. No graphing calculators allowed.

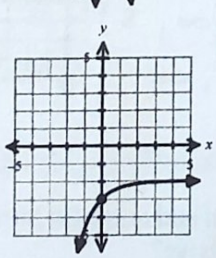
1) $y = e^x - 2$ D
2) $y = e^{x-2}$ F
3) $y = -e^x + 2$ A
4) $y = -e^{x+2}$ G
5) $y = \left(\frac{1}{e}\right)^x - 2$ B
6) $y = e^{-x} + 2$ H
7) $y = \left(\frac{1}{e}\right)^{x+2}$ C
8) $y = -e^{-x} - 2$ E

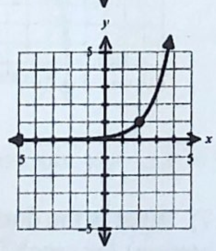
A) 

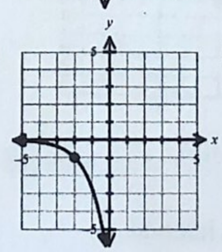
B) 

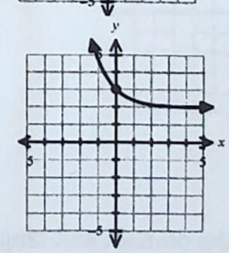
C) 

D) 

E) 

F) 

G) 

H) 

9) What is the domain and range (in interval notation) for graph E above?

D: $(-\infty, \infty)$
R: $(-\infty, -2)$

10) What is the domain and range (in interval notation) for graph B above?

D: $(-\infty, \infty)$
R: $(-2, \infty)$

Transformations from $y = e^x$ to $y = a \cdot b^{x-h} + k$

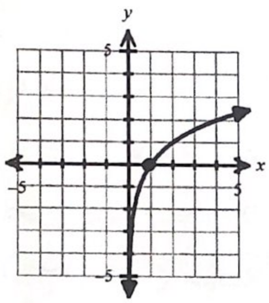
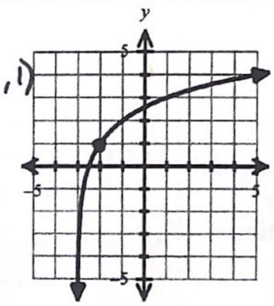
$a < 0$	$\lvert a \rvert > 1$	h	k
vert	vert	$\leftrightarrow$	$\updownarrow$
reflect	stretch	opp.	

11) Describe the transformations from $y = e^x$ to $y = -3e^{x+1} + 5$.

vert. reflected, vert. stretched by 3

$\leftarrow 1$ $\uparrow 5$

Parent Functions for Logarithmic Functions

<table>
<tr><th>$y = \log_b x$ where $b > 1$</th><th>$y = \log_b(x - h) + k$ where $b > 1$</th></tr>
<tr><td>VA: $x = 0$
Anchor point: (1, 0)
As $x \to \infty, y \to$ ∞
Domain: $(0, \infty)$
Range: $(-\infty, \infty)$

*A table of values can also be used.
Sample: $y = \log_2 x$</td><td>VA: $y = h$ $x = -3$
Anchor point: $(h + 1, k)$ $(-2, 1)$
As $x \to \infty, y \to$ ∞
Domain: $(-3, \infty)$
Range: $(-\infty, \infty)$

*A table of values can also be used.
Sample: $y = \log_2(x + 3) + 1$</td></tr>
</table>

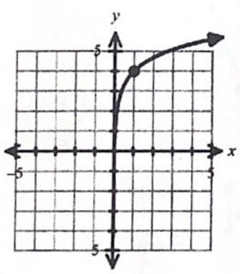
Examples 12 – 15: Match each equation with its graph without using a graphing calculator.

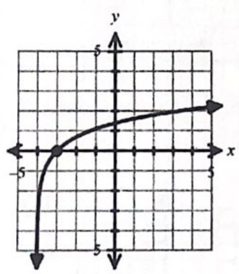
12) $y = \ln(x + 4)$ B

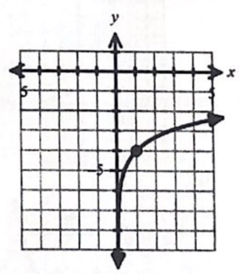
13) $y = \ln(x - 4)$ D

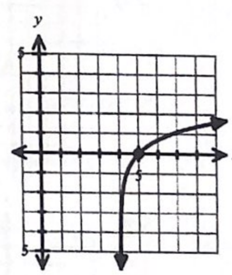
14) $y = \ln x + 4$ A

15) $y = \ln x - 4$ C

A) 

B) 

C) 

D) 

16) What is the domain and range (in interval notation) for graph B above?

D: $(-4, \infty)$

R: $(-\infty, \infty)$

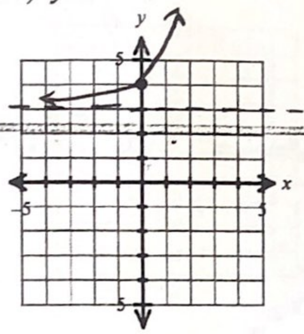
17) What is the domain and range (in interval notation) for graph C above?

D: $(0, \infty)$

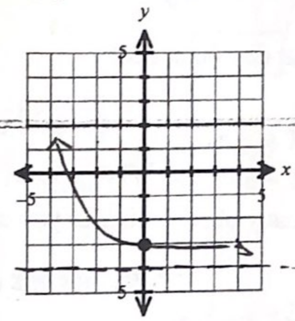
R: $(-\infty, \infty)$

For #18 – 20: Sketch each exponential function without a calculator. Include the HA and anchor point.

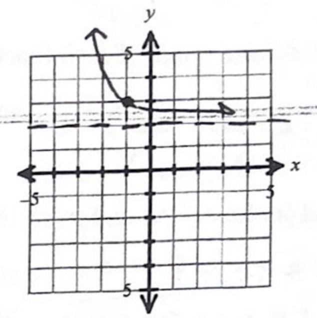
18) $y = e^x + 3$



19) $y = e^{-x} - 4$



20) $y = -e^{x+1} + 2$



21) What is the domain and range of #20? What are the transformations from $y = e^x$?

D: $(-\infty, \infty)$

R: $(-2, \infty)$

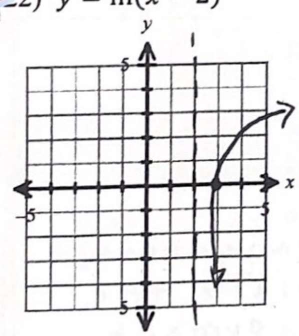
• vert reflection

• ← 1

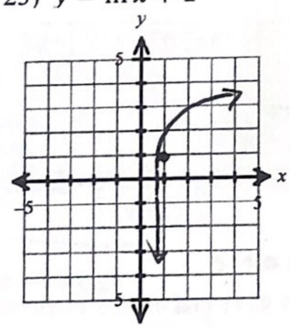
• ↑ 2

For #22 – 24: Sketch each logarithmic function without a calculator. Include the VA and anchor point.

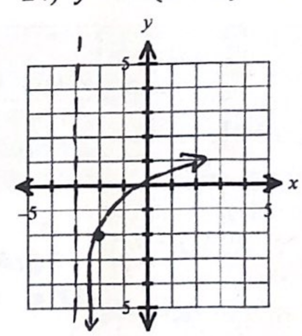
22) $y = \ln(x - 2)$



23) $y = \ln x + 1$



24) $y = \ln(x + 3) - 2$



25) What is the domain and range of #22?

D: $(2, \infty)$

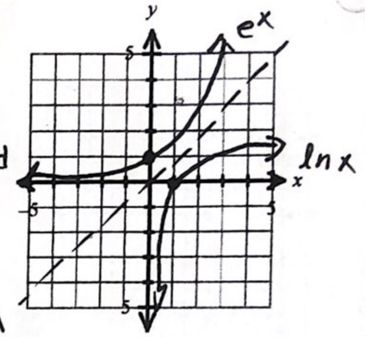
R: $(-\infty, \infty)$

26) What are the transformations from $y = \ln x$ to the equation from #24?

← 3 ↓ 2

3.2 and 3.3 Notes: Properties of Logarithmic and Exponential Expressions

Exploration: Use a graphing calculator to graph $f(x) = \ln x$ and $g(x) = e^x$. Draw a sketch on the provided coordinate systems.



Find the domain and range of each function. What do you notice?

$\left\{\begin{array}{l} e^x \\ D: (-\infty, \infty) \\ R: (0, \infty) \end{array}\right.$ $\left\{\begin{array}{l} \ln x \\ D: (0, \infty) \\ R: (-\infty, \infty) \end{array}\right.$ $\Big\{$ switched

Compare and contrast $f(x)$ and $g(x)$. Make as many observations as you can.

inverses!

VA versus HA

- reflection in $y = x$
- $(0,1) \rightarrow (1,0)$

anchor pts

Logarithmic and Exponential Equations and Expressions	
Logarithmic and Exponential functions are inverses of each other, as long as they use the same base.	
Parts of Logarithmic and Exponential Expressions	$\log_b a = x$ and $b^x = a$. **Base:** b **Argument (of log):** a **Exponent:** x
Converting Equations to Logarithmic or Exponential Form	$\log_b a = x \longrightarrow b^x = a$. logs isolate the exponents; exponentials isolate the argument
Inverse Properties with Logs and Exponentials of the Same Base	$\boxed{\log_b b}^x = x$ and $\boxed{b^{\log_b}}^x = x$ same base $\rightarrow$ inverses!

For #1 – 3: Re-write each equation in logarithmic or exponential form.

1) $\log_x 81 = 4$

$x^4 = 81$

2) $\log a = x$

$10^x = a$

3) $2^3 = x$

$\log_2 x = 3$

For #4 – 13: Simplify each expression without a calculator.

4) $\log_3 81$

$\log_3 3^4$

$\boxed{= 4}$

5) $\log_5 \sqrt[7]{5}$

$\log_5 5^{1/7} = \boxed{\frac{1}{7}}$

6) $\log_{1/4} 256$

$\log_{\frac{1}{4}} \left(\frac{1}{4}\right)^{-4}$ $\boxed{= -4}$

7) $\ln e^5$

$\boxed{5}$

8) $\log \frac{1}{10}$ $\leftarrow 10^0$

$\log_{10} 10^0 = \boxed{0}$

9) $\log_{36} 6$

$\sqrt{36}$

$\log_{36} 36^{\frac{1}{2}} = \boxed{\frac{1}{2}}$

10) $\log_3 27^x$

$\log_3 3^{3x}$

$\boxed{= 3x}$

11) $\log_5 25^{-2x}$

$\log_5 5^{2(-2x)}$

$\boxed{= -4x}$

12) $\ln e^{8-x}$

$\boxed{8 - x}$

13) $\log_3 27 + \ln 1 - \log 10^4 - 5\ln e^4 - \log_2 32 + e^{\ln 6}$

$3 + 0 - 4 - 5 \cdot 4 - 5 + 6$

$-1 - 25 + 6 = \boxed{-20}$

Note: important values you *must know:* $\ln e =$ 1 ; $\ln 1 =$ 0 ; $\ln 0 =$ undef

Compound Interest Formulas	
Compounded *n* times per year	**Compounded continuously**
$A = P\left(1 + \frac{r}{n}\right)^{nt}$ A → final amount; P → principle; r → rate(decimal); n → # times compounded per yr.; t → t in years	$A = Pe^{rt}$ A → final; P → principle (starting amount); r → rate (decimal); t → t = time (yrs)

14) A person wants to invest \$3,000 in a saving account for 4 years. The bank has two options. The first option compounds interest weekly at a rate of 5.4%. The second option compounds interest continuously at a rate of 5%. Which option should you choose? Explain your choice.

$P = 3000 \quad t = 4$

Weekly $n = 52 \quad r = .054$

$A = 3000\left(1 + \frac{.054}{52}\right)^{52 \cdot 4}$

$= \$3,722.89$

$P = 3000 \quad t = 4$

$r = .05$ %

$A = 3000e^{.05 \cdot 4}$

$= \$3664.21$

option 1 → more \$

Properties of Logarithmic Expressions			
Product Property	$\log_b(m \cdot n) = \log_b m + \log_b n$	**Power Property**	$\log_b(m^n) = n \cdot \log_b m$
Quotient Property	$\log_b\left(\frac{m}{n}\right) = \log_b m - \log_b n$	**Change - of - Base Formula**	$\log_b m = \frac{\ln m}{\ln b}$ $\log_b m = \frac{\log_a m}{\log_a b}$
Condensing Log Expressions	1. Move coefficients to powers of the argument. 2. Change addition to multiplication of the arguments. 3. Change subtraction to multiplication of the arguments.	**Expanding Log Expressions**	1. Change multiplication of the argument to addition of two logs of the same base. 2. Change division of the argument to subtraction of two logs of the same base. 3. Move any powers of the argument to coefficients of the log.

Examples 15 – 19: Condense each logarithmic expression.

15) $\ln 4 + 3\ln 3 - \ln 12$

$\ln \frac{4 \cdot 3^3}{12} \rightarrow \boxed{\ln 9}$

16) $\frac{1}{3}[2\ln(x+5) - \ln x - \ln(x^2-4)]$ 2/3

$\frac{1}{3}\left[\ln \frac{(x+5)^2}{x(x-4)}\right] \rightarrow \boxed{\ln\left(\frac{(x+5)}{x^{\frac{1}{3}}(x-4)^{1/3}}\right)}$

17) $\log_2(6x+1) - \frac{1}{2}\log_2 2y + \log_2 24 - \log_2 3z$

$\log_2 \frac{(6x+1)\cdot 24}{\sqrt{2y}\cdot 3z} = \boxed{\log_2\left(\frac{8(6x+1)}{z\sqrt{2y}}\right)}$

18) $-\frac{1}{2}\ln 16 + 2\ln 3$

$\ln \frac{3^2}{\sqrt{16}} \rightarrow \boxed{\ln\left(\frac{9}{4}\right)}$

19) $\ln 5 + \frac{1}{3}\ln(a+2) - 2\ln b - \ln c$

$\boxed{\ln\left(\frac{5(a+2)^{\frac{1}{3}}}{b^2 c}\right)}$

For #20 – 22, expand each logarithmic expression.

20) $\log_7 \frac{3\sqrt[4]{x}}{5y^3}$

$\boxed{\log_7 3 + \frac{1}{4}\log_7 x - \log_7 5 - 3\log_7 y}$

21) $\ln \frac{2x^3 y}{7z^4}$

$\boxed{\ln 2 + 3\ln x + \ln y - \ln 7 - 4\ln z}$

22) $\log \frac{7ab^4}{(c+d)^5}$

$\boxed{\log 7 + \log a + 4\log b - 5\log(c+d)}$

For #23 – 24: Use the change-of-base formula to evaluate each logarithm. Give an exact solution and an approximate solution to 3 decimals.

23) $\log_5 8 = \frac{\ln 8}{\ln 5}$ or $\frac{\log 8}{\log 5}$

$\boxed{\approx 1.292}$

24) $\log_8 14 = \frac{\ln 14}{\ln 8}$ or $\frac{\log 14}{\log 8}$

$\boxed{\approx 1.269}$

3.4 Notes: Solving Exponential and Log Equations

Solving Exponential Equations			
Option 1: **Both sides can be written with the same base.**	1. Write both sides with the same base. 2. Set the exponents equal and solve.	**Option 2:** **Both sides cannot be written with the same base.**	1. If possible, isolate the base and exponent term. a. Use inverse operations or b. Factor the expression 2. Take the ln (or another log) to both sides. 3. Use properties of logs to solve.

For #1 – 8: Solve each exponential equation for the variable. If needed, round to one decimal place.

1) $e^{x+6} = e^{3x}$

$x + 6 = 3x$

$6 = 2x$

$\boxed{3 = x}$

2) $9^{2x} = 27^{x-1}$

$3^{2(2x)} = 3^{3(x-1)}$

$4x = 3x - 3$

$\boxed{x = -3}$

3) $\frac{1}{343} = 7^{2x+5}$

$7^{-3} = 7^{2x+5}$

$-3 = 2x + 5$

$-8 = 2x$

$\boxed{-4 = x}$

4) $2^{x-4} = 5.3$

$\ln 2^{x-4} = \ln 5.3$

$(x-4)\ln 2 = \ln 5.3$

$x - 4 = \frac{\ln 5.3}{\ln 2}$

$x = \frac{\ln 5.3}{\ln 2} + 4 \boxed{\approx 6.4}$

5) $5 \cdot 3^{x-5} + 1 = 21$

$5 \cdot 3^{x-5} = 20$

$\ln 3^{x-5} = \ln 4$

$(x-5)\ln 3 = \ln 4$

$x - 5 = \frac{\ln 4}{\ln 3}$

$x = \frac{\ln 4}{\ln 3} + 5 \boxed{\approx 6.3}$

6) $3^{x-4} = 2^{3x+1}$

$\ln 3^{x-4} = \ln 2^{3x+1}$

$(x-4)\ln 3 = (3x+1)\ln 2$

$x\ln 3 - 4\ln 3 = 3x\ln 2 + \ln 2$

$x\ln 3 - 3x\ln 2 = \ln 2 + 4\ln 3$

$x(\ln 3 - 3\ln 2) = \ln 2 + 4\ln 3$

$x = \frac{\ln 2 + 4\ln 3}{\ln 3 - 3\ln 2} \approx \cancel{3.8}$ -5.2

7) $e^{2x} - 8e^x + 7 = 0$

$(e^x - 7)(e^x - 1) = 0$

$e^x = 7$ | $e^x = 1$

$\ln e^x = \ln 7$ | $\ln e^x = \ln 1$

$x = \ln 7$ or $x \approx 1.9$ | $x = 0$

8) $5e^{2x} - 5e^x = 0$

$5e^x(e^x - 1) = 0$

$5e^x = 0$ | $e^x = 1$

$e^x = 0$ | $x = 0$

$\ln e^x = \ln 0$

undef.

Solving Logarithmic Equations			
Only one logarithmic term	1. Isolate the log term, if possible. 2. Convert the equation to an exponential expression (or exponentialize each side) 3. Solve. Sample: $\ln(5x) + 6 = 8$ $\ln(5x) = 2$ OR $e^{\ln(5x)} = e^2$ $e^2 = 5x$; $5x = e^2$ $\boxed{\frac{e^2}{5} = x}$; $\boxed{x = \frac{e^2}{5}}$	**One log term on each side (with the same base)**	1. Note: this strategy only works if there are no additional terms/constants in the equation. 2. Set the arguments equal and solve. Sample: $\ln(3x - 2) = \ln(4 - x)$ $3x - 2 = 4 - x$ $4x = 6$ $\boxed{x = \frac{3}{2}}$
More than two logarithmic terms on one side.	1. Condense to one log term on each side. 2. Move any powers of the argument to coefficients of the log. 3. Use the row above to find an appropriate strategy to solve. Samples: $\ln x + \ln(x - 2) = \ln 8$ $\ln x(x-2) = \ln 8$ $x^2 - 2x = 8$ $x^2 - 2x - 8 = 0$ $(x-4)(x+2) = 0$ $\boxed{x = 4}$; $\cancel{x = -2}$ $\log_4 x + \log_4(x + 6) = 2$ $\log_4 x(x+6) = 2$ $\log_4(x^2+6x) = 2$ $4^2 = x^2 + 6x$ $x^2 + 6x = 16$ $x^2 + 6x - 16 = 0$ $(x+8)(x-2) = 0$ $\cancel{x = -8}$ $\boxed{x = 2}$		
Extraneous Solutions	All arguments for all log expressions must be positive values. Reject any values that would make the argument a negative value or zero.		

For #9 – 12: Solve each logarithmic equation and check for extraneous solutions

9) $\log_5(4x - 7) = \log_5(x + 5)$

$4x - 7 = x + 5$

$3x = 12$

$x = 4$

verify → arguments both positive ✓

10) $\log_4(5x - 1) = 3$

$4^3 = 5x - 1$

$64 = 5x - 1$

$65 = 5x$

$x = 13$

pos. argue. ✓

11) $\log_4(x + 12) + \log_4 x = 3$

$\log_4 x(x + 12) = 3$

$4^3 = x^2 + 12x$

$0 = x^2 + 12x - 64$

$0 = (x + 16)(x - 4)$

$\cancel{x = -16}$ neg. $x = 4$ ✓

12) $\log x + \log(x + 5) = \log 24$

$\log x(x + 5) = \log 24$

$x^2 + 5x = 24$

$x^2 + 5x - 24 = 0$

$(x + 8)(x - 3) = 0$

$\cancel{x = -8}$ neg. $x = 3$ ✓

More Examples: Solve each problem. Round your final answer to one decimal place.

13) The population of deer in a forest preserve can be modeled by the equation $P(t) = 50 + 200\ln(t+1)$, where t is the time in years from the present. In how many years will be deer population reach 500?

$P = 500$

Find t

$500 = 50 + 200\ln(t+1)$

$450 = 200\ln(t+1)$

$2.25 = \ln(t+1)$

$e^{2.25} = t+1$

$t = e^{2.25} - 1$

$t \approx 8.4877$

$\boxed{\approx 8.5 \text{ years}}$

14) How long would you have to invest \$30,000 in an account earning 6% interest compounded continuously so that you have a total of \$40,000?

Pe^{rt}

P $\quad r = .06$ $\quad A$

find t

When solving equations with logs and exponential functions, if you choose to write your intermediate values (non-final answers) as decimals, **use at least 6 decimal places** for accuracy. Alternatively, write values as fractions.

$A = Pe^{rt}$

$40000 = 30000e^{.06t}$

$1.33333 = e^{.06t}$

$\ln 1.33333 = .06t$

$t = 4.7947$

$\boxed{t \approx 4.8 \text{ years}}$

15) Sally invests \$350 in an account earning 5% interest, compounded annually. How long will it take her to earn \$50 in interest?

P

$r = .05$

$n = 1$

$t = ?$

$A = 350 + 50 = 400$

$A = P\left(1 + \frac{r}{n}\right)^{nt}$

$400 = 350\left(1 + \frac{.05}{1}\right)^{1 \cdot t}$

$1.142857143 = 1.05^{t}$

$\ln 1.142857143 = t\ln 1.05$

$\frac{\ln 1.142857143}{\ln 1.05} = t$

$t \approx 2.737 \approx \boxed{2.7 \text{ years}}$

3.5 Exponential Growth and Decay

Solving Exponential Growth/Decay Equations			
Basic Formula	$A = A_0 e^{kt}$ — time (yrs); constant; Amount; starting amount Growth: $k > 0$ Decay: $k < 0$ Note: Half-life problems are exponential decay problems.	**Hints and Strategies**	1. Write ordered pairs in the form (t, A) ... (time, amount) 2. Use substitution with one ordered to solve for k. Either use an exact answer, or include 6 decimal places. 3. Re-write the customized equation (with the known value for k). 4. Use substitution with another ordered pair to solve the problem.

1) In 1990, the population of Africa was 643 million, and by 2006 it had grown to 906 million.

a) Use the exponential growth model, where t is the number of years after 1990, to find the exponential growth function that models the data.

(time, amount)
$(0, \underbrace{643}_{A_0})$
$(16, 906)$
$(t, 2100)$

$A = A_0 e^{kt}$
$A = 643e^{k \cdot 16}$
$\frac{906}{643} = e^{16k}$
$\ln\left(\frac{906}{643}\right) = \cancel{\ln e}^{16k}$

$\frac{\ln\left(\frac{906}{643}\right)}{16} = k$

$k \approx 0.021431$

b) In which year will Africa's population reach 2100 million (2.1 billion)?

$A = 643e^{0.021431t}$

$\frac{2100}{643} = e^{0.021431t}$

$\ln\left(\frac{2100}{643}\right) = \cancel{\ln e}^{0.021431t}$

$\frac{\ln\left(\frac{2100}{643}\right)}{0.021431} = t$

$t = 55.23$

$1990 + 55 = \boxed{\text{year } 2045}$

2) Strontium-90 is a waste product from nuclear reactors. The half-life of strontium-90 is 28 years, meaning that after 28 years a given amount of the substance will have decayed to half the original amount. Find the exponential decay model for strontium-90.

$(0, A_0)$
$(28, \frac{1}{2}A_0)$

$A = A_0 e^{kt}$
$\frac{1/2 \cancel{A_0}}{\cancel{A_0}} = \frac{\cancel{A_0} e^{k(28)}}{\cancel{A_0}}$
$\frac{1}{2} = e^{28k}$
$\ln\left(\frac{1}{2}\right) = \cancel{\ln e}^{28k}$

$\ln\left(\frac{1}{2}\right) = 28k$

$\frac{\ln\left(\frac{1}{2}\right)}{28} = k$

model: $A = A_0 e^{\left(\frac{\ln \frac{1}{2}}{28}\right)t}$ or $A = A_0 e^{-0.024755t}$

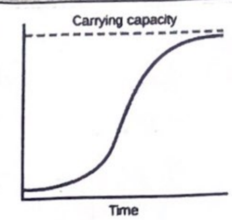
Then suppose that a nuclear accident occurs and releases 60 grams of strontium-90 into the atmosphere. How many years (rounded to one decimal place) will it take for strontium-90 to decay to a level of 10 grams?

$(0, \underbrace{60}_{A_0})$
$(t, \underbrace{10}_{A})$

$A = 60e^{-0.024755t}$
$10 = 60e^{-0.024755t}$
$\ln\left(\frac{1}{6}\right) = \cancel{\ln e}^{-0.024755t}$

$t = \frac{\ln(1/6)}{-0.024755}$

$t \approx 72.38$
$\approx \boxed{72.4 \text{ yrs}}$

Logistic Growth Models			
Formula	$A = \dfrac{c}{1 + ae^{-bt}}$ $a, b,$ and c are constants with $c < 0$ and $b > 0$.	**Graph and Features**	• input is t (often "time") • c is a horizontal asymptote • c is called the "carrying capacity" or "limiting amount," and it is the maximum height (or amount) of the function. 

3) In a learning theory project, psychologists discovered that $f(t) = \frac{0.8}{1+e^{-0.2t}}$ is a model for describing the proportion of correct responses after t learning trials.

# of trials

a) Find the proportion of correct responses prior to learning trials taking place ($t = 0$)

$$f(t) = \frac{0.8}{1+e^{-0.2 \cdot 0}} = 0.4$$

proportion

b) Find the proportion of correct responses after 10 learning trials

$$f(t) = \frac{0.8}{1+e^{-0.2 \cdot 10}} = 0.7046$$

$$\approx 0.7$$

c. What is the limiting size of $f(t)$, the proportion of correct responses, as continued learning trials take place? $c = 0.8 \Rightarrow$ numerator of logistic growth model

carrying capacity (max amount)

Newton's Law of Cooling			
Formula and variables	$T = C + (T_0 - C)e^{-kt}$ $T =$ Final temp $C =$ constant temp (room temp) $T_0 =$ Starting temp of item $k =$ Constant $t =$ Time (often minutes)	**When is this used?**	• CSI investigators and coroners use Newton's Law of Cooling to determine the time of death for a deceased person. • Newton's Law of Cooling is used in business, industry, medicine, etc…

Newton's Law of Cooling: $T = C + (T_0 - C)e^{-kt}$

4) An object is heated to 100°C. It is left to cool in a room that has a temperature of 30°C. After 5 minutes, the temperature of the object is 80°C. $T_0 = 100$ $C = 30$ $T = 5$ $T = 80$

a) Use Newton's Law of Cooling to find a model for the temperature of the object, T, after t minutes.

$80 = 30 + (100-30)e^{-k \cdot 5}$

$50 = 70e^{-5k}$

$\ln\left(\frac{50}{70}\right) = \cancel{\ln e}^{-5k}$

$\ln\left(\frac{50}{70}\right) = -5k$

$k = -5\ln\left(\frac{50}{70}\right)$

$k \approx 0.06729$

$T = 30 + 70e^{-0.06729t}$

$\uparrow$

$(100-30)$

b) What is the temperature of the object after 20 minutes? Round your answer to one decimal place.

$t = 20$ Find T

$T = 30 + 70e^{-0.06729(20)}$

$\boxed{T = 48.2°C}$

c) When will the temperature of the object be 35°C? Round your answer to one decimal place.

$T = 35$ find t

$35 = 30 + 70e^{-0.06729t}$

$5 = 70e^{-0.06729t}$

$\ln\left(\frac{5}{70}\right) = \cancel{\ln e}^{-0.06729t}$

$t = \dfrac{\ln\left(\frac{5}{70}\right)}{-0.06729} \approx 39.2$ min.

5) Rewrite $y = 4(7.8)^x$ in terms of base e. Express the answer in terms of natural logarithm and then round to three decimal places. (Hint: $b^{\log_b x} = x$)

$y = 4(7.8)^x$

$\frac{y}{4} = 7.8^x$

$\ln\left(\frac{y}{4}\right) = x\ln(7.8)$

e e

$\left(\frac{y}{4}\right) = e^{x \cdot \ln(7.8)}$

$y = 4e^{x \cdot \ln 7.8}$ or $y = 4e^{2.054x}$

alt. solution

$y = 4(\underbrace{e^{\ln 7.8}}_{\text{same as } 7.8})^x$

$y = 4e^{x\ln 7.8}$