Ch. 7 Notes: Right Triangles

7.1 Notes: The Pythagorean Theorem

Objectives:

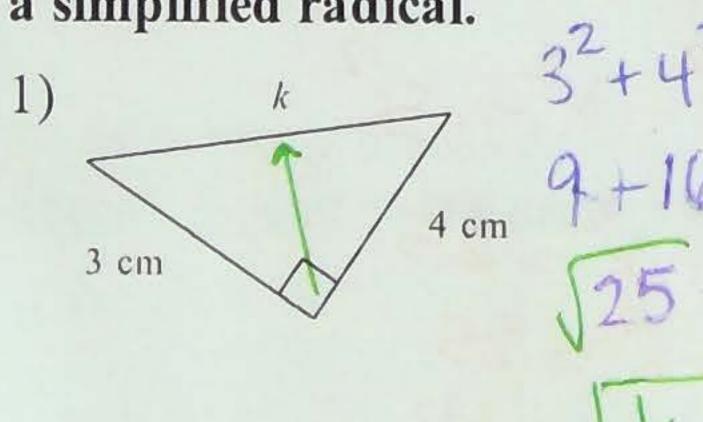
- Students will be able to find the missing side of a right triangle.
- Students will use the Pythagorean Theorem to solve problems.

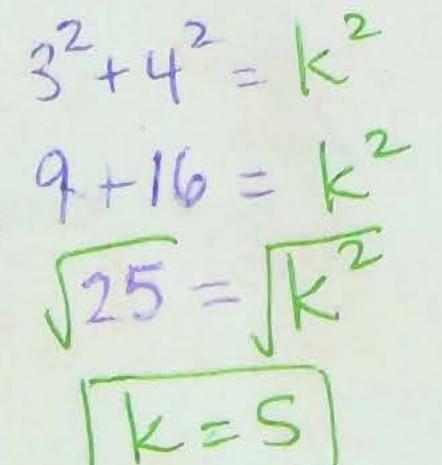
Video Demonstration of the Pythagorean Theorem:
https://knpb.pbslearningmedia.org/resource/mgbh-math-ee-gshreepythag/pythagorean-theorem/

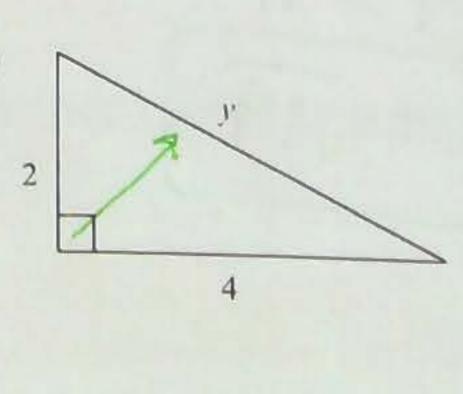
• Why does the Pythagorean Theorem work? Explain it in your own words.

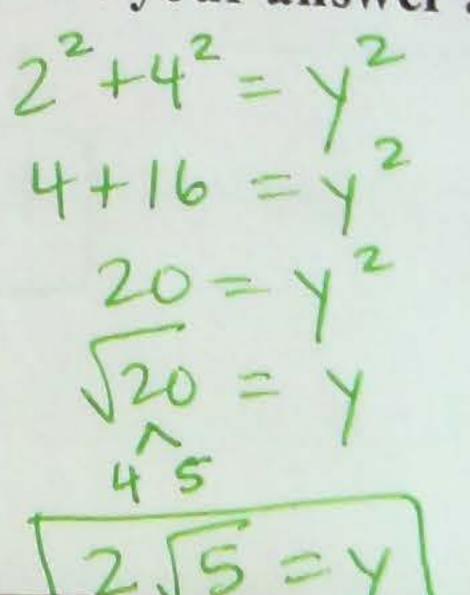
Right Triangle	A right triangle has right angle. The side opposite the right angle is called the, it is the longest side of the triangle. The other two sides are called the	leg hypotenuse right angle leg
The Pythagorean Theorem	The Pythagorean Theorem states that the sum of the Squares of the legs equals the square of the hypotenuse. Note: This only works for RIGHT triangles!	$a = c$ b $a^2 + b^2 = c^2$

For #1-8: Find the missing side for each right triangle. If needed, write your answer as a simplified radical.





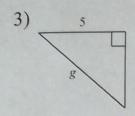




Ch. 7 Notes: Right Triangles

DRHS

You try #3 - 4!

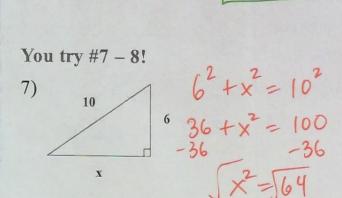


$$5^2+5^2=9^2$$

25+25=9²

$$25+25=9^{2}$$
 $50=9^{2}$
 $9=\sqrt{50}$
 $9=5\sqrt{2}$
 25^{2}

4)
$$\frac{9^{2}+12}{81+14!}$$
 $\frac{81+14!}{225}$ $\frac{12}{12}$



$$8^{2} + y^{2} = |2|$$

$$64 + y^{2} = |44|$$

$$64 + y^{2} = |44|$$

$$64 + y^{2} = |44|$$

$$y^{2} = |80|$$

$$y = |65|$$

$$y = |89|$$

8) Write your answer as a decimal, to one place.

$$x^{2}+15^{2}=21^{2}$$
 x
$$x^{2}+225=441$$

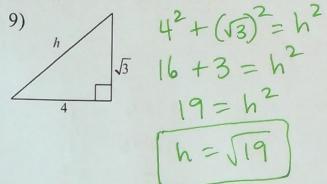
$$-225$$

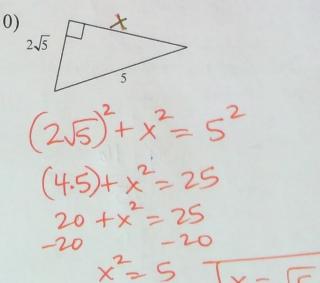
$$-225$$

$$x^{2}=5216$$

$$x=14.7$$

For #9 – 12: Find the missing side in each right triangle. If needed, write your answer as a simplified radical.

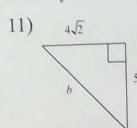




Ch. 7 Notes: Right Triangles

DRHS

You try #11 - 12!



 $\begin{bmatrix} (4\sqrt{2})^2 + 5^2 = b^2 \\ (4\sqrt{2})^2 + 25 = b^2 \\ 32 + 25 = b^2 \end{bmatrix}$

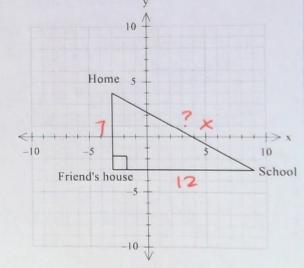
$$57 = 6^{2}$$
 $57 = 6^{2}$
 $6 = \sqrt{57}$

13) A triangle has side lengths of 6, 8, and 10. Is the triangle a right triangle? Explain your reasoning.

Toes $6^2 + 8^2 = 10^2$? If so, it is a right Δ

$$36+64=100$$
 $100=100$
 $yes, it is$

14) Each day, Amy walks to school. She leaves her home and walks south 7 blocks to her friend's house. They then turn to the west and walk 12 blocks to the school. At the end of the day, Amy walks directly from the school to her home.



a) To the nearest tenth (one decimal place), how many blocks does Amy walk on her way home?

$$7^{2} + 12^{2} = x^{2}$$
 $49 + 144 = x^{2}$
 $\sqrt{193} = \sqrt{x^{2}}$
 $\sqrt{13.9}$ blocks

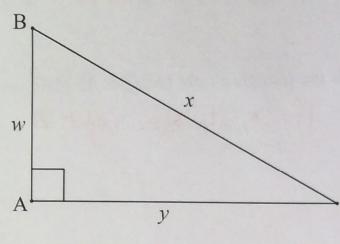
b) How many blocks does Amy walk in one day for her round trip to the school and back home? Round to the nearest tenth.

7.1 Remediation

Part 1: When to use an upper-case letter or lower-case letter in Geometry.

- Lower case letters are usually used to represent numbers and variables:
 - o unknown side lengths, scale factors & parts of polygons
- Upper case letters are usually used to name points (vertices) and answers in a formula, capital letters are used in naming segments, rays, and lines:
 - o area formula, line name, point name

Example 1: Use the following triangle to complete the statements below.



a) Write the Pythagorean Theorem for $\triangle ABC$.

$$W^2 + y^2 = x^2$$

b) Name the hypotenuse and right angle.

c) Write the area formula using the variables (sides) of this triangle.

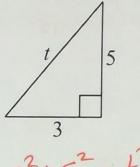
Part 2: Setting up the Pythagorean Theorem Practice

- The Pythagorean Theorem is $a^2 + b^2 = c^2$,
 - \circ a & b are the legs and c is the hypotenuse
- Using the parts of the triangle (legs and hypotenuse) to represent the Pythagorean Theorem:

$$o leg^2 + leg^2 = hypotenuse^2$$

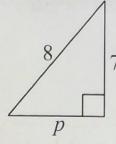
Example 2: Set up the Pythagorean Theorem for each triangle below. Do not solve.

a)

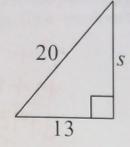


$$3^2 + 5^2 = t^2$$

b)



C



Part 3: Squaring a Radical

1.
$$(\sqrt{x})^2 = \sqrt{x} \cdot \sqrt{x} = x$$

$$\circ (\sqrt{9})^2 = \sqrt{9} \cdot \sqrt{9}$$

$$3 \cdot 3 = 9$$

2.
$$(a\sqrt{b})^2 = (a\sqrt{b})(a\sqrt{b})$$

= $(a \cdot a \cdot \sqrt{b} \cdot \sqrt{b})$
= $a^2 \cdot b$

Example 3: Solve.

a)
$$(\sqrt{11})^2$$

$$\sqrt{11} \cdot \sqrt{11} = \boxed{1}$$

b)
$$(\sqrt{21})^2 = 21$$

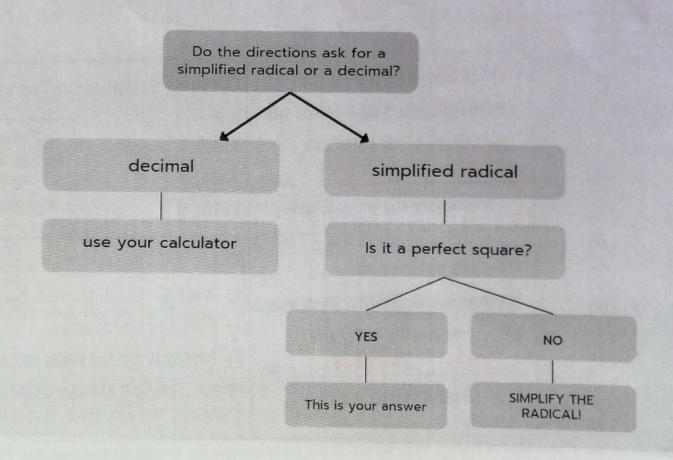
$$(\sqrt{11})^2$$
 b) $(\sqrt{21})^2 = 21$ c) $(2\sqrt{7})^2$ d) $(3\sqrt{8})^2$ $2^2 \cdot 1 = 4 \cdot 7 = 3^2 \cdot 8 = 7$

d)
$$(3\sqrt{8})^2$$

 $3^2 \cdot 8 = 12$
 $9 \cdot 8 = 12$

Part 4: When to simplify a radical and when to give a decimal answer.

Simplified Radical or Decimal Answer?



7.2 Notes: Pythagorean Theorem Triples

Objectives:

- Students will be able to use Pythagorean Triples to find missing sides in right triangles.
- Students will be able to solve problems involving Pythagorean Triples.

Exploration:

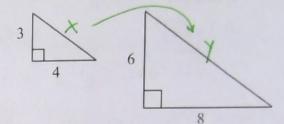
Consider the two triangles shown. Are they similar? What theorem or postulate did you use to make this decision? (Consider AA~, SAS~, and SSS~.)

Small

brg

Find the missing hypotenuse of the smaller triangle



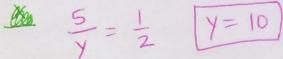


Find the missing hypotenuse of the smaller triangle. Hint: use the Pythagorean Theorem.

 $3^{2}+4^{2}=x^{2}$ $\sqrt{25}=x^{2}$ $9+16=x^{2}$ x=5

Without using the Pythagorean Theorem, find the missing hypotenuse of the larger triangle. What do you notice about the sides of these triangles?





Pythagorean Theorem, as well.

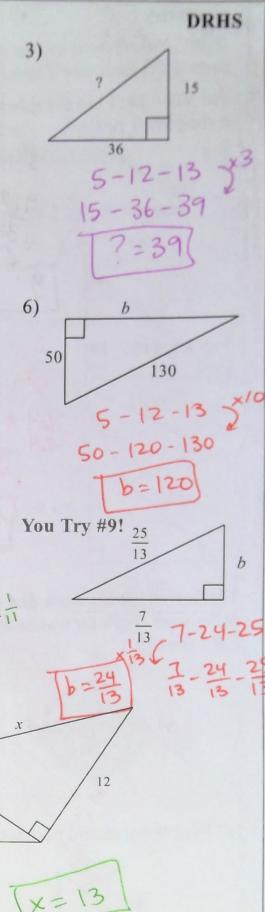
	If a right triangle is a	Common Pythagore	an Triples:
	Pythagorean Triple, then all	3, 4, _5_	
Pythagorean Triples	three of its sides are	5, 12, 13 Note: the largest num	Note: the
2775765	positive integers.	7, 24, 25	is the
	Memorize the ones listed!	8, 15,	hypotenuse.
	If a triangle is a dilation of a	Steps:	
Using	Pythagorean Triple, then you	1) Identify which tri	ple is being used.
Pythagorean Triples	can multiply by the	2) Find the scale factor.	
	Scale factor to	3) Multiply by the scale factor.	
	find a missing side.	Note: You can always use the	

For #1 - 9: Find the missing side of each right triangle by using Pythagorean Triples.

Ch. 7 Notes: Right Triangles Geometry 1) 40 50 14 30 You try #4 - 6! 5) 15 11

10) Use Pythagorean Triples to find x.

3

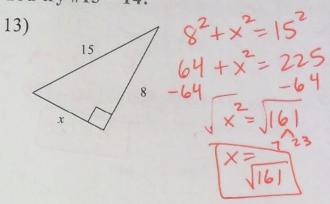


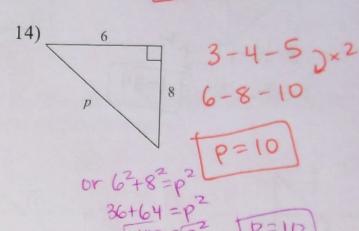
Note: Not all right triangles are Pythagorean Triples. If the sides of a right triangle do not form a Pythagorean Triple, then use the Pythagorean Theorem to find a missing side.

For #11-14: Find the missing side for each right triangle. If needed, write your answer as a simplified radical.

11) $3^{2}+5^{2}=9^{2}$ $3^{2}+5^{2}=9^{2}$ $3^{3}+5^{2}=9^{2}$ $3^{3}+5^{2}=9^{2}$ $3^{3}+5^{2}=9^{2}$ $3^{3}+5^{2}=9^{2}$ $3^{3}+5^{2}=9^{2}$ $3^{3}+5^{2}=9^{2}$ $3^{3}+5^{2}=9^{2}$

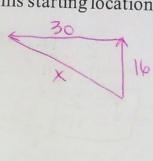
You try #13 - 14!





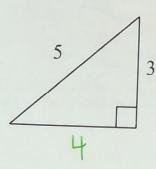
15) Garrett left his work and drove 16 miles north. He then turned 90 degrees and drove 30 miles west. How far was he from his starting location? (Hint: label the compass then draw a right triangle.)

Tangle.)



$$8-15-17$$
 $16-30-34$
 $X=34$ miles away

16) Find the area and perimeter of the right triangle shown below.



$$P = 5 + 4 + 3 = 12$$

$$A = \frac{1}{2} \cdot 4 \cdot 3 = 6$$

7.3 Notes: Special Right Triangles

Objectives:

- Students will be able to identify special right triangles.
- Students will be able to find missing sides of special right triangles.

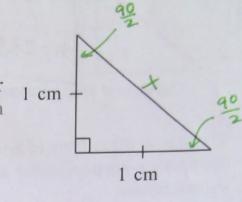
Exploration: Consider an isosceles right triangle, with each side of 1 cm, as shown.

a) Find the measures of all angles of the triangle.

90°, 45°, 45°

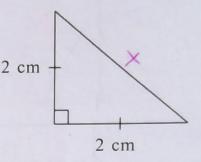
b) Find the length of the hypotenuse of the triangle. Write your answer as a simplified radical. Hint: Use the Pythagorean Theorem.

 $|x^2 + |^2 = x^2$ $|x^2 + |^2 = x^2$ $|x^2 + |^2 = x^2$ $|x^2 + |^2 = x^2$



c) Change the length of the legs to 2 cm each. Do the angles change?

d) Using the new length of 2cm for each leg, find the length of the hypotenuse. Write your answer as a simplified radical. Hint: Use the Pythagorean Theorem.



 $2^{2} + 2^{2} = x^{2}$ $4 + 4 = x^{2}$ $\sqrt{8 = x^{2}}$

If a triangle is a 45-45-90 triangle

(these are the angle measurements),

then the triangle is an isosceles

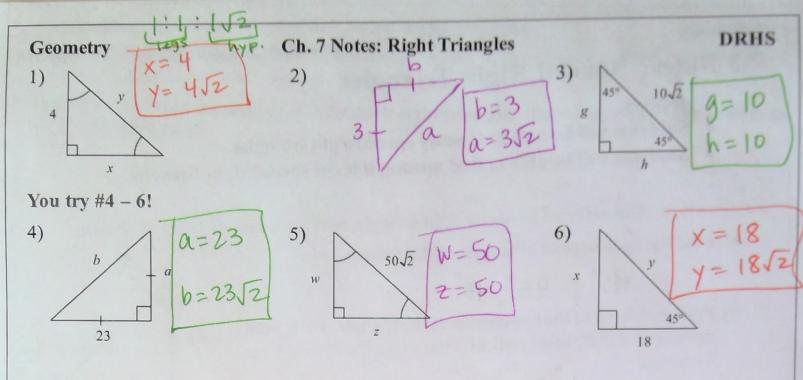
right triangle.

45-45-90 Triangles The sides of a 45-45-90 triangle are in the ratio of

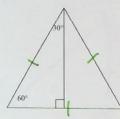
1:11:152

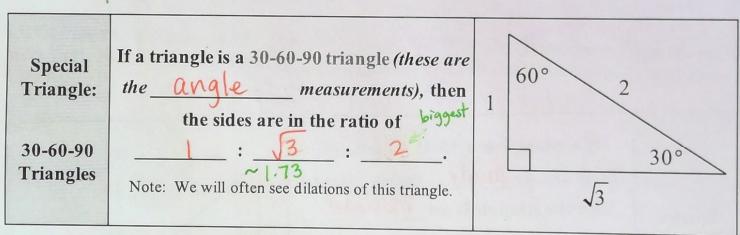
Note: We will often see dilations of this triangle.

For #1-6: For each 45-45-90 triangle, find the lengths of the missing sides.

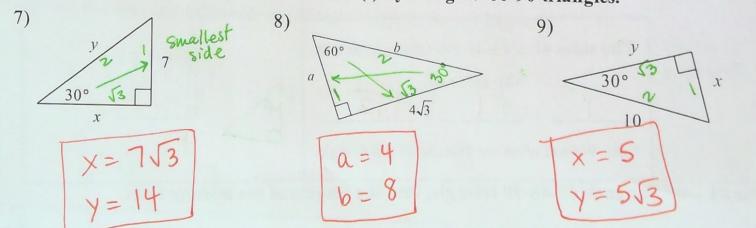


There is another type of special right triangle, whose side relationships can be found by dropping a perpendicular segment from one vertex of an equilateral triangle to the base, as shown below.





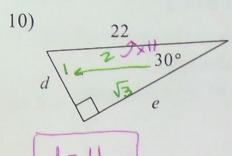
For #7 - 9: Find the value of the variable(s) by using 30-60-90 triangles.

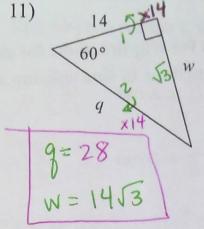


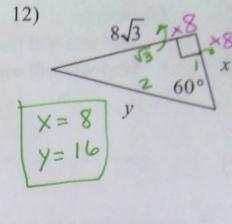
Ch. 7 Notes: Right Triangles

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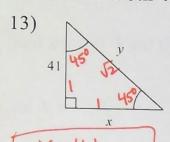
You try #10 - 12! Find the value of the variable(s) by using 30-60-90 triangles.

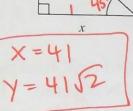


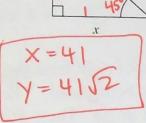


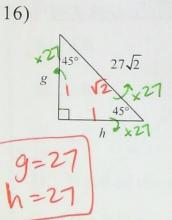


You try #13 - 18! Find the value of the variable(s) by using special right triangles. Be careful! Both 45-45-90 and 30-60-90 triangles are below.

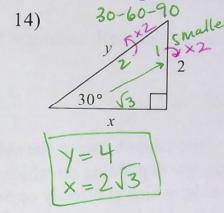


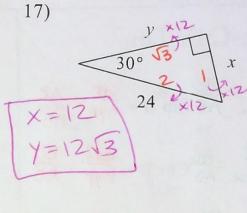


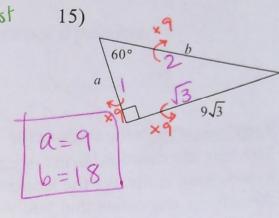


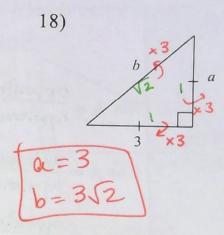


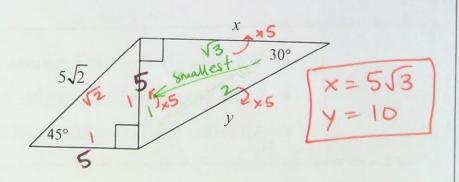
19) Challenge: Find x and y.











7.4 Notes: Right Triangle Trigonometry, Part I

Objectives:

- Students will find ratios for trig functions for right triangles.
- Students will use trig functions to find missing sides of right triangles.

<u>Trigonometry</u> is the study of the relationships between the sides and angles of right triangles. The legs are called *adjacent or opposite* depending on which acute angle is being used. The hypotenuse is always the longest side, which is directly opposite the right angle. There are three basic trig ratios that you will need to know: sine, cosine, and tangent.

Trig Functions	Soh-Cah-Toa Sin A = $\frac{\text{opposite}}{\text{hypotenuse}} = \frac{BC}{AB} \sin B = \frac{\text{opp.}}{\text{hyp}} = \frac{AC}{AB}$ Cos A = $\frac{\text{adjacent}}{\text{hypotenuse}} = \frac{AC}{AB} \cos B = \frac{adj.}{\text{hyp}} = \frac{BC}{AB}$ Tan A = $\frac{\text{opposite}}{\text{adjacent}} = \frac{BC}{AC} \tan B = \frac{\text{opp.}}{\text{adj.}} = \frac{AC}{BC}$	B C A
Helpful hints	 We will only find trig functions of acute Label your sides as Opposite, Adjacent, All ratios must be reduced and rationalized 	and Hypotenuse.

SOH – CAH – TOA

1) Find the requested trig ratio. Reduce your answers, if possible.

$$\sin A = \frac{OPP}{hyp} = \frac{6}{10} = \frac{3}{5} \cos A = \frac{adj}{hyp} = \frac{8}{10} = \frac{4}{5}$$

$$\tan A = \frac{OPP}{adj} = \frac{6}{8} = \frac{3}{4}$$

$$\sin C = \frac{8}{10} = \frac{4}{5} \cos C = \frac{6}{10} = \frac{3}{5} \tan C = \frac{8}{6} = \frac{4}{34}$$

$$\tan A = \frac{\text{opp.}}{\text{adj.}} = \frac{6}{8} = \frac{3}{4}$$

$$\tan C = \frac{8}{4} = \frac{4}{3}$$

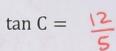
Reminder: We only find trig functions for the <u>acute</u> angles of a right triangle this year!

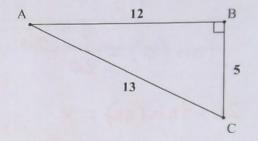
You try #2! Find the requested trig ratio. Reduce your answers, if possible.

$$\sin A = \frac{5}{13} \qquad \cos A = \frac{12}{13}$$

$$\sin C = \frac{12}{13} \qquad \cos C = \frac{5}{13}$$

$$\tan A = \frac{5}{12}$$



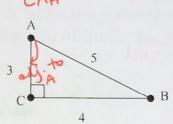


3) Which angle has a cosine of $\frac{3}{5}$?



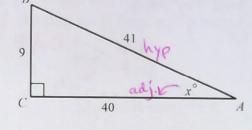


D. None of the above.



You try #4! What is **cos** x in the triangle?





5) Find the following values by using your calculator. Round to 3 decimal places.

Your calculator must be in Degrees!

a. sin 18°

b. cos 68°

c. tan 32°

d. tan 80°

- sin(18)= 0.309
 - 0 375

0.625

5.671

Malculator









Ch. 7 Notes: Right Triangles

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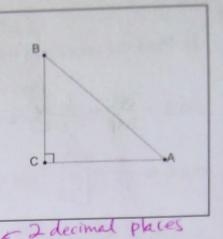
Writing Equations with Trig Functions

Soh-Cah-Toa

- 1) Decide what trig function matches your picture.
- 2) Make an equation.

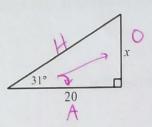
Trig function (angle) = side ratio

3) Solve for the variable by using inverse operations.



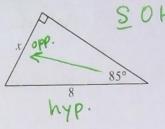
For #3 – 8: find the variable. Round to the nearest hundredth.

3)

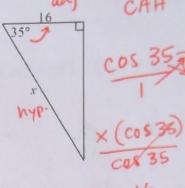


$$+an(31) = \frac{\times}{20.20}$$

4)



adj call

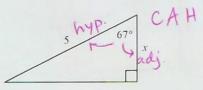


$$\chi = \frac{16}{\cos 35}$$

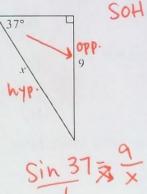
$$X = 19.53$$

You Try #6-8!

6)



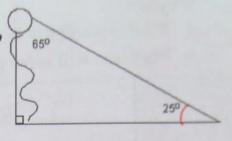
7)



Nyp.

Angle of Elevation: the "line of sight" angle made between an object and the ground.

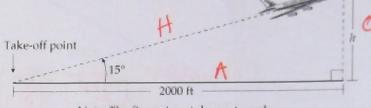
• What is the angle of elevation in the picture shown?



[25°]

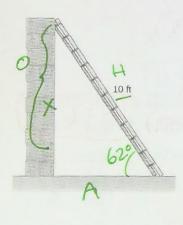
9) An airplane has an angle of elevation of 15 degrees from the runway when it takes off. The airplane pictured below is 2,000 feet along the ground from its take-off point. Find the height, h, of the airplane (round answer to nearest foot).

$$70A$$
 2000 , $tan 15 = h$
 2000



Note: The figure is not drawn to scale.

10) A 10-foot ladder is leaning against the side of a house, with an angle of elevation of 62 degrees. How far up the side of the house does the ladder reach? Round to 2 decimal places.



SoH
10. Sin
$$62 = \frac{x}{10}$$
 ato
 $10(\sin 62) = x$
 $x = 8.8 \text{ ft}$

7.5 Notes: Right Triangle Trigonometry, Part II

Objectives:

- Students will use inverse trig functions to find missing angles of a right triangle.
- Students will solve problems involving trig and inverse trig functions.

Inverse Trig Functions	Soh-Cah-Toa If the variable in an equation involving a trig function is in the angle position, then use an inverse trig function to isolate the variable. sin-1 cos-1 tan-1 tan-1 tan-1 sin-1 cos-1 tan-1 tan-1 sin-1 cos-1 tan-1	B $Sin x = \frac{4}{5}$
Helpful hints	 Use an inverse trig function to find a miss. Most calculators require you to use SHIFT of function. Trig function (angle) = s 	or 2 nd to access an inverse trig

1) Use your calculator to find the angle (round to nearest whole angle):

a. $(\cos B) = (0.5)$

$$(\cos B) = (0.5)$$

 $B = \cos^{-1}(0.5)$ $B = 60^{\circ}$

You try c and d!

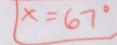
c.
$$(\sin A) = (8990)$$

 $A = Sih^{-1} (.8990)$

b. $(\tan x) = (1.33)$

$$x = \tan^{-1}(1.33)$$
 $x = 53^{\circ}$
 $\cos^{-1}(\cos x) = (0.397)$
 $x = \cos^{-1}(0.391)$

$$\frac{\mathrm{d}}{\cos x} = 0.397$$



2) Use your calculator to find x: Note: decide if you should use the trig function or its inverse. Round to two decimal places.

a. $\sin 60 = x$

just put in calculator = .87

You try c and d!

c. tan 30 = x

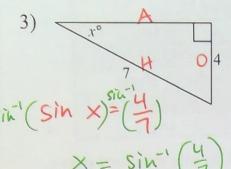
b. $(\cos x) = (.75)$

For #3 – 6: Find x in each picture. Round to one decimal place. $\times = +\infty$

Ch. 7 Notes: Right Triangles

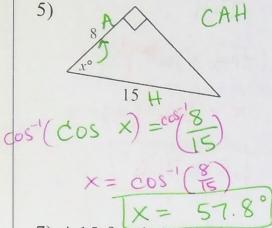
 $Trig\ function\ (angle)\ =\ side\ ratio.$

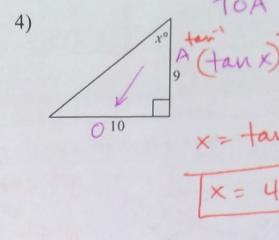
SOH

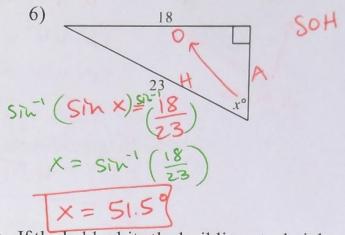


$$x = \sin^{-1}(\frac{4}{5})$$

You Try #5-6!





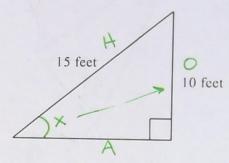


7) A 15-foot ladder is leaning against a building. If the ladder hits the building at a height of 10 feet, find the angle of elevation to two decimal places.

Soft

$$\sin^{2}(\sin x) = (10)$$

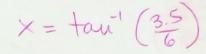
 $x = \sin^{2}(10)$
 $x = \sin^{2}(10)$
 $x = 41.8^{\circ}$

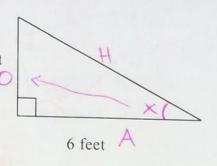


You try #8! A skateboard ramp is 3.5 feet high and 6 feet long along the horizontal. To the nearest degree, what is the measure of the angle of elevation for the ramp?

- A. 27°
- **B**. 60°
- C. 30°
- D. 63°

to A tan' (tan x)= (3.5) 3.5 fee

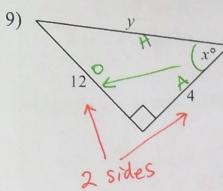




 $x = 30^{\circ}$

Summary of finding parts of a right triangle				
If I know	I should use	to find the		
2 sides	the Pythagorean Theorem	3rd side		
2 51005	inverse trig function	related angle		
A side and an angle	trig function	related side		
/ 11 2 angles	sum of 180 degrees	3rd angle		

For #9 - 11: Solve for the variable(s) in each triangle. Round to one decimal place.



$$3^{rd}$$
 side = $\frac{1}{2^2 + 4^2} = \frac{1}{2^2 + 4^2} = \frac{1}{2^2}$
 $\frac{1}{160} = \frac{1}{2^2}$
 $\frac{1}{160} = \frac{1}{2^2}$

$$x = \frac{\text{related}}{\text{an angle}}$$

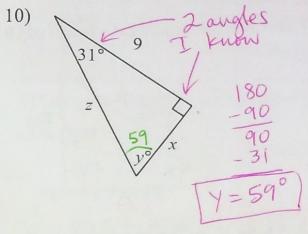
$$X = \frac{\text{Toth}}{\text{fan'}(12)}$$

$$X = \frac{\text{fan'}(12)}{\text{4}}$$

$$X = \frac{\text{Toth}}{\text{4}}$$

$$X = \frac{\text{Toth}}{\text{4}}$$

$$X = \frac{\text{Toth}}{\text{4}}$$



SOH

$$\sin 59 = \frac{9}{2}$$
 9. $\tan 31 = \frac{x}{9}$. 9
 $\frac{2 \sin 59 = 9}{\sin 59}$ $x = 5.4$
 $\frac{9}{\sin 59}$
 $\frac{9}{2} = \frac{9}{\sin 59}$
 $\frac{1}{2} = \frac{9}{\sin 59}$