6.0 Notes: Review of Radicals and Proportions

Ratio	**A __________________ of two numbers.** **Ratios can be written as a ____________,** **in words, or with a ____________.**	**Sample:**
Proportion	**Two or more __________ set ___________** **to each other.**	**Sample:**
Solving Proportions	**Steps for Solving Proportions:** 1) ___________-multiply 2) Set the products from step 1 ____________ to each to each other. 3) Solve for the variable.	

For #1 – 4, solve each proportion for the variable.

1) $\frac{4}{9} = \frac{x}{7}$

2) $\frac{b-2}{6} = \frac{3}{5}$

You Try #3 – 4!

3) $\frac{5}{d} = \frac{4}{3}$

4) $\frac{11}{2} = \frac{8}{3+y}$

5) Solve the proportion for h: $\frac{4+h}{3} = \frac{h-2}{10}$

Simplifying Radicals		**Sample:**

For #6 – 9, simplify each radical.

6) $\sqrt{54}$

7) $-7\sqrt{300}$

You Try #8 – 9!

8) $\sqrt{90}$

9) $5\sqrt{48}$

Dividing Radicals		Sample:

For #10 – 15, simplify each radical.

10) $\sqrt{\frac{16}{25}}$

11) $\frac{\sqrt{4}}{\sqrt{49}}$

12) $\frac{\sqrt{50}}{\sqrt{18}}$

You Try #13 – 15!

13) $\sqrt{\frac{100}{49}}$

14) $\frac{\sqrt{121}}{\sqrt{64}}$

15) $\frac{\sqrt{3}}{\sqrt{12}}$

Rationalizing Radicals		Sample:

For #16 – 23, simplify each radical. Rationalize, if needed.

16) $\frac{1}{\sqrt{3}}$

17) $\frac{4}{\sqrt{5}}$

18) $\frac{6}{\sqrt{2}}$

19) $\frac{10}{\sqrt{10}}$

You Try #20 – 23!

20) $\frac{1}{\sqrt{13}}$

21) $\frac{7}{\sqrt{6}}$

22) $\frac{18}{\sqrt{3}}$

23) $\frac{14}{\sqrt{14}}$

6.1 Notes: Dilations and Scale Factor

Objectives:
- **Students will be able to classify dilations as enlargements or reductions.**
- **Students will be able to find the scale factor of dilation.**

Exploration #1: Use the following link to explore dilations:
https://www.geogebra.org/m/waP9naNC
Follow the directions below.

- Click on the slider to make the $scale\ factor = 2$. What do you notice about the size of the sides of the image and preimage?

- Move the slider of the scale factor to ½. What do you notice?

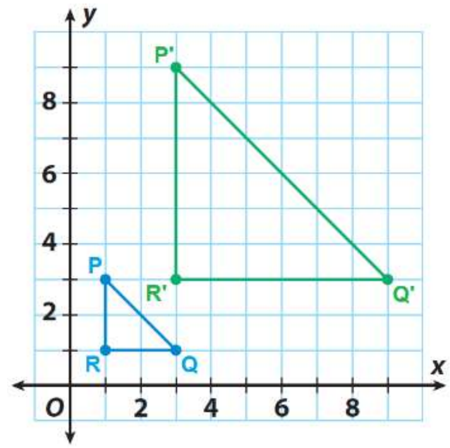
- Make a **conjecture** ("guess") about what scale factor tells you about the image of a dilation.

Dilation (centered at the origin)	**If a figure is dilated, then the image has the same ______________ as the pre-image, but the __________ can be different.**	
Scale Factor (k) of a Dilation	**The scale factor of a dilation is a __________ that is the multiplier for the sides of the pre-image to get the lengths of the sides of the image. We use k for scale factor.**	Scale Factor: $k = \underline{\qquad\qquad}$ Reduction: Enlargement:

For #1 – 2, use the diagram of the dilation shown, which is centered at the origin.

1) Is this dilation a *reduction* or an *enlargement?*

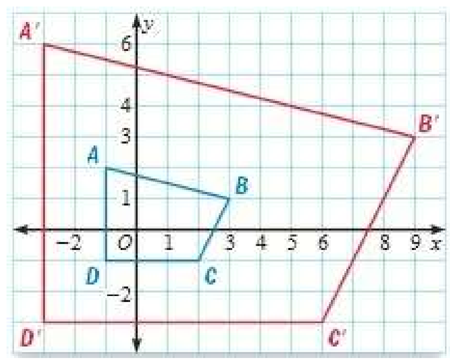
2) Find the scale factor of the dilation: $\frac{\text{image}}{\text{pre-image}}$



You try #3 – 4! **Use the diagram of the dilation shown, which is centered at the origin.**

3) Is this dilation a *reduction* or an *enlargement?*

4) Find the scale factor of the dilation: $\frac{\text{image}}{\text{pre-image}}$



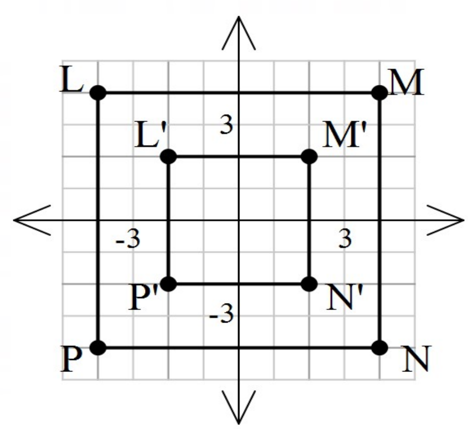
5) Multiple choice. Which statement below is true for the dilation shown?

A) It is an enlargement; the scale factor is $k = 2$.

B) It is an enlargement; the scale factor is $k = \frac{1}{2}$.

C) It is a reduction; the scale factor is $k = 2$.

D) It is a reduction; the scale factor is $k = \frac{1}{2}$.



You try #6!

6) Multiple choice. A figure is dilated. Which scale factor below shows a reduction?

A) $k = 6$ B) $k = \frac{7}{6}$ C) $k = \frac{1}{2}$ D) $k = -3$

For #7 – 10, ΔABC is dilated. Given the lengths below, find the scale factor of the dilation. Also, classify each dilation as a *reduction* or an *enlargement*.

7) $AB = 8; A'B' = 32$

8) $BC = 24; B'C' = 9$

You Try #9-10!

9) $AC = 14; A'B' = 7$

10) $AB = 18; A'B' = 12$

Exploration #2: Use the following link to explore the coordinates of a dilation centered at the origin. https://www.geogebra.org/m/d6HBmDNZ

- Click on the slider and set the scale factor to 2. Compare the coordinates of the image and the pre-image. What do you notice?

- Click on the slider and set the scale factor to $\frac{1}{2}$. Compare the coordinates of the image and the pre-image. What do you notice?

Coordinates of a dilation centered at the origin	**For any dilation centered at the origin, the coordinates of the image can be found by __________________ the coordinates of the pre-image by the ____________ _____________.**	

For #11 – 14, find the coordinates of the image of ΔABC after a dilation with the given scale factor. Given: $\boldsymbol{A(-6,4); B(3,-2); C(-1,0)}$

11) $k = 2$

12) $k = \frac{1}{3}$

You try #13-14!

13) $k = \frac{1}{2}$

14) $k = -5$

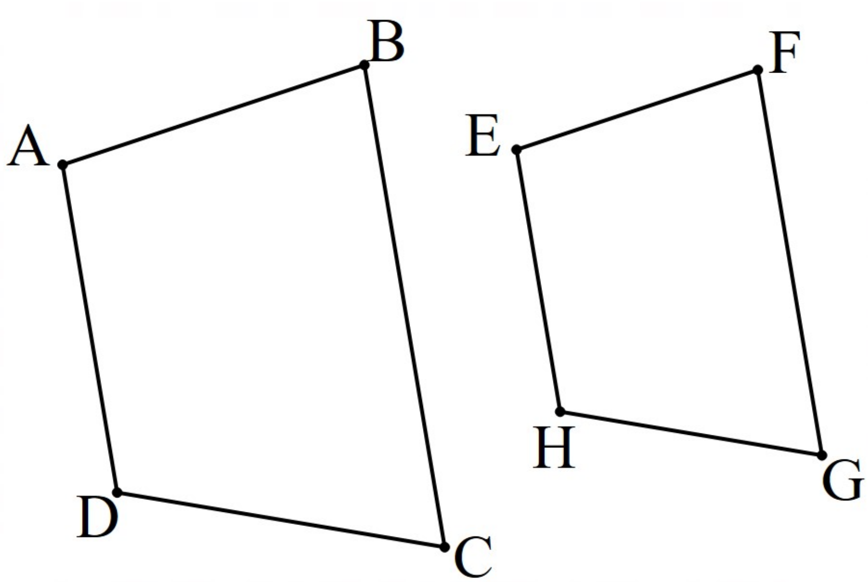
6.2 Day 1 Notes: Similar Figures

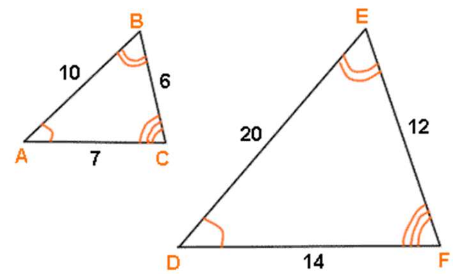
Objectives:

- **Students will be able to identify similar figures.**
- **Students will be able to find the scale factor between similar figures.**

Exploration #1: Use the following link to explore similar figures:
https://www.geogebra.org/m/mVYrt5u9

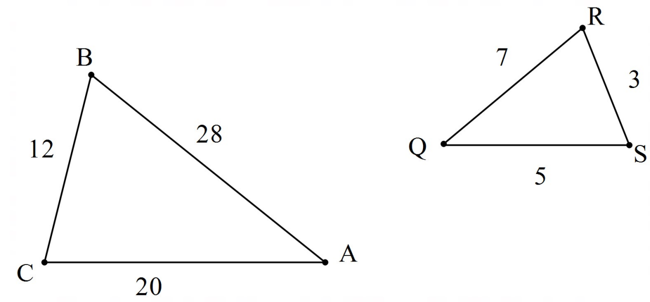
- Click on the slider to *change the size of* the image of the similar figures.
- As you adjust the size of the image, what do you notice about the angles?
- Compare the ratios made by corresponding sides on the right side of the screen. What do you notice?
- In the space below, draw what you learned in the exploration. Specifically, the ratios.



Similar Figures **~**	**If two figures are similar, then they have the same ____________, but not *necessarily* the same ____________.**	
Corresponding Angles of Similar Figures	**With similar figures, the corresponding angles are ____________.**	**Given: $\Delta ABC \sim \Delta DEF$.** **A *similarity statement* correlates the congruent angles and proportional sides.**
Corresponding Sides of Similar Figures	**The corresponding sides of similar figures are ____________________.** Note that each ratio is equivalent to the ____________ ____________.	 **Angles:**
Scale Factor of Similar Figures	**Use the simplified ______________ of two corresponding _________, in the order of triangles given or requested.** $\frac{\text{1st named shape}}{\text{2nd named shape}}$ or $\frac{\text{small shape}}{\text{large shape}}$ We could think of similar figures as the result of a dilation, but we DO NOT know which is the image and which is the pre-image.	**Sides:** **Scale Factor of ΔABC to ΔDEF:** **Scale Factor of ΔDEF to ΔABC:**

For #1 – 4: Given that $\triangle ABC \sim \triangle QRS$.

1) Write a proportion comparing the sides of the two triangles.



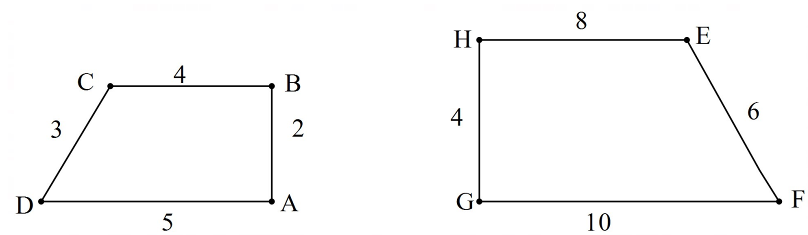
2) What is the scale factor of ΔABC to ΔQRS? $\dfrac{\text{1st named }\Delta}{\text{2nd named }\Delta}$

3) What is the scale factor of ΔQRS *to* ΔABC? $\dfrac{\text{1st named }\Delta}{\text{2nd named }\Delta}$

4) Complete the statement: $\angle B \cong$_______.

You Try #5 – 8! Given that ABCD ~ GHEF.

5) Write a proportion comparing the sides of the quadrilaterals.



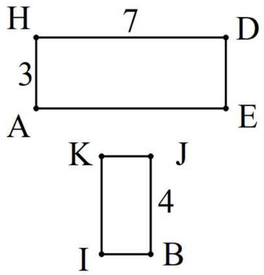
6) What is the scale factor of ABCD to GHEF? $\dfrac{\text{1st named quad}}{\text{2nd named quad}}$

7) What is the scale factor of GHEF to ABCD? $\dfrac{\text{1st named quad}}{\text{2nd named quad}}$

8) Complete the statement: $\angle C \cong$_______.

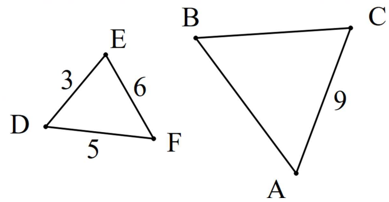
For #9 – 10, find the scale factor (small figure to large figure) for each set of similar figures below.

9) Given: AEDH ~ KIBJ $\frac{\text{small figure}}{\text{large figure}}$



You try!

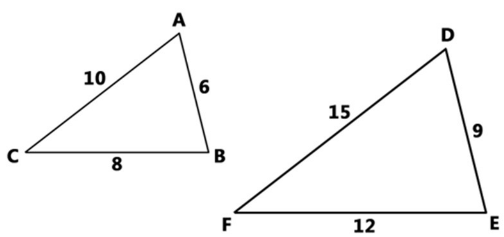
10) Given: ΔDEF~ΔBAC $\frac{\text{small figure}}{\text{large figure}}$

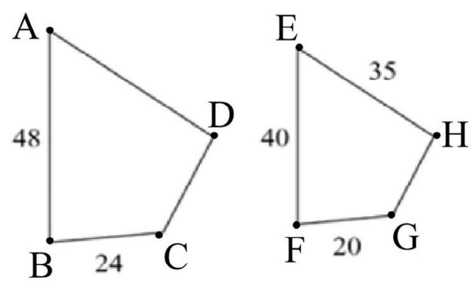


For #11 – 12: Find the scale factor of each pair of similar figures. Use the requested order.

11a) large: small 11b) small: large

12a) ABCD to EFGH 12b) EFGH to ABCD





You Try #13 – 15!

13) Which statements below are **TRUE**, given that **ΔPQR~ΔHKG**? Select all that apply.

A) $\angle P \cong \angle H$

B) $\angle G \cong \angle Q$

C) $\frac{PQ}{HK} = \frac{QR}{KG} = \frac{PR}{HG}$

D) $PQ \cong HK$

14) Given that $\Delta\mathbf{WXY} \sim \Delta\mathbf{DFE}$, then complete each statement below.

$\angle Y \cong$ ________

b. $\frac{\mathrm{WX}}{\mathrm{DF}} = \frac{?}{\mathrm{DE}}$

c. $\angle D \cong$ ________

15) Given that $\Delta\boldsymbol{CDE} \sim \Delta\boldsymbol{KLM}$, and the scale factor of $\Delta CDE\ to\ \Delta KLM$ is $\frac{3}{7}$.

a. Find the scale factor of $\Delta KLM\ to\ \Delta CDE$.

b. Which triangle is larger? How do you know?

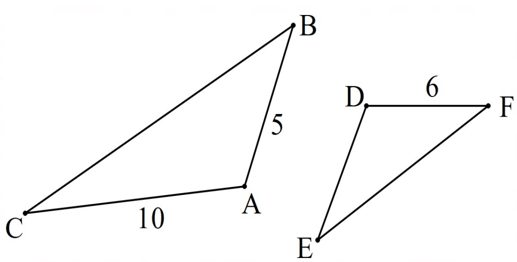
6.2 Day 2 Notes: Similar Figures

Objectives:
- **Students will be able to identify similar figures.**
- **Students will be able to solve problems involving similar figures.**
- **Students will be able to find the scale factor between similar figures.**

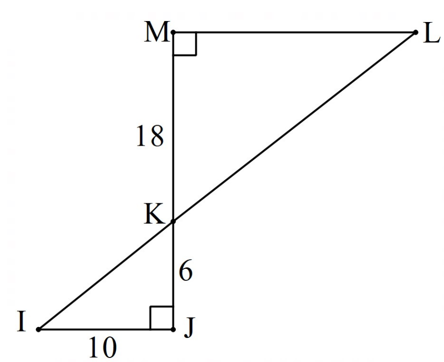
Finding sides with similar figures	**If two figures are similar, then the sides are ________________________.**	1. Use the similarity statement to write a proportion with the sides. 2. Substitute values from the diagram. 3. Cross-multiply to solve.
Finding angles with similar figures	**If two figures are similar, then the corresponding angles are _______________.**	1. Use the similarity statement to decide which angles are congruent. 2. Write an equation setting the corresponding congruent angles equal. 3. Solve using algebra.

For #1 – 4: Given each pair of similar figures, find the requested side.

1) Find DE if $\Delta CBA \sim \Delta EFD$.

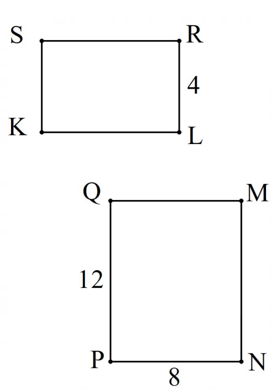


2) Find ML if $\Delta IJK \sim \Delta LMK$.

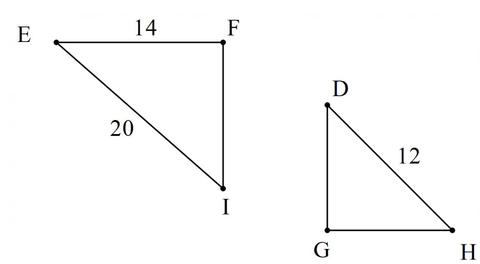


You try #3 – 4!

3) Find KL if $KLRS \sim QPNM$.



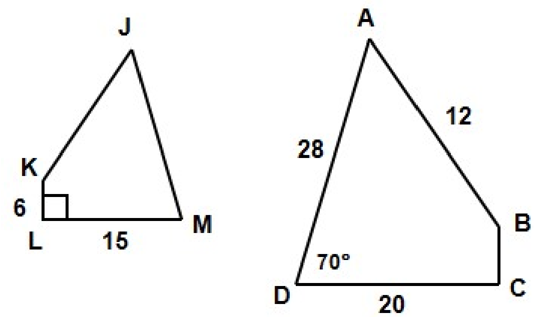
4) Find GH if $\Delta EFI \sim \Delta HGD$.



For #5 – 7: Given that $JMLK \sim ADCB$.

5) Find $m\angle M$.

6) Find $m\angle C$.

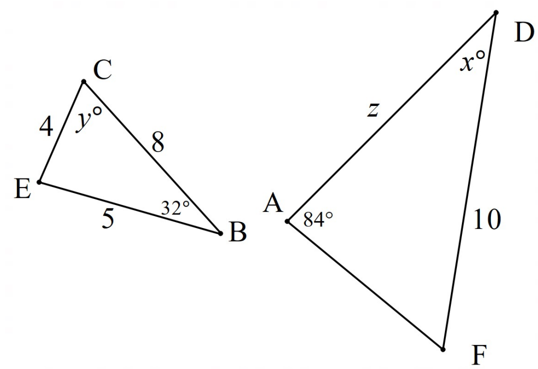


7) Find the length of JM.

You Try #8 – 10! **Given that $\Delta BCE \sim \Delta DFA$.**

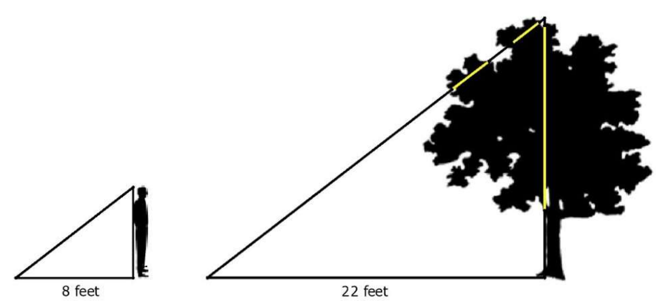
8) Find x.

9) Find y.

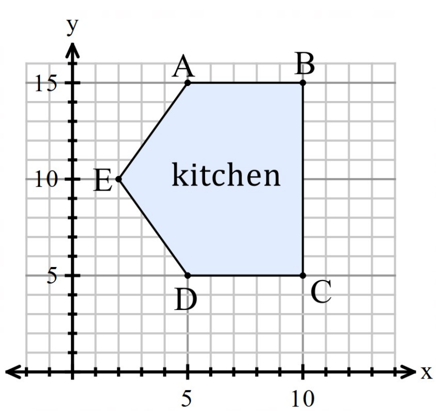


10) Find z.

11) A man who is 6 feet tall casts a shadow that is 8 feet long. At the same time, a tree's shadow is 22 feet long. Assuming the triangles shown below are similar, find the height of the tree.



12) An architect's blueprint of a home being constructed is a scaled drawing with all similar figures to the actual home. The ratio of drawings on the blueprint to the home being built is 1 inch to 2.1 feet. The diagram below shows the blueprint of the kitchen. Find the length of the longest wall in the kitchen if each square on the grid represents one inch.



6.3 Notes: Proving Similar Triangles

Objectives:

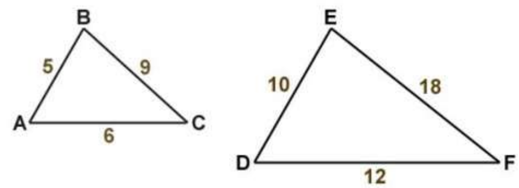
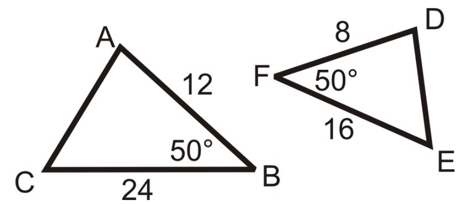
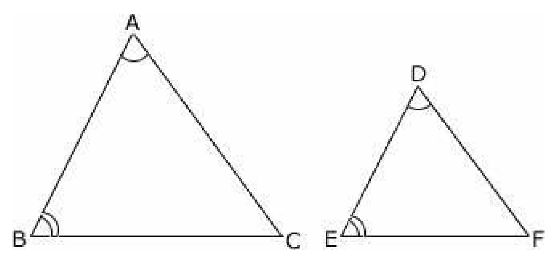
- **Students will be able to decide if triangles are similar.**
- **Students will be able to solve problems using similar triangles.**

Explorations: **Use the following links to explore similar triangles: Follow the directions below.**

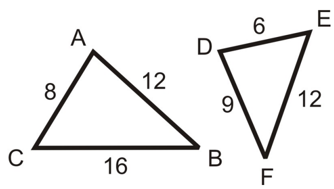
- **#1:** https://www.geogebra.org/m/DfZCQQAa What happens when two triangles have proportional sides?

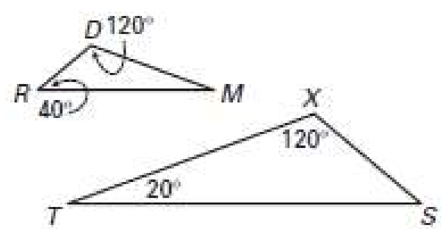
- **#2:** https://www.geogebra.org/m/Ksvpuvds What happens when two triangles have two pairs of congruent angles?

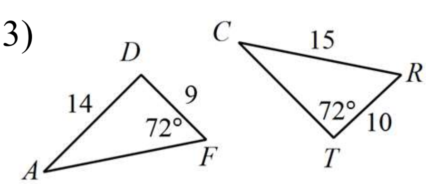
- **#3:** https://www.geogebra.org/m/jE9AKzZp What happens when two triangles have two pairs of proportional sides and one pair of congruent included angles?

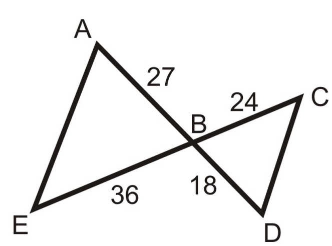
SSS~ Postulate	**If two triangles have three pairs of ______________________ sides, then the triangles are similar.**	
SAS~ Theorem	**If two sides in one triangle are ____________________ to two sides in another triangle, and if their included angles are _______________, then the triangles are similar.**	
AA~ Postulate	**If two angles in one triangle are _____________ to two angles in another triangle, then the triangles are similar.**	

For #1 – 8: Are the pair of triangles shown similar? If so, by what postulate or theorem? Choose from SSS ~, SAS ~, or AA ~.

1) 

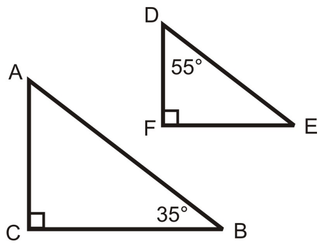
2) 

3) 

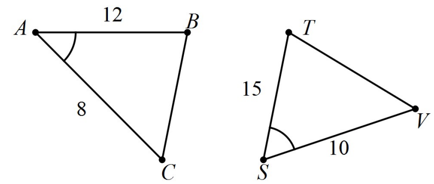
4) 

You try #5 – 8!

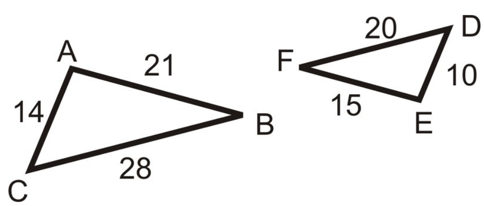
5)



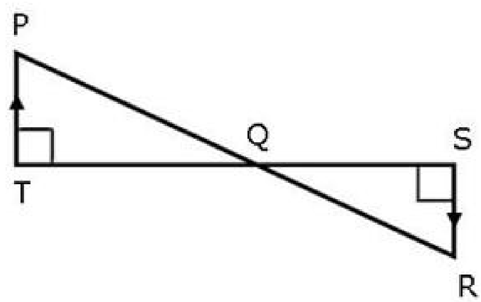
6)



7)

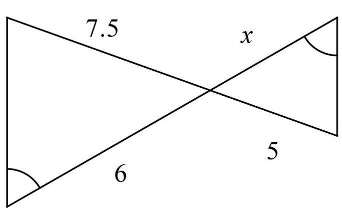


8)

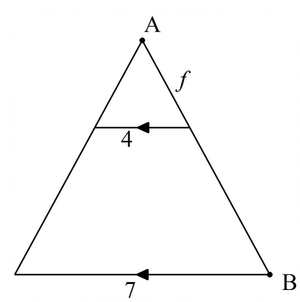


For #9 – 14: Find the value of the variable.

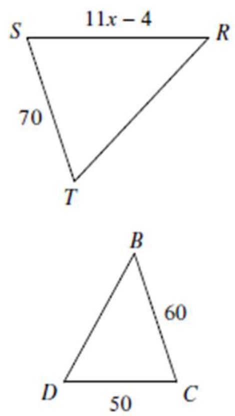
9)



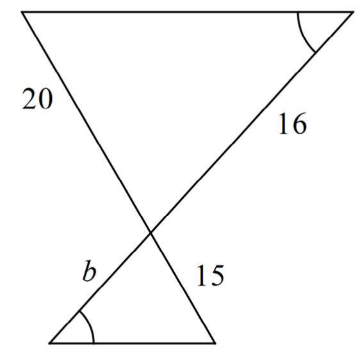
10) If $AB = 10$, find f.



11) Given that $\Delta RST \sim \Delta DCB$, find x.

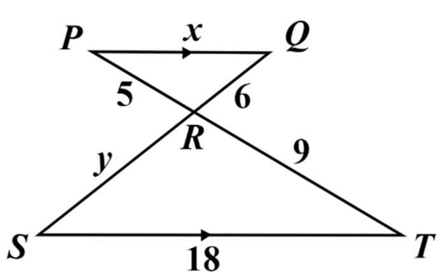


You try #12! 12) Find b.

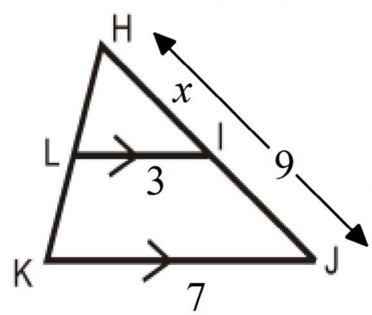


You try #13 – 14!

13) Find the missing variables.

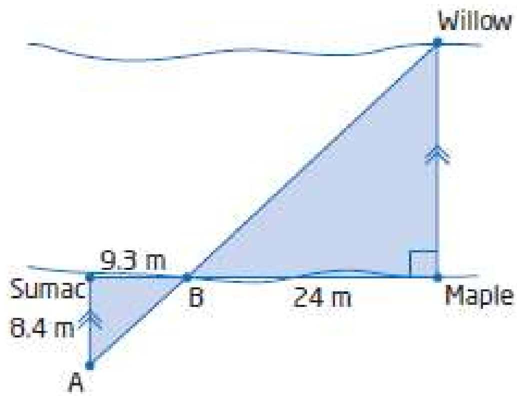


14) Find x.



15) To determine the width of a river, Naomi finds a willow tree and a maple tree that are directly across from each other on opposite shores. Using a third tree on the shoreline, Naomi plants two stakes, A and B, and measures the distances shown.

Find the width of the river using the information that Naomi found.

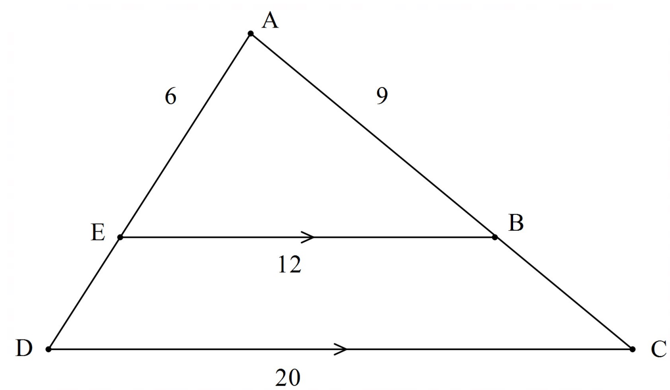


6.4 Notes: Proportional Parts

Objectives:

- **Students will use proportional parts of triangles to solve problems.**
- **Students will use similar triangles to solve problems.**

Guided Exploration #1: Consider the diagram below. Reminder: If two lines are //, then corresponding angles are congruent.



a) Mark any congruent angles.

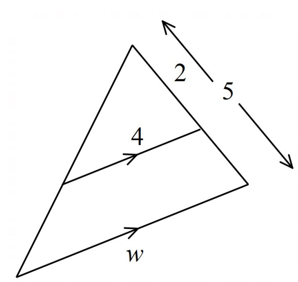
b) Which triangles are similar in the diagram?

c) Find the scale factor (small to big).

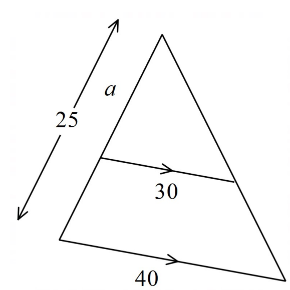
d) Use a proportion to find the lengths of AD and AC.

For #1 – 4: Find the missing variable.

1)

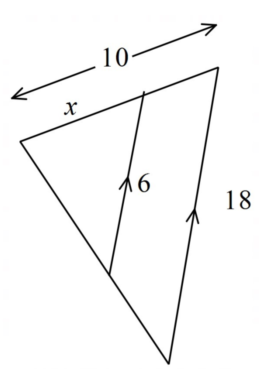


2)

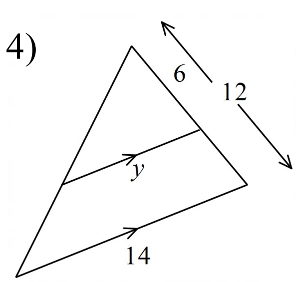


You try #3 - 4!

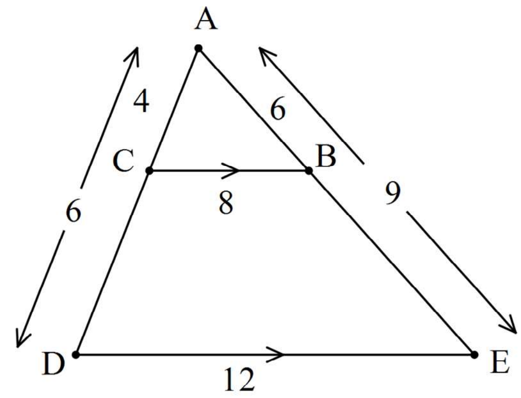
3)



4)



Guided Exploration #2: Consider the diagram below, which has two similar triangles by AA~.

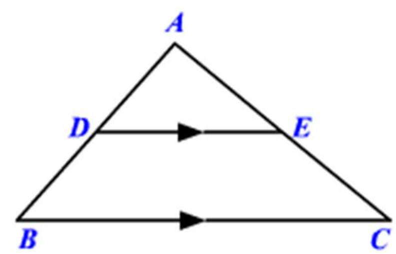


a) Find the scale factor of ΔABC to ΔAED.

b) Find the lengths of CD and BE.

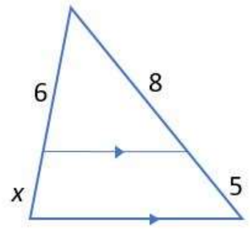
c) Find the ratio of $\frac{AC}{CD}$ and $\frac{AB}{BE}$. What do you notice?

d) Find the ratio of $\frac{BC}{DE}$? Is it the same as your answer for part c? For part a?

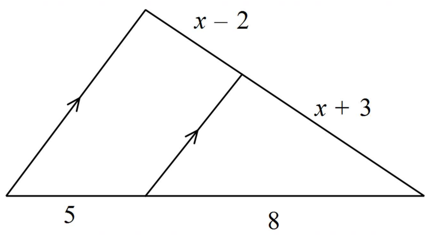
Triangle Proportionality Theorem	**If a triangle has two sides intersected by a line parallel to the 3rd side of the triangle, then the intersected sides are split into ______________________ ___________________.**	 If $DE \parallel BC$, then $\frac{AD}{DB} = \frac{AE}{EC}$. Note: The parallel sides are *not* proportional to these parts. The parallel sides are proportional to the *scale factor* of the similar triangles, which is not the same ratio.

For #5 – 11: Find each missing variable.

5)

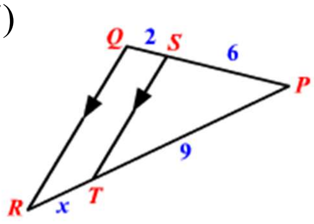


6)

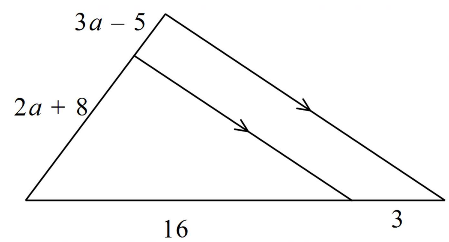


You try #7 - 8!

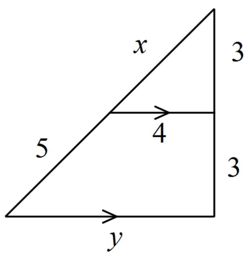
7)



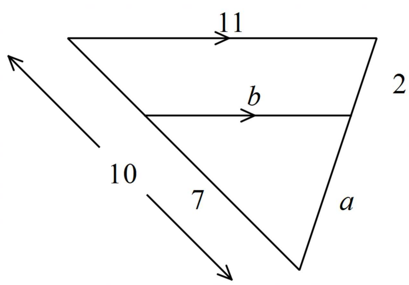
8)



9)

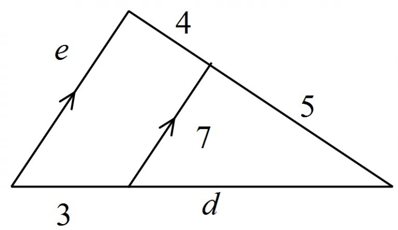


10)



You try #11!

11) Find *d* and *e*.

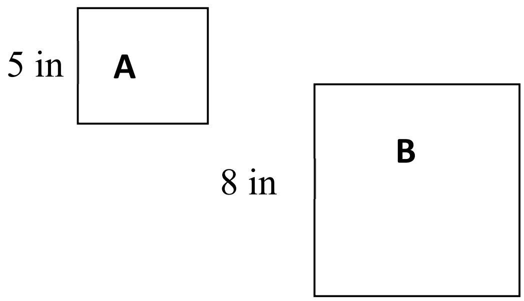


6.5 Notes: Perimeter & Area of Similar Polygons

Objectives:

- **Students will be able to write ratios for perimeters and areas of similar figures.**
- **Students will be able to find the area and perimeter of similar figures using proportions.**

Exploration: **Consider the squares below. Find the perimeter and area of each square.**



	Square A	Square B
Perimeter		
Area		

1) What is the scale factor (A to B)?

2) What is the ratio of the perimeters (A to B)? Make sure you <u>reduce your ratio</u>.

3) What is the ratio of the areas (A to B)?

4) What do you notice about your answers from #1 and #2?

$$\textit{scale factor} = \frac{\quad}{\quad}$$

$$\textit{ratio of perimeters} = \frac{\quad}{\quad}$$

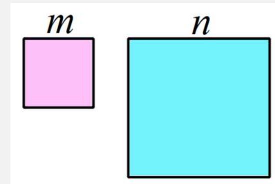
5) Is there a relationship between your answers for #1 and #3?

$$\textit{scale factor} = \frac{\quad}{\quad}$$

$$\textit{ratio of areas} = \frac{\quad}{\quad}$$

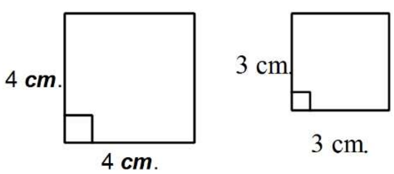
***Reminder*:** **Scale factor is the ratio of two corresponding sides of similar figures.**

$$\boldsymbol{scale\ factor}: \frac{\boldsymbol{side}}{\boldsymbol{side}}$$

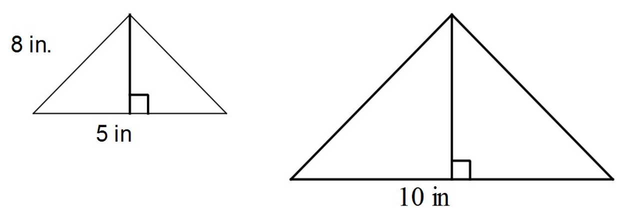
<table>
<tr><td colspan="3">Given that two similar figures have a scale factor of $\frac{m}{n}$, then…
</td></tr>
<tr><td>Ratio of Perimeters of Similar Figures</td><td>If the scale factor of two similar figures is $\frac{m}{n}$, then the ratio of their perimeters is also ______.</td><td>Ratio of perimeters = scale factor
$\frac{perimeter}{perimeter} = \frac{m}{n} = scale\ factor$</td></tr>
<tr><td>Ratio of Areas of Similar Figures</td><td>If the ratio of areas of two similar figures is $\frac{m}{n}$, then the ratio of their areas is $(\quad\quad)^2$.</td><td>Ratio of areas = $(\boldsymbol{scale\ factor})^2$
$\frac{area}{area} = \left(\frac{m}{n}\right)^2 = (scale\ factor)^2$</td></tr>
</table>

For #1 a-b: For each pair of similar figures, find the ratio of the perimeter and the ratio of the area (small to big).

a)



b)



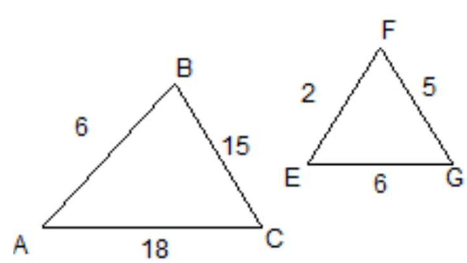
For #2 a-d: Given that $\Delta ABC \sim \Delta EFG$. Find the request ratios.

a) scale factor (ΔABC to ΔEFG)

b) ratio of perimeters (ΔABC to ΔEFG)

c) scale factor (ΔEFG to ΔABC)

d) ratio of areas (ΔEFG to ΔABC)



You try #3 a-c! Given that two similar figures have corresponding sides of $6in$ and $7in$.

a) Find the scale factor (big to small).

b) Find the ratio of the perimeters (big to small).

c) Find the ratio of the areas (big to small).

Missing Triangle Parts?!?! ***** You CAN find them! *****		
To find a missing perimeter	**Set up a proportion with the scale factor equal to the perimeters.**	**Ratio of perimeters = *scale factor*** $\frac{m}{n} = \frac{perimeter}{perimeter}$
To find a missing area	**Set up a proportion with the scale factor ________________ equal to the areas.**	**Ratio of areas =** $(scale\ factor)^2$ $\left(\frac{m}{n}\right)^2 = \frac{area}{area}$

4) The perimeter of ΔABC is 12 and $\Delta \boldsymbol{ABC} \sim \Delta \boldsymbol{XYZ}$. Find the perimeter of ΔXYZ if the scale factor of ΔABC to ΔXYZ is 2:3.

You try #5! The perimeter of ΔXYZ is 27 and $\Delta \boldsymbol{ABC} \sim \Delta \boldsymbol{XYZ}$. Find the perimeter of ΔABC if the scale factor of ΔABC to ΔXYZ is 5:4.

6) Two rectangles are similar, and their scale factor is 3:5. What is the area of the smaller rectangle if the larger rectangle has an area of 50 m^2?

You try #7! Two similar figures have corresponding sides of lengths 9 inches and 11 inches. The smaller figure has an area of 35 inches squared. Find the area of the larger figure.

8) Two triangles are similar and the ratio of each pair of corresponding sides is $2:1$. Which statement regarding the two triangles is <u>not true</u>? Assume that each ratio represents the larger triangle first.

A. Their areas have a ratio of $4:1$

B. The scale factor is a ratio of $2:1$

C. Their perimeters have a ratio of $2:1$

D. Their corresponding angles have a ratio of $2:1$

For #9 a-b: Two similar figures have areas of 25 mm^2 and 36 mm^2. Find the requested ratios.

a) Scale factor (small to big)

b) Ratio of perimeters (big to small)

Challenge Problems! These are problems that could prepare you for a *bonus* on the test.

For #10 a-c: Two similar figures have areas of 8 mm^2 and 18 mm^2. Find the requested ratios.

a) Scale factor (small to big)

b) Ratio of perimeters (big to small)

c) If the perimeter of the smaller figure is 24 mm, then find the perimeter of the larger figure.

You Try #11! The ratio of the areas of square A to square B is $\frac{16}{25}$. If square B has one side of 10 cm, then what is the length of a side of square A?

A. 4 cm

B. 8 cm

C. 10 cm

D. 64 cm

Ch 6 Study Guide

- 6.1:
  - Dilations of figures.
    - Scale factor = $\frac{\text{image}}{\text{pre-image}}$
    - Coordinates of the image of a dilation centered at the origin can be found by multiplying the coordinates of the pre-image by the scale factor.
    - Enlargement: $scale\ factor > 1$
    - Reduction: $0 < scale\ factor < 1$
- 6.2:
  - Similar figures have the same shape, but not necessarily the same size.
  - Corresponding angles of similar figures are congruent.
  - Corresponding sides of similar figures are proportional.
  - Scale factor = ratio of corresponding sides (reduced)
- 6.3:
  - Methods to prove that triangles are similar are:
    - SSS~ (all three pairs of corresponding sides are proportional)
    - SAS~ (two pairs of corresponding sides are proportional, and the included angles are congruent)
    - AA~ (two pairs of angles are congruent.)
  - To solve for a missing side: set up a proportion with the corresponding sides
  - To solve for a missing angle: set it equal to its corresponding angle
- 6.4:
  - Similar triangles are formed when a triangle is intersected by a line parallel to one side.
  - Triangle Proportionality Theorem: If two sides of a triangle are intersected by a line parallel to the 3rd side, then the intersected sides are split into proportional parts.
    - Note: the parallel sides are not proportional to those parts.
    - Note: the parallel sides are proportional to the scale factor of the similar triangles.
- 6.5:
  - Scale factor is equal to the ratio of sides which is equal to the ratio of perimeters
  - The ratio of areas is equal to the ratio of perimeters squared
  - Set up a proportion to solve for missing side, perimeter, or area