Essential Understanding: Can you represent linear situations with equations, graphs, and inequalities, and model constraints to optimize solutions?

Teacher Notes, 1.1:

- **15 min:** Go over the syllabus (talk about $3 lab fee, 2% bonus, etc…)
- **10 min:** Pass out the student notes, which has a reading on functional notation.
- **20 min:** Goal: Students will teach each other how to use functional notation and composition of functions.

C day: (Reading and also 1.1: Partner Discovery Worksheet)

Note: only the reading is provided in the student notes. Students must write down the activity on their own paper. Display the examples and have the students use those to help each other.

Directions:

- Have each student complete the reading. Ask them to identify two important pieces of information and at least one question. Give them time to discuss in groups (10 min total for reading/discussion)
- Break students into groups. Display each topic's examples, and ask students to work in groups to do the assigned problems. Walk around and provided support/feedback.
- Move to the next topic and continue the process until all three topics have been completed.

Topic #1: Use functional notation to evaluate a function with a given input.

Example: Find $f(3)$ if $f(x) = 8x - 1$.

$$= 8(3) - 1$$
$$= 24 - 1$$
$$f(3) = 23$$

1) Find $h(5)$ if $h(x) = -2x^2 + 23$

$-2(5)^2 + 23$

$-2(25) + 23$

$-50 + 23$

$\boxed{h(5) = -27}$

2) Find $d(-8)$ if $d(x) = \frac{3}{4}x + 6$

$\frac{3}{\cancel{4}_1}(\cancel{-8}^{-2}) + 6$

$-6 + 6$

$\boxed{d(-8) = 0}$

Topic #2: Use functional notation to solve for the input with a given output.

Example: Solve for x if $f(x) = 5x - 3$ and $f(x) = 17$.

$$17 = 5x - 3$$
$$20 = 5x$$
$$4 = x$$

3) Solve for x if $b(x) = -2x - 3$ and $b(x) = 6$.

$6 = -2x - 3$

$+3 \quad\quad +3$

$\frac{9}{-2} = \frac{-2x}{-2}$

$\boxed{-\frac{9}{2} = x}$

4) Solve for x if $p(x) = \frac{1}{4}x - 8$ and $p(x) = -2$

$-2 = \frac{1}{4}x - 8$

$+8 \quad\quad +8$

$4 \cdot 6 = \frac{1}{4}x \cdot 4$

$\boxed{24 = x}$

Topic #3: Use functional notation to perform operations on functions.

Examples: If $f(x) = 6x + 18$, $g(x) = 9x - 1$, and $h(x) = 3$, then find the following.

a) $f(x) + g(x)$
$= 6x + 18 + 9x - 1$
$= 17x + 17$

b) $f(x) - g(x)$
$= 6x + 18 - (9x - 1)$
$= 6x + 18 - 9x + 1$
$= -3x + 19$

c) The product of $f(x)$ and $g(x)$
$= (6x + 18)(9x - 1)$
$= 54x^2 - 6x + 162x - 18$
$= \mathbf{54x^2 + 156x - 18}$

d) $\frac{f(x)}{h(x)}$
$= \frac{6x+18}{3}$
$= 2x + 6$

4) Find $f(x) - g(x) + h(x)$

$6x + 18 - (9x - 1) + 3$
$6x + 18 - 9x + 1 + 3$
$\boxed{-3x + 22}$

5) Find the product of $h(x)$ and $f(x)$.

$3(6x + 18)$
$\boxed{18x + 54}$

6) Find $\frac{g(x)}{h(x)}$.

$\frac{9x-1}{3} = \frac{9x}{3} - \frac{1}{3}$
$\boxed{3x - \frac{1}{3}}$

1.1 Notes (next day): Functions and Compositions

- **Syllabus signatures (Teams or Forms?) and Lab Fee Receipt reminders**
- **Pre-assessment:**
- **Notes**

Composition: When an expression is substituted as the input for a function.

Given that $f(x) = 3x + 5$ and $g(x) = 2x^2$. Find the following compositions.

Example 1: Find $f(g(x))$.

$3(2x^2) + 5$
$\boxed{6x^2 + 5}$

Example 2: Find $g(f(x))$.

$2(3x+5)^2$
$2(3x + 5)(3x + 5)$
$2(9x^2 + 30x + 25)$
$\boxed{18x^2 + 60x + 50}$

$f(x) = 3x + 5$ $g(x) = 2x^2$

Example 3: Find $f(x+3)$.

$3(x+3)+5$

$3x+9+5$

$\boxed{3x+14}$

Example 4: Find $-f(x)$.

$-(3x+5)$

$\boxed{-3x-5}$

Example 5: Find $f(x) - 4$

$3x+5-4$

$\boxed{3x+1}$

Example 6: Find $f\left(-\frac{1}{2}x\right)$

$3\left(-\frac{1}{2}x\right)+5$

$\boxed{-\frac{3}{2}x+5}$

Example 7: Find $f(f(x))$.

$3(3x+5)+5$

$9x+15+5$

$\boxed{9x+20}$

Example 8: Find $f(h(g(x)))$ if $h(x) = -4x + 1$.

$3(-4(2x^2)+1)+5$

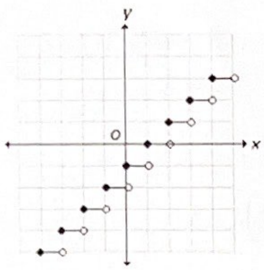
$3(-8x^2+1)+5$

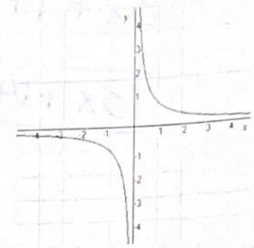
$-24x^2+3+5$

$\boxed{-24x^2+8}$

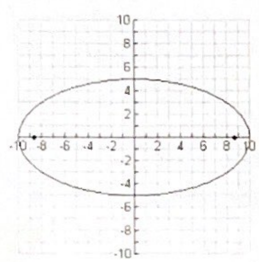
1.2 Notes: Equations of Lines

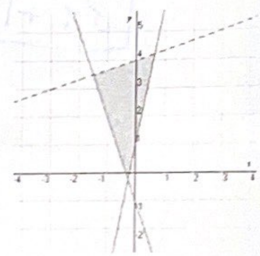
Function Families

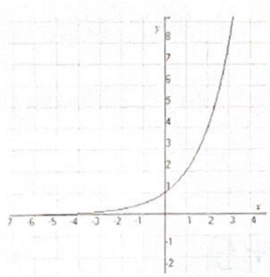


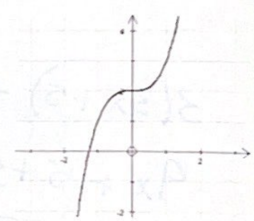


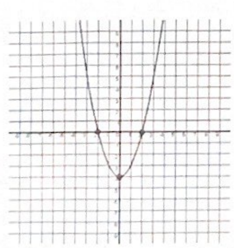
- Table of Values
- Graph
- Domain and Range
- *x*- and *y*-intercepts
- Max/Min values
- Increasing/Decreasing?
- Turning points
- Asymptotes or holes
- Vertex or center
- End behavior
- Transformations
- Symmetry
- Odd or Even
- Periodic
- Modeling/context

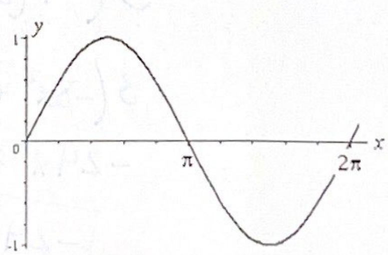












Input	Output
4	12
9	27
6	18
8	24

Exploration: In groups, students should explore the equation $y = 3x - 2$ by looking at all its key features. Using the Function Families handout as a guide, create a table of values and a graph, and try to name any features for this function that they recognize.

Domain: All real #'s

Range: All real #'s

Transformations: ↓2, stretch by 3

x-intercept: $2/3$

y-intercept: -2

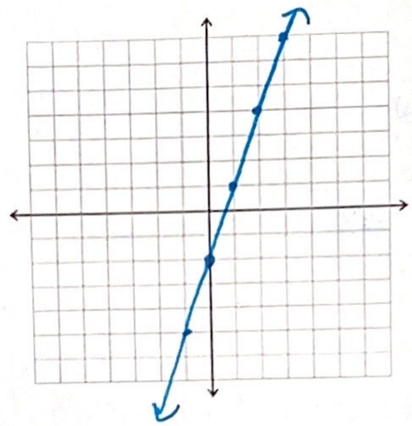
max/min: None

increasing: $(-\infty, \infty)$

decreasing: no

odd/even: odd

x	y
3	7
2	4
1	1
0	-2
-1	-5
-2	-8



x-int

$y=0$

$0=3x-2$

$+2 \quad +2$

$\frac{2}{3}=\frac{3x}{3}$

$\frac{2}{3}=x$

Graphing and Writing Equations of Lines

Slope-intercept Form $y=mx+b$ (slope: m; y-int: b)

Standard Form $ax+by=c$ • a, b, and c are integers • good for x- and y-int.

Point-Slope Form $y-y_1=m(x-x_1)$ (slope: m) (x_1, y_1) point on line

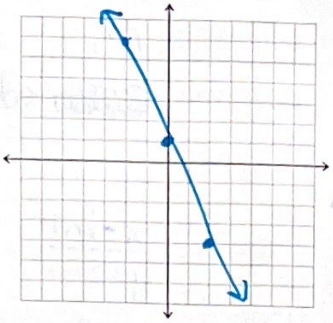
(h,k) Form $y=m(x-x_1)+y_1$ (based on above)

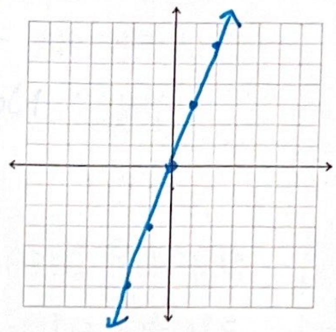
(using (h,k)) $y=m(x-h)+k$

slope (h,k) point on line

Slope-Intercept Form $y=mx+b$

Example: Graph $f(x) = -\frac{5}{2}x + 1$ and $g(x) = 3x$





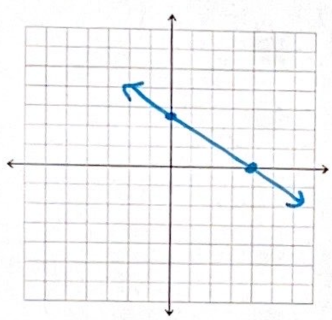
What is the domain and range for each line?

All real #'s

Standard Form

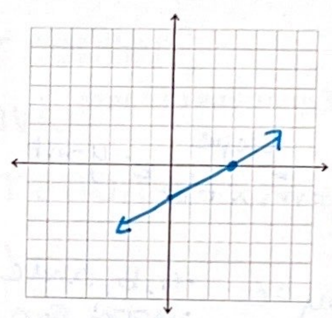
$ax+by=c$

Example: Graph $4x + 6y = 16$ and $3x - 5y = 9$



x-int
$4x=16$
$x=4$

y-int
$6y=16$
$y=\frac{8}{3}$



x-int
$3x=9$
$x=3$

y-int
$-5y=9$
$y=9/-5$

Example: A candy bar costs \$1.25 and a soda costs \$0.75. If Angie spends \$7.50 on soda and candy bars, then write an equation that models this situation.

$1.25c + 0.75s = 7.50$

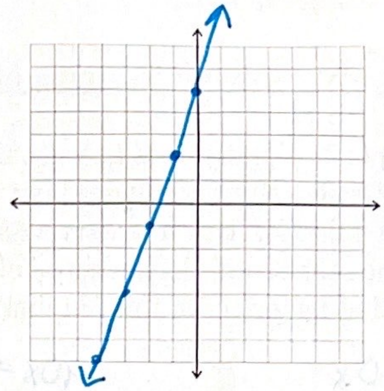
Point-Slope Form

$y - y_1 = m(x - x_1)$

Example: Graph $y - 2 = 3(x + 1)$

point: $(-1, 2)$

slope: 3



Example: Write the equation of a line in slope-intercept form with a slope of $\frac{2}{3}$ passing through (5, -3).

$y + 3 = \frac{2}{3}(x - 5)$

$y + 3 = \frac{2}{3}x - \frac{10}{3} - \frac{9}{3}$

$\boxed{y = \frac{2}{3}x - \frac{19}{3}}$

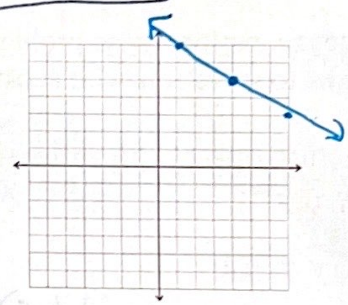
(h, k) Form

$y = m(x - h) + k$

Example: Graph $y = -\frac{2}{3}(x - 4) + 5$

point: $(4, 5)$

slope: $-2/3$



Example: Write the equation in slope-intercept form of a line parallel to $y = 5x + 6$, passing through (-2, -1).

Same slope

$y = 5(x + 2) - 1$

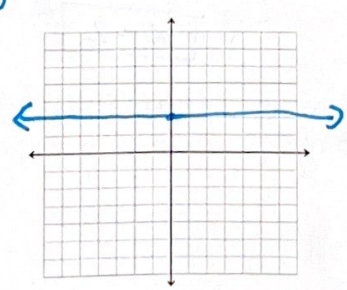
$y = 5x + 10 - 1$

$\boxed{y = 5x + 9}$

Special Lines

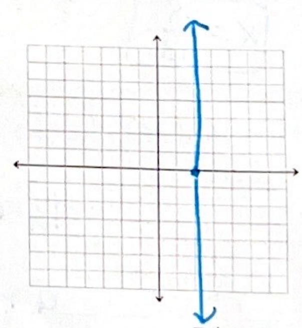
Horizontal Lines $y = 2$

$y = ?$



Vertical Lines $x = 2$

$x = ?$



1.3 Notes: Solving Systems of Equations

Example 1: Felicia lives near an amusement park. She has two options to pay for entrance.

Option A: Pay \$40 per visit.

Option B: Buy a season pass for \$140 and then pay \$5 per visit.

Find the number of visits where Felicia's cost is the same, regardless of the option she chooses. Also, find the price for this number of visits.

A: $40x$

B: $5x+140$

$40x = 5x + 140$

$35x = 140$

$x = 4$

$40(4)$

160

$\boxed{\text{4 visits at \$160}}$

Take 5 minutes to explore these problems in your groups, then share your strategies. (Then show them how to solve this with substitution and/or elimination.)

Example: Pick one of the following to solve using any method you choose.

2. $\begin{cases} y = \frac{1}{2}x + 3 \\ 4x + 5y = 11 \end{cases}$

$4x + 5\left(\frac{1}{2}x + 3\right) = 11$

$4x + \frac{5}{2}x + 15 = 11$

$\frac{8}{2}x + \frac{5}{2}x = -4$

$\frac{2}{13} \cdot \frac{13}{2}x = -4 \cdot \frac{2}{13}$

$\boxed{x = \frac{-8}{13}}$

$y = \frac{1}{2}\left(\frac{-8}{13}\right) + 3$

$y = \frac{-4}{13} + \frac{39}{13}$

$\boxed{y = \frac{35}{13}}$

3. $\begin{cases} 3.1x + 2.45y = -1.395 \\ -3.1(x - 1.6y = 6.96) \end{cases}$

$\cancel{3.1x} + 2.45y = -1.395$

$\cancel{-3.1x} + 4.96y = -21.576$

$7.41y = -22.971$

$\boxed{y = -3.1}$

$x - 1.6(-3.1) = 6.96$

$x + 4.96 = 6.96$

$\boxed{x = 2}$

Example 4: Set up a system of equations for the following. **DO NOT SOLVE!**

A movie theater charges \$9 for adults and \$7 for children. A group of seven people spend \$57 on tickets.

$9a + 7c = 57$

$a + c = 7$

Example 5: Frankie's cell phone plan charges \$30 per month, plus \$0.10 per text message. Elizabeth's cell phone plan charges \$12 per month, plus \$0.25 per text message. How many text messages would they each send in order to have the same monthly bill? How much would the bill be?

F: $y = 0.10x + 30$

E: $y = 0.25x + 12$

$0.10x + 30 = 0.25x + 12$

$18 = 0.15x$

$120 = x$

$y = 0.10(120) + 30$

$y = 42$

At 120 texts, they will each spend \$42

Example 6: Solve the system.

$$\begin{array}{l} A \\ B \\ C \end{array}\begin{cases} 2x + 3y - z = -5 \\ 8y - 4 = 12 \\ -x + 4y - 2z = 1 \end{cases}$$

B $8y - 4 = 12$

$8y = 16$

$\boxed{y = 2}$

A $2x + 3(2) - z = -5$

$2x + 6 - z = -5$

$2x - z = -11$

C $-x + 4(2) - 2z = 1$

$-x + 8 - 2z = 1$

$-x - 2z = -7$

A $-2(2x - z = -11)$

B $-x - 2\cancel{z} = -7$

$-4x + \cancel{2z} = +22$

$-5x = 15$

$\boxed{x = -3}$

A $2(-3) - z = -11$

$-6 - z = -11$

$-z = -5$

$\boxed{z = 5}$

Example 7: Solve the system.

$$\begin{array}{l} A \\ B \\ C \end{array}\begin{cases} 2x - y + 2z = -7 \\ -x + 2y - 4z = 5 \\ x + 4y - 6z = -1 \end{cases}$$

B $-\cancel{x} + 2y - 4z = 5$

C $\cancel{x} + 4y - 6z = -1$

D $6y - 10z = 4$

A $2x - y + 2z = -7$

B $2(-x + 2y - 4z = 5)$

$2\cancel{x} - y + 2z = -7$

$-2\cancel{x} + 4y - 8z = 10$

E $3y - 6z = 3$

D $6y - 10z = 4$

E $-2(3y - 6z = 3)$

$\cancel{6y} - 10z = 4$

$-\cancel{6y} + 12z = -6$

$2z = -2$

$\boxed{z = -1}$

E $3y - 6(-1) = 3$

$3y + 6 = 3$

$3y = -3$

$\boxed{y = -1}$

A $4x - 2\cancel{y} + 4\cancel{z} = -14$

B $-x + 2\cancel{y} - 4\cancel{z} = 5$

$3x = -9$

$\boxed{x = -3}$

$2y - 4(-1) = 2$

$2y + 4 = 2$

$2y = -2$

$\boxed{y = -1}$

B $3 + 2y - 4z = 5$

$-2(2y - 4z = 2)$

C $-3 + 4y - 6z = -1$

$4y - 6z = 2$

$4y - 6z = 2$

$-4y + 8z = -4$

$2z = -2$

$\boxed{z = -1}$

C $x + 4(-1) - 6(-1) = -1$

$x - 4 + 6 = -1$

$x + 2 = -1$

$\boxed{x = -3}$

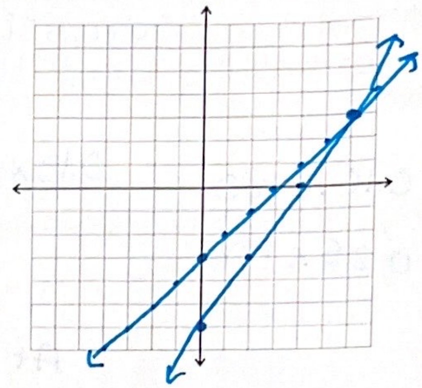
Example 8: Solve the following system by graphing.

$$\begin{cases} y = x - 3 \\ 3x - 2y = 12 \end{cases}$$

$-2y = -3x + 12$

$y = \frac{3}{2}x - 6$

$(6, 3)$



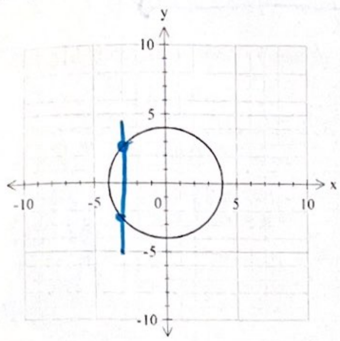
1.4 Notes: Set and Interval Notation for Domain and Range

Function:	Domain:	Range:
$f(x) =$ $y =$ To decide if a graph is a function use vertical line test. It should intersect the graph only once.	All possible inputs (x)	All possible outputs (y)

	Set Notation	Interval Notation	Graph	Anything goes
Open Interval	$\{x \mid a < x < b\}$	(a, b)		$x > a$ and $x < b$ x is greater than a and x is less than b
Closed Interval	$\{x \mid a \le x \le b\}$	[a, b]		$x \ge a$ and $x \le b$ x is greater than or equal to a and x is less than or equal to b
Infinite Interval	$\{x \mid x > a\}$	(a, ∞)		$x > a$ x is greater than a
Infinite Interval	$\{x \mid x \le b\}$	$(-\infty, b]$		$x \le b$ x is less than or equal to b

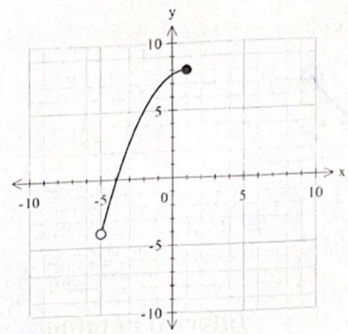
Examples: State the Domain, Range, and whether or not each graph is a function. Find the x- and y-intercepts identify the intervals where the function is increasing and/or decreasing.

1)



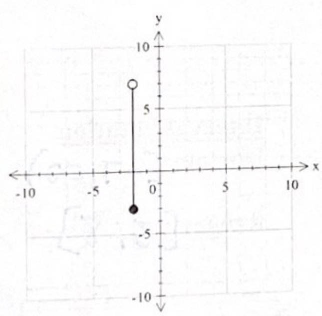
Set notation	**Interval notation**
Domain: $\{x \mid -4 \le x \le 4\}$	Domain: $[-4, 4]$
Range: $\{y \mid -4 \le y \le 4\}$	Range: $[-4, 4]$
Function? No	
x-int: $-4, 4$	~~Increasing:~~
y-int: $-4, 4$	~~Decreasing:~~

2)



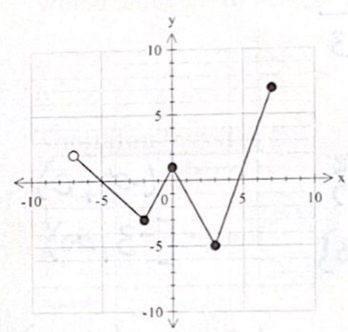
Set notation	**Interval notation**
Domain: $\{x \mid -5 < x \le 1\}$	Domain: $(-5, 1]$
Range: $\{y \mid -4 < y \le 8\}$	Range: $(-4, 8]$
Function? yes	
x-int: 4	Increasing: $(-5, 1)$
y-int: 7.5	Decreasing: No

3)



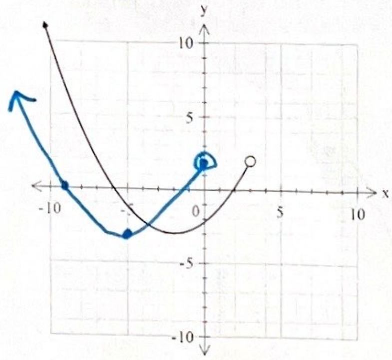
Set notation	**Interval notation**
Domain: $\{x \mid x = -2\}$	Domain: $[-2, -2]$
Range: $\{y \mid -3 \le x < 7\}$	Range: $[-3, 7)$
Function? No	
x-int: -2	~~Increasing:~~
y-int: None	~~Decreasing:~~

4)



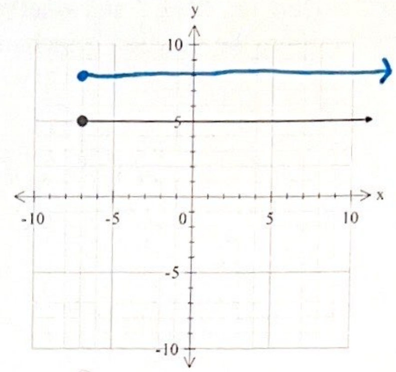
Set notation	**Interval notation**
Domain: $\{x \mid -7 < x \le 7\}$	Domain: $(-7, 7]$
Range: $\{y \mid -5 \le y \le 7\}$	Range: $[-5, 7]$
Function? yes	
x-int: $-5, -\frac{1}{2}, \frac{1}{2}, 4.5$	Increasing: $(-2, 0) \cup (3, 7)$
y-int: 1	Decreasing: $(-7, -2) \cup (0, 3)$

5)



Set notation	Interval notation
Domain: $\{x \mid x < 3\}$	Domain: $(-\infty, 3)$
Range: $\{y \mid y \geq -3\}$	Range: $[-3, \infty)$
Function? yes	
x-int: 2, -6	Increasing: $(-2, 3)$
y-int: -2	Decreasing: $(-\infty, -2)$

6)



Set notation	Interval notation
Domain: $\{x \mid x \geq -7\}$	Domain: $[-7, \infty)$
Range: $\{y \mid y = 5\}$	Range: $[5, 5]$
Function? yes	
x-int: None	~~Increasing:~~
y-int: 5	~~Decreasing:~~

7) If $f(x)$ is the graph represented in #5 above and $g(x) = f(x+3)$, fill in the table below for g(x).

← 3

Set notation	Interval notation
Domain: $\{x \mid x < 0\}$	Domain: $(-\infty, 0)$
Range: $\{y \mid y \geq -3\}$	Range: $[-3, \infty)$
Function? yes	
x-int: -1, -9	Increasing: $(-5, 0)$
y-int: 2	Decreasing: $(-\infty, -5)$

8) If $h(x)$ is represented in #6 above and $b(x) = h(x) + 3$, fill in the table below for b(x).

↑3

Set notation	Interval notation
Domain: $\{x \mid x \geq -7\}$	Domain: $[-7, \infty)$
Range: $\{y \mid y = 8\}$	Range: $[8, 8]$
Function? yes	
x-int: None	~~Increasing:~~
y-int: 8	~~Decreasing:~~

9) Describe the transformation associated with $f(x)$ and $f(-x)$.

- Reflection in the y-axis
- Turns your graph
  $+ \to -$ or $- \to +$

1.1 Intro: Simplifying Radicals & Rational Exponents (After Unit 1 Test)

Examples: Simplifying each radical. Make a Factor Tree for simplifying square roots!

1) $\sqrt{180}$

10 18

5 2 2 9

3 3

$\boxed{6\sqrt{5}}$

2) $5\sqrt{24} \cdot 3\sqrt{10}$

$15\sqrt{24 \cdot 10} = \boxed{60\sqrt{15}}$

12 2 2 5

6 2

3 2

3) $4\sqrt{5} \cdot \sqrt{18}$

3 6

3 2

$\boxed{12\sqrt{10}}$

4) $\sqrt{\frac{32}{49}} = \frac{\sqrt{32}}{7}$

32

8 4

2 4

$\boxed{\frac{4\sqrt{2}}{7}}$

5) $\frac{7}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$\boxed{\frac{7\sqrt{3}}{3}}$

6) $\frac{4\sqrt{6}}{\sqrt{12}} \cdot \frac{\sqrt{12}}{\sqrt{12}}$

6 12

6 2

$\frac{4\sqrt{6 \cdot 12}}{12}$

$\frac{24\sqrt{2}}{12} = \boxed{2\sqrt{2}}$