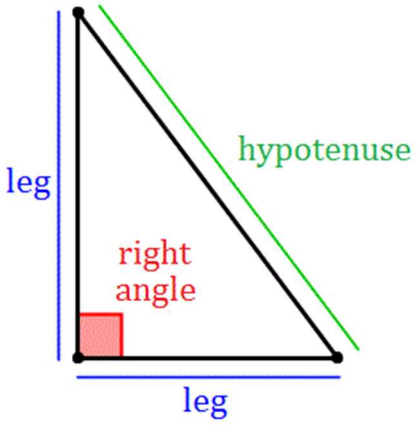
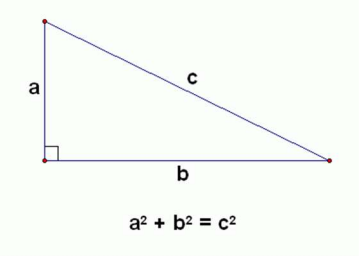
7.1 Notes: The Pythagorean Theorem

Objectives:

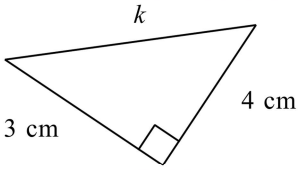
- **Students will be able to find the missing side of a right triangle.**
- **Students will use the Pythagorean Theorem to solve problems.**

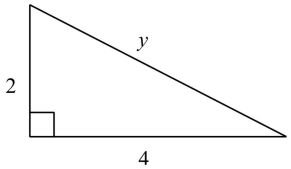
Video Demonstration of the Pythagorean Theorem:
https://knpb.pbslearningmedia.org/resource/mgbh-math-ee-gshreepythag/pythagorean-theorem/

- Why does the Pythagorean Theorem work? Explain it in your own words.

Right Triangle	**A right triangle has _____ right angle. The side opposite the right angle is called the _____, it is the longest side of the triangle. The other two sides are called the _____.**	
The Pythagorean Theorem	**The Pythagorean Theorem states that the sum of the _____ of the legs equals the square of the _____. Note: This only works for _____ triangles!**	

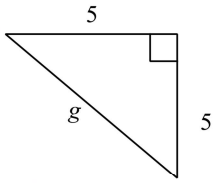
For #1 – 8: Find the missing side for each right triangle. If needed, write your answer as a simplified radical.

1) 

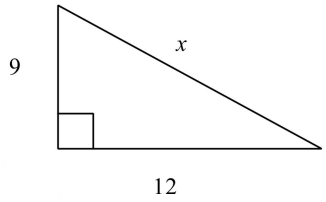
2) 

You try #3 – 4!

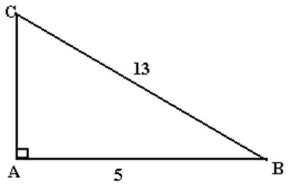
3)



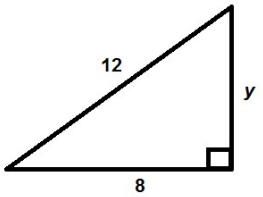
4)



5)

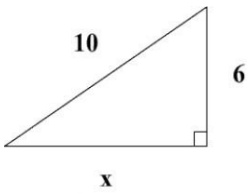


6) Write your answer as a decimal, to one place.

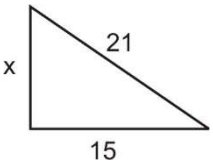


You try #7 – 8!

7)

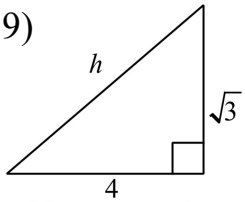


8) Write your answer as a decimal, to one place.

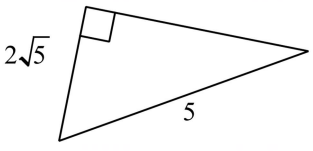


For #9 – 12: Find the missing side in each right triangle. If needed, write your answer as a simplified radical.

9)

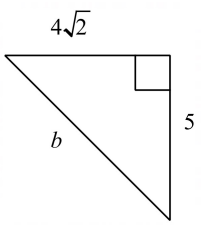


10)

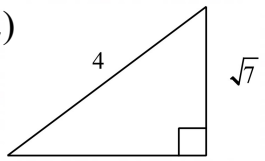


You try #11 – 12!

11)

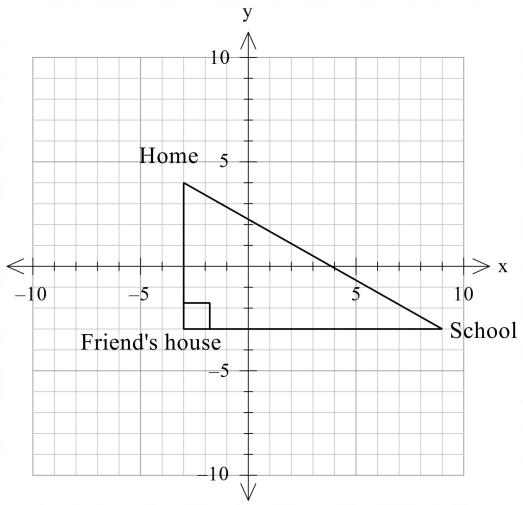


12)



13) A triangle has side lengths of 6, 8, and 10. Is the triangle a right triangle? Explain your reasoning.

14) Each day, Amy walks to school. She leaves her home and walks south 7 blocks to her friend's house. They then turn to the west and walk 12 blocks to the school. At the end of the day, Amy walks directly from the school to her home.



a) To the nearest tenth (one decimal place), how many blocks does Amy walk on her way home?

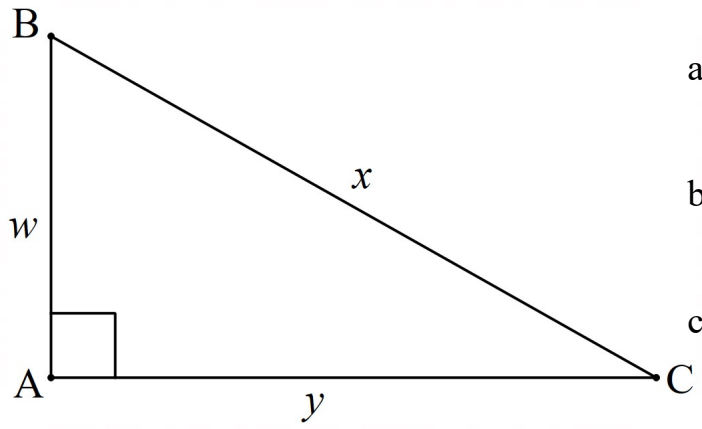
b) How many blocks does Amy walk in one day for her round trip to the school and back home? Round to the nearest tenth.

7.1 Remediation

Part 1: When to use an *upper-case* letter or *lower-case* letter <u>in Geometry</u>.

- Lower case letters are usually used to represent numbers and variables:
  - unknown side lengths, scale factors & parts of polygons
- Upper case letters are usually used to name points (vertices) and answers in a formula, capital letters are used in naming segments, rays, and lines:
  - area formula, line name, point name

Example 1: Use the following triangle to complete the statements below.



a) Write the Pythagorean Theorem for ΔABC.

b) Name the hypotenuse and right angle.

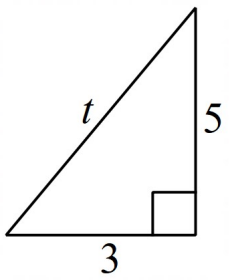
c) Write the area formula using the variables (sides) of this triangle.

Part 2: Setting up the Pythagorean Theorem Practice

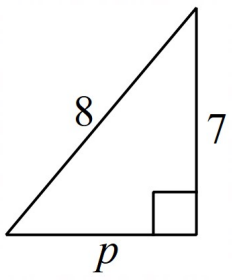
- The Pythagorean Theorem is $a^2 + b^2 = c^2$,
  - a & b are the legs and c is the hypotenuse
- Using the parts of the triangle (legs and hypotenuse) to represent the Pythagorean Theorem:
  - $leg^2 + leg^2 = hypotenuse^2$

Example 2: Set up the Pythagorean Theorem for each triangle below. *Do not solve.*

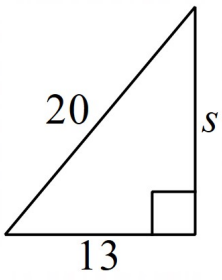
a)



b)



c)



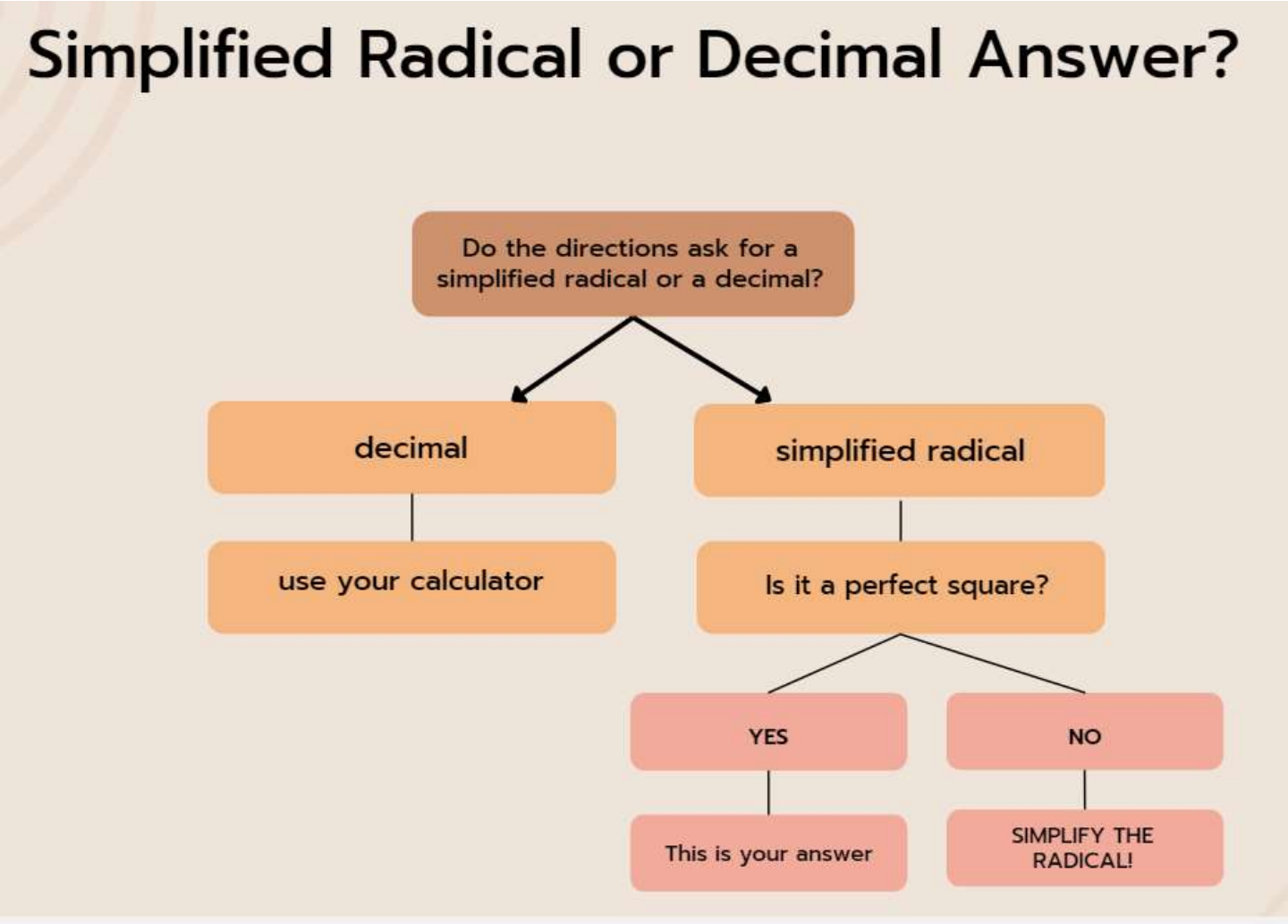
Part 3: **Squaring a Radical**

1. $\left(\sqrt{x}\right)^2 = \sqrt{x} \cdot \sqrt{x} = x$
   - $\left(\sqrt{9}\right)^2 = \sqrt{9} \cdot \sqrt{9}$
     $3 \cdot 3 = 9$
2. $\left(a\sqrt{b}\right)^2 = \left(a\sqrt{b}\right)\left(a\sqrt{b}\right)$
   $= \left(a \cdot a \cdot \sqrt{b} \cdot \sqrt{b}\right)$
   $= a^2 \cdot b$

Example 3: Solve.

a) $\left(\sqrt{11}\right)^2$ b) $\left(\sqrt{21}\right)^2$ c) $\left(2\sqrt{7}\right)^2$ d) $\left(3\sqrt{8}\right)^2$

Part 4: **When to simplify a radical and when to give a decimal answer.**



7.2 Notes: Pythagorean Theorem Triples

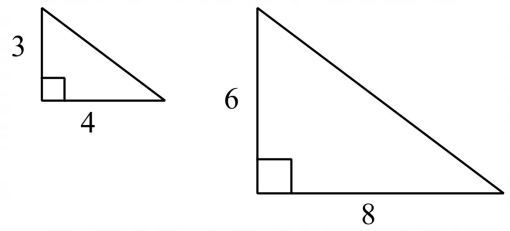
Objectives:

- **Students will be able to use Pythagorean Triples to find missing sides in right triangles.**
- **Students will be able to solve problems involving Pythagorean Triples.**

Exploration:

- Consider the two triangles shown. Are they similar? What theorem or postulate did you use to make this decision? (Consider AA~, SAS ~, and SSS ~.)

- Find the missing hypotenuse of the smaller triangle. Hint: use the Pythagorean Theorem.

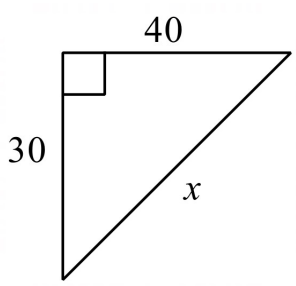


- Without using the Pythagorean Theorem, find the missing hypotenuse of the larger triangle. What do you notice about the sides of these triangles?

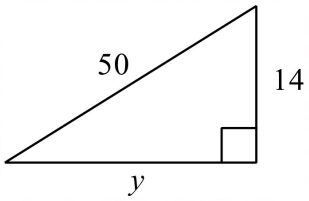
Pythagorean Triples	**If a right triangle is a Pythagorean Triple, then all three of its sides are** _____ _____. Memorize the ones listed!	Common Pythagorean Triples: 3, 4, ____ 5, 12, ____ 7, 24, ____ 8, 15, ____ Note: the largest number is the **hypotenuse**.
Using Pythagorean Triples	**If a triangle is a dilation of a Pythagorean Triple, then you can multiply by the** _____ _____ **to find a missing side.**	Steps: 1) Identify which triple is being used. 2) Find the scale factor. 3) Multiply by the scale factor. Note: You can always use the Pythagorean Theorem, as well.

For #1 – 9: Find the missing side of each right triangle by using Pythagorean Triples.

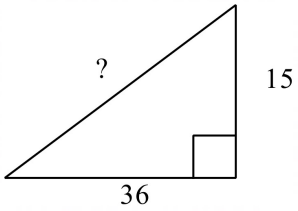
1)



2)

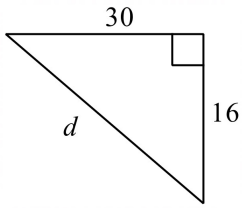


3)

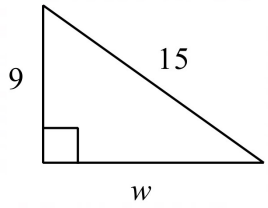


You try #4 – 6!

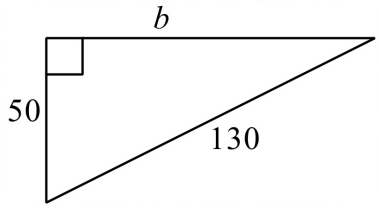
4)



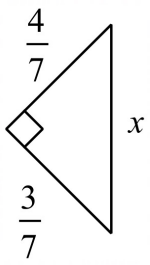
5)



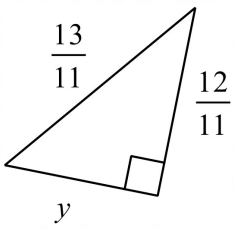
6)



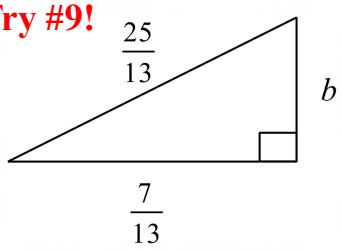
7)



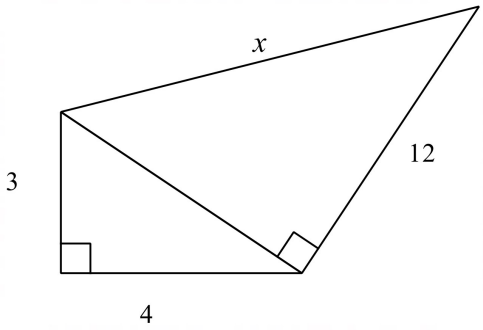
8)



You Try #9!



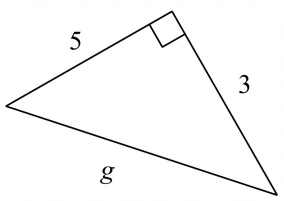
10) Use Pythagorean Triples to find x.



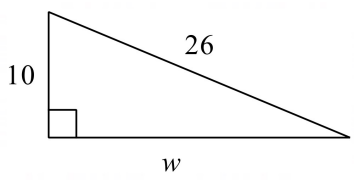
Note: Not all right triangles are Pythagorean Triples. If the sides of a right triangle do not form a Pythagorean Triple, then use the Pythagorean Theorem to find a missing side.

For #11 – 14: Find the missing side for each right triangle. If needed, write your answer as a simplified radical.

11)

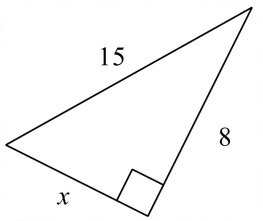


12)

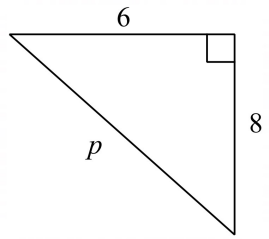


You try #13 – 14!

13)

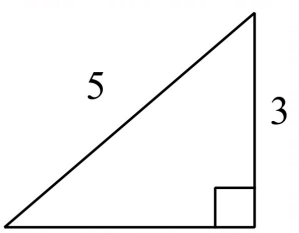


14)



15) Garrett left his work and drove 16 miles north. He then turned 90 degrees and drove 30 miles west. How far was he from his starting location? (Hint: label the compass then draw a right triangle.)

16) Find the area and perimeter of the right triangle shown below.



7.3 Notes: Special Right Triangles

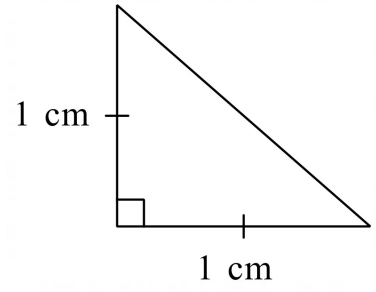
Objectives:

- **Students will be able to identify special right triangles.**
- **Students will be able to find missing sides of special right triangles.**

Exploration: Consider an isosceles right triangle, with each side of 1 cm, as shown.

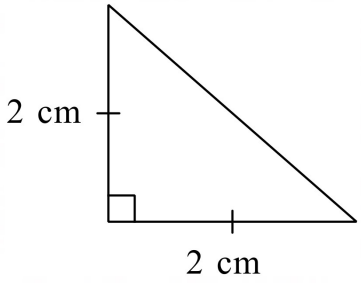
a) Find the measures of all angles of the triangle.

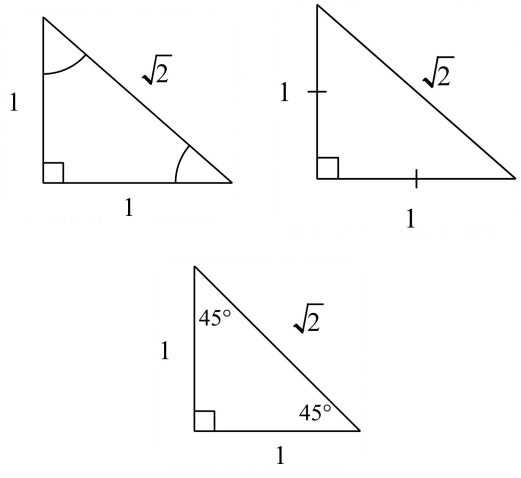
b) Find the length of the hypotenuse of the triangle. Write your answer as a simplified radical. Hint: Use the Pythagorean Theorem.



c) Change the length of the legs to 2 cm each. Do the angles change?

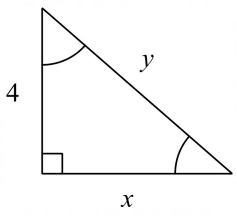
d) Using the new length of 2cm for each leg, find the length of the hypotenuse. Write your answer as a simplified radical. Hint: Use the Pythagorean Theorem.



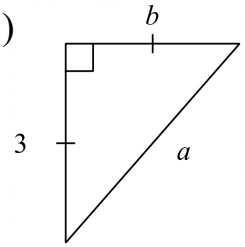
Special Triangle:	**If a triangle is a 45-45-90 triangle *(these are the _____ measurements),* then the triangle is an _____ right triangle.**	
45-45-90 Triangles	**The sides of a 45-45-90 triangle are in the ratio of _____ : _____ : _____.** Note: We will often see dilations of this triangle.	

For #1 – 6: For each 45-45-90 triangle, find the lengths of the missing sides.

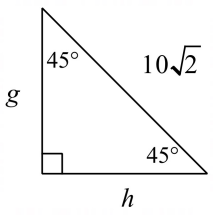
1)



2)

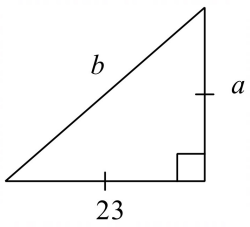


3)

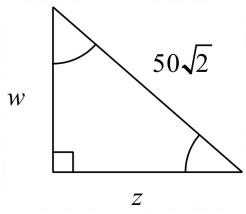


You try #4 – 6!

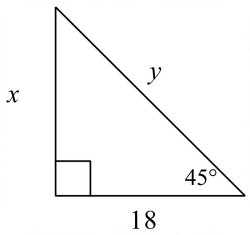
4)



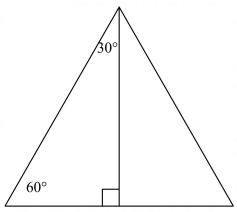
5)

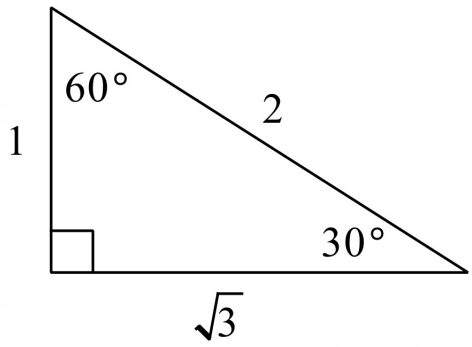


6)



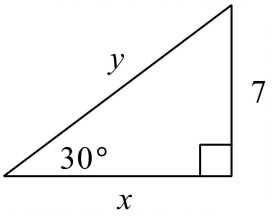
There is another type of special right triangle, whose side relationships can be found by dropping a perpendicular segment from one vertex of an equilateral triangle to the base, as shown below.



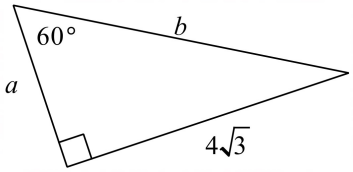
Special Triangle: **30-60-90 Triangles**	**If a triangle is a 30-60-90 triangle *(these are the _____ measurements)*, then the sides are in the ratio of** **_____ : _____ : _____.** Note: We will often see dilations of this triangle.	

For #7 – 9: Find the value of the variable(s) by using 30-60-90 triangles.

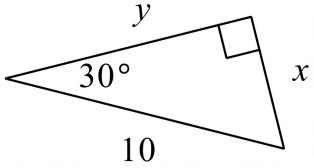
7)



8)

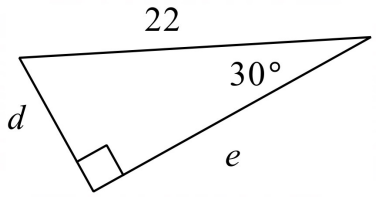


9)

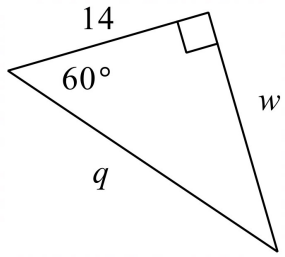


You try #10 – 12! Find the value of the variable(s) by using 30-60-90 triangles.

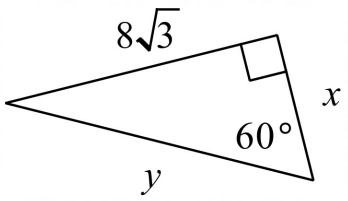
10)



11)

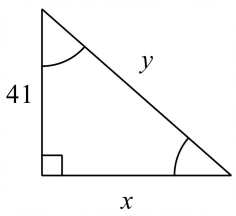


12)

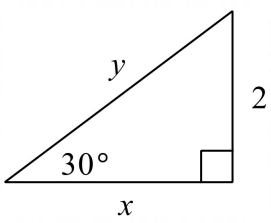


You try #13 – 18! Find the value of the variable(s) by using special right triangles.
Be careful! Both 45-45-90 and 30-60-90 triangles are below.

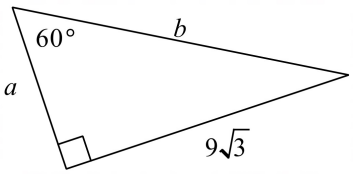
13)



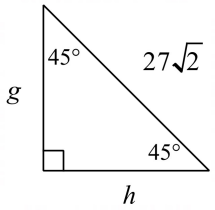
14)



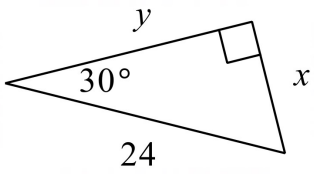
15)



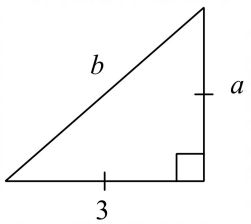
16)



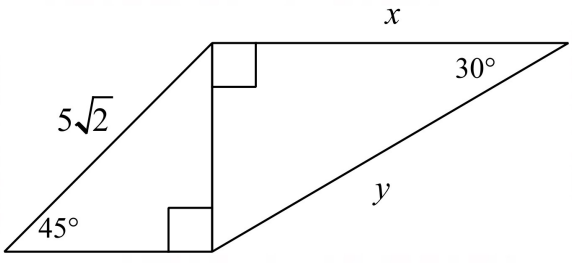
17)



18)



19) **Challenge: Find x and y.**

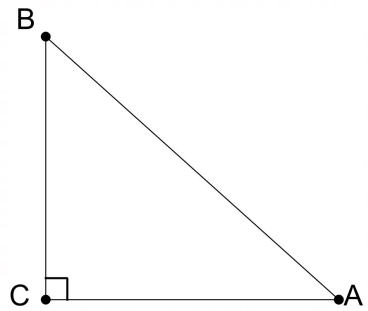


7.4 Notes: Right Triangle Trigonometry, Part I

Objectives:

- **Students will find ratios for trig functions for right triangles.**
- **Students will use trig functions to find missing sides of right triangles.**

Trigonometry is the study of the relationships between the sides and angles of right triangles. The legs are called *adjacent or opposite* depending on which acute angle is being used. The hypotenuse is always the longest side, which is directly opposite the right angle. There are three basic trig ratios that you will need to know: sine, cosine, and tangent.

Trig Functions	**Soh-Cah-Toa**	
	Sin A = **Sin B =**	
	Cos A = **Cos B =**	
	Tan A = **Tan B =**	

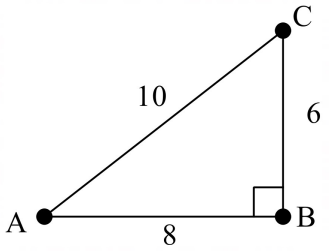
Helpful hints	• We will only find trig functions of acute angles this year. • Label your sides as Opposite, Adjacent, and Hypotenuse. • All ratios must be reduced and rationalized, if needed.

SOH – CAH – TOA

1) Find the requested trig ratio. Reduce your answers, if possible.

$\sin A =$ $\cos A =$ $\tan A =$

$\sin C =$ $\cos C =$ $\tan C =$

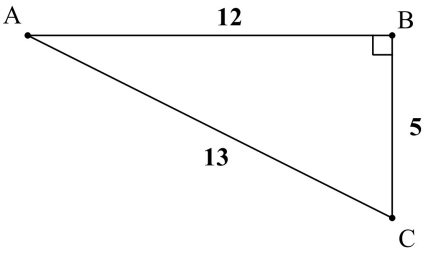


Reminder: We only find trig functions for the _____ angles of a right triangle this year!

You try #2! Find the requested trig ratio. Reduce your answers, if possible.

$\sin A =$ $\cos A =$ $\tan A =$

$\sin C =$ $\cos C =$ $\tan C =$



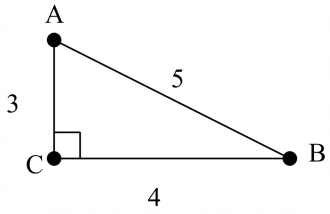
3) Which angle has a cosine of $\frac{3}{5}$?

A. ∠A

B. ∠B

C. ∠C

D. None of the above.



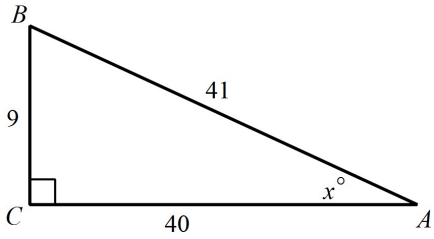
You try #4! What is $\boldsymbol{\cos x}$ in the triangle?

A. $\frac{41}{40}$

B. $\frac{41}{9}$

C. $\frac{40}{9}$

D. $\frac{40}{41}$



5) Find the following values by using your calculator. Round to 3 decimal places.

Your calculator must be in Degrees!

a. sin 18° b. cos 68° c. tan 32° d. tan 80°

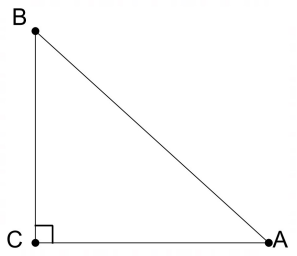
Writing Equations with Trig Functions

Soh-Cah-Toa

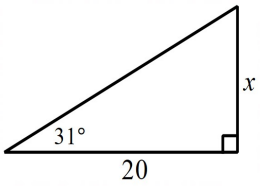
1) Decide what trig function matches your picture.
2) Make an equation.

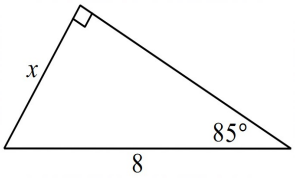
$$\textit{Trig function}\,(\textit{angle}) = \textit{side ratio}$$

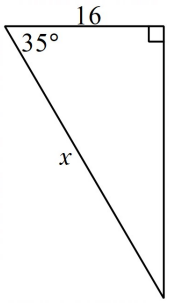
3) Solve for the variable by using inverse operations.



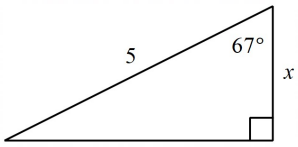
For #3 – 8: find the variable. Round to the nearest hundredth.

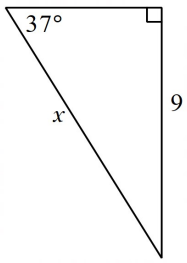
3) 

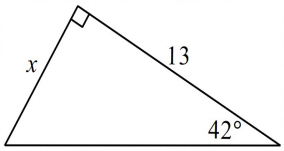
4) 

5) 

You Try #6-8!

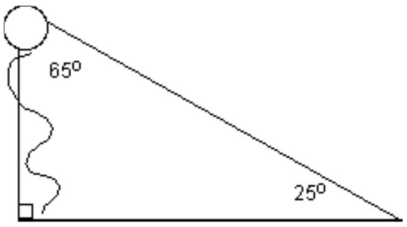
6) 

7) 

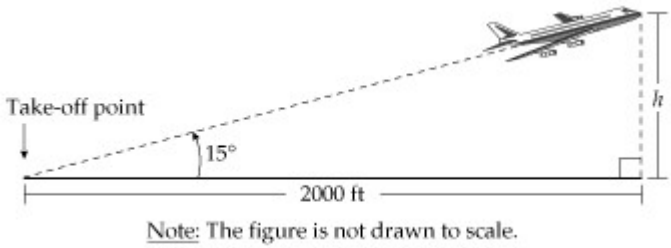
8) 

Angle of Elevation: the "line of sight" angle made between an object and the ground.

- What is the angle of elevation in the picture shown?

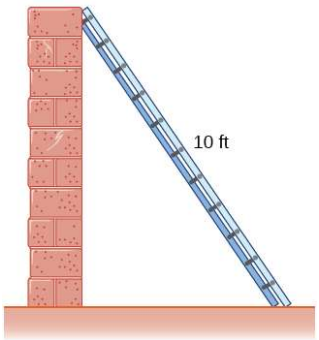


9) An airplane has an angle of elevation of 15 degrees from the runway when it takes off. The airplane pictured below is 2,000 feet along the ground from its take-off point. Find the height, h, of the airplane (round answer to nearest foot).



Note: The figure is not drawn to scale.

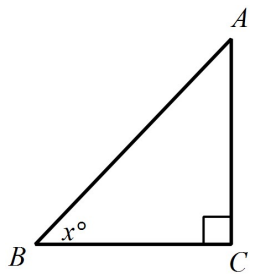
10) A 10-foot ladder is leaning against the side of a house, with an angle of elevation of 62 degrees. How far up the side of the house does the ladder reach? Round to 2 decimal places.



7.5 Notes: Right Triangle Trigonometry, Part II

Objectives:

- **Students will use inverse trig functions to find missing angles of a right triangle.**
- **Students will solve problems involving trig and inverse trig functions.**

Inverse Trig Functions	**Soh-Cah-Toa** **If the variable in an equation involving a trig function is in the *angle* position, then use an** _____ _____ _____ **to isolate the variable.** $\sin^{-1}$ $\cos^{-1}$ $\tan^{-1}$	
Helpful hints	• Use an inverse trig function to find a missing angle in a right triangle. • Most calculators require you to use SHIFT or 2nd to access an inverse trig function. $Trig\ function\ (angle) = side\ ratio$	

1) Use your calculator to find the angle (round to nearest whole angle):

a. $\cos B = 0.5$

b. $\tan x = 1.33$

You try c and d!

c. $\sin A = .8990$

d. $\cos x = 0.397$

2) Use your calculator to find x: *Note:* decide if you should use the trig function or its inverse. Round to two decimal places.

a. $\sin 60 = x$

b. $\cos x = .75$

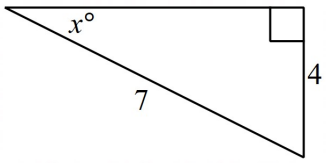
You try c and d!

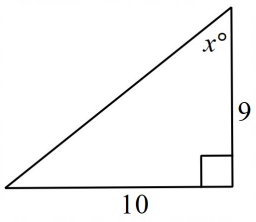
c. $\tan 30 = x$

d. $\tan x = 1$

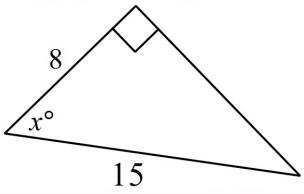
For #3 – 6: Find x in each picture. Round to one decimal place.

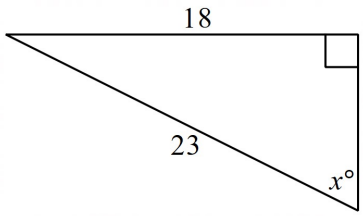
$$Trig\ function\ (angle)\ =\ side\ ratio.$$

3) 

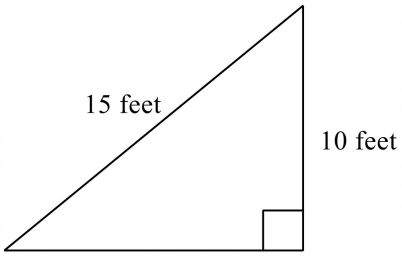
4) 

You Try #5-6!

5) 

6) 

7) A 15-foot ladder is leaning against a building. If the ladder hits the building at a height of 10 feet, find the angle of elevation to two decimal places.



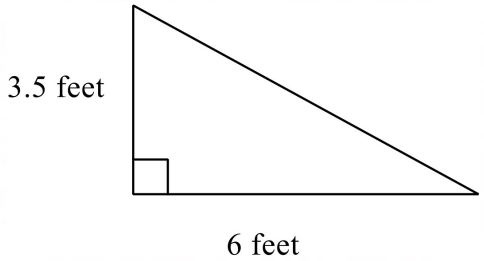
You try #8! A skateboard ramp is 3.5 feet high and 6 feet long along the horizontal. To the nearest degree, what is the measure of the angle of elevation for the ramp?

A. 27°

B. 60°

C. 30°

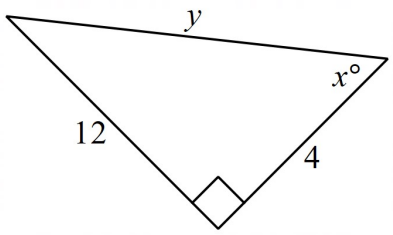
D. 63°



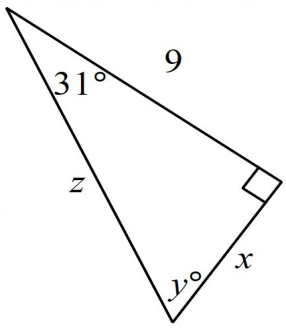
Summary of finding parts of a right triangle…		
If I know…	I should use…	to find the…
2 sides	the Pythagorean Theorem	3rd side
2 sides	inverse trig function	related angle
A side and an angle	trig function	related side
2 angles	sum of 180 degrees	3rd angle

For #9 – 11: Solve for the variable(s) in each triangle. Round to one decimal place.

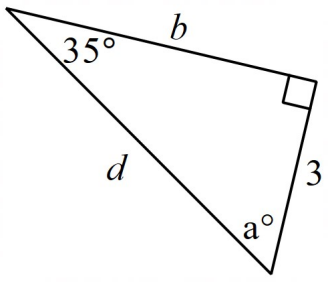
9)



10)



11)



Ch 7 Study Guide

- 7.1:
  - The Pythagorean Theorem: $a^2 + b^2 = c^2$
    - Only use for right triangles.
    - c is the hypotenuse (side opposite the right angle)
- 7.2:
  - Pythagorean Triples are right triangles with sides that are whole numbers.
  - We can have dilations of Pythagorean Triples.
  - Common triples include the following:
    - 3-4-5
    - 5-12-13
    - 7-24-25
    - 8-15-17
    - And many, many more…
- 7.3:
  - Special Right triangles have the following ratio of sides:
    - $45 - 45 - 90$: $1, 1, \sqrt{2}$
    - $30 - 60 - 90$: $1, \sqrt{3}, 2$
  - We can have dilations of this patterns.
- 7.4:
  - Trig Functions:
    - Sin = $\frac{\text{opposite}}{\text{hypotenuse}}$
    - Cos = $\frac{\text{adjacent}}{\text{hypotenuse}}$
    - Tan = $\frac{\text{opposite}}{\text{adjacent}}$
  - Use trig functions to find missing sides in a right triangle.
    - Trig function (angle) = ratio
  - Angle of elevation
    - Angle made by the line of sight of an object and a horizontal line (typically the ground.)
- 7.5:
  - Inverse Trig functions are used to find a missing angle of a right triangle.
  - "Summary of finding parts of a right triangle…" chart on pg. 16.