Extra Knowledge: Scientific Notation and Dimensional Analysis

OBJECTIVES

1. Express numbers in scientific notation.
2. Convert numbers in scientific notation to numbers without exponents.
3. Use scientific notation in calculations.

Scientific Notation: Used to express very small/large values

form: $a \times 10^n$ where: $1 < |a| < 10$, n is an integer

EXAMPLE 1 Using Scientific Notation

Write each number in scientific notation.

(a) 93,000,000 9.3×10^7

(b) 63,200,000,000 6.32×10^{10}

(c) 0.00462 4.67×10^{-3}

(d) −0.0000762 -7.62×10^{-5}

EXAMPLE 2 Writing Numbers without Exponents

Write each number without exponents.

(a) 6.2×10^3 6200

(b) 4.283×10^6 4283000

(c) 7.04×10^{-3} 0.00704

EXAMPLE 3 Multiplying and Dividing with Scientific Notation

Perform each calculation.

(a) $(7 \times 10^3)(5 \times 10^4)$

$(7 \cdot 5)(10^3 \cdot 10^4)$

35×10^7

3.5×10^8

(b) $\dfrac{4 \times 10^{-5}}{2 \times 10^{3}}$ $\left(\frac{4}{2}\right) \times \left(\frac{10^{-5}}{10^{3}}\right)$

$2 \times (10^{-5-3})$

$\boxed{2 \times 10^{-8}}$

(c) $(3 \times 10^{4})(8 \times 10^{7})$

$(3 \cdot 8) \times (10^{4} \cdot 10^{7})$

$\boxed{24 \times 10^{11}}$

(d) $\dfrac{6 \times 10^{-2}}{2 \times 10^{8}}$ $\left(\frac{6}{2}\right) \times \left(\frac{10^{-2}}{10^{8}}\right)$

$\boxed{3 \times 10^{-10}}$

EXAMPLE 4 Using Scientific Notation to Solve an Application

A *nanometer* is a very small unit of measure that is equivalent to about 0.00000003937 in. About how much would 700,000 nanometers measure in inches? (*Source: World Almanac and Book of Facts.*)

$\dfrac{700{,}000 \text{ nm} \mid 0.00000003937}{1 \text{ nm}} = \boxed{0.027559 \text{ or } 2.7559 \times 10^{-2}}$

EXAMPLE 5 Using Scientific Notation to Solve an Application

In 2008, the national debt was $\$1.0025 \times 10^{13}$ (which is more than $10 trillion). The population of the United States was approximately 304 million that year. About how much would each person have had to contribute in order to pay off the national debt? (*Source*: Bureau of Public Land; U.S. Census Bureau.)

$\dfrac{1.0025 \times 10^{13}}{304 \times 10^{6}} = \boxed{\$\ 32{,}976.97 \text{ per person}}$

Dimensional Analysis

What is Dimensional Analysis?

How to express an equivalent value in a different unit.

Conversions:

1 foot = 12 inches	1000 mm = 1 m	60 sec = 1 min
3 ft = 1 yard	100 cm = 1 m	60 min = 1 hr
5280 ft = 1 mile	10 mm = 1 cm	24 hr = 1 day
1 inch = 2.54 cm	1 km = 1000 m	365 days = 1 yr
1 km = 0.62 miles		

Using the above conversion factors, make the following conversions.

1. Convert 100 mm into inches.

$$\frac{100\text{mm}}{1}\left|\frac{1\text{cm}}{10\text{mm}}\right|\frac{1\text{inch}}{2.54\text{cm}} = 3.94\text{in}$$

2. Convert 32 years into hours.

$$\frac{32\text{yrs}}{1}\left|\frac{365\text{ days}}{1\text{ yrs}}\right|\frac{24\text{ hrs}}{1\text{ day}} = 280320\text{ hrs}.$$

3. Convert 12,345 mm into km.

$$\frac{12345\text{ mm}}{1}\left|\frac{1\text{ m}}{1000\text{ mm}}\right|\frac{1\text{Km}}{1000\text{ m}} = 0.012345\text{ km}$$

or 1.2345×10^{-2} km

4. Convert 12,345 seconds into years.

$$\frac{12345\text{ sec}}{1}\left|\frac{1\text{ min}}{60\text{ sec}}\right|\frac{1\text{ hr}}{60\text{ min}}\left|\frac{1\text{day}}{24\text{hr}}\right|\frac{1\text{ yr}}{365\text{ days}} = 3.91457 \times 10^{-4}\text{ years}$$

5. Convert 35 inches into meters.

$$\frac{35\text{ in}}{1}\left|\frac{2.54\text{cm}}{1\text{ in}}\right|\frac{1\text{ m}}{100\text{cm}} = 0.889\text{ m}$$

6. Convert 1234 mm into feet.

$$\frac{1234\text{mm}}{1}\left|\frac{1\text{cm}}{10\text{mm}}\right|\frac{1\text{in}}{2.54\text{cm}}\left|\frac{1\text{ft}}{12\text{in}}\right. = 4.049\text{ ft}.$$

7. Convert 27 km/hr into m/s.

$$\frac{27\text{ Km}}{1\text{hr}}\left|\frac{1000\text{m}}{1\text{ Km}}\right|\frac{1\text{hr}}{60\text{min}}\left|\frac{1\text{min}}{60\text{sec}}\right. = 7.5\text{m/s}$$

8. Convert 35 miles/hr into ft/sec. $\frac{35\text{ miles}}{1\text{ hr}} \cdot \frac{5280\text{ ft}}{1\text{ mile}} \cdot \frac{1\text{ hr}}{3600\text{ sec}} = 51.33\ldots$

9. Convert 120 ft/min into km/hr. $\frac{120\text{ ft}}{\text{min}} \cdot \frac{60\text{ min}}{1\text{ hr}} \cdot \frac{1\text{ mile}}{5280\text{ ft}} \cdot \frac{1\text{ km}}{0.62\text{ mile}} = 2.19\ldots$

10. Convert 420 ft/sec into miles/hr. $\frac{420\text{ ft}}{\text{sec}} \cdot \frac{3600\text{ sec}}{1\text{ hr}} \cdot \frac{1\text{ mile}}{5280\text{ ft}} = 286.36\text{ mi/hr}$

11. You are traveling on the freeway going 70 mph and you get a text that you must look at right away. It takes you 5 seconds to look down and read the text. How far have you traveled while looking down at your phone? in feet

$\frac{70\text{ miles}}{\text{hr}} \cdot \frac{5280\text{ ft}}{1\text{ mile}} \cdot \frac{1\text{ hr}}{3600\text{ sec}} = 102.67\text{ ft/sec} \cdot 5\text{ sec}$

$\approx 513.33\text{ ft.!!!}$

Converting Units: Currency Exchange

A great example of simple dimensional analysis involves converting form one unit of currency to another. Consider, for example, the table of values below, using data current on 12 September 2011 rounded to the second decimal place.

	USD	GBP
	1	1.58762
	0.62987	1
	1.00088	1.58902
	0.732279	1.16258
	0.968439	1.53752

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www.x-rates.com

The table shows us, for example, that \$1.00 = £0.63 British pounds = € 0.73 (euros) rounded to two decimal places.

Suppose, for example, while traveling through France you find an mp3 player priced at € 89.99. What is the equivalent number of US Dollars?

1. Start with what is given: € 89.99

2. Write the conversion that $1.00 = € 0.73, and multiply it by what is given so that the units you start with cancel out.

$$€\,89.99 \times \frac{\$1.00}{€\,0.73} = \$123.27$$

Now, answer these questions in a similar manner.

1. How much does the mp3 player cost in Canadian dollars?

$\$123.27 \mid 1.00088\ \text{Can.}$
$\overline{\qquad\qquad 1\ \$}$

2. Which currency listed is closest to the value of the US dollar? Which is the most "valuable" on a one to one comparison?

- Closest - Candian dollar
- Valuable - GBP

3. While visiting an international bazaar, you find three ice cream vendors side by side. One sells ice cream cones for $2.49, one sells the identical product for £1.19, and the third sells them for 1.99 Australian dollars. Which is the most expensive? Which is the cheapest? Show your work

option 1 $2.49 ↑ USD ↑ GBP

option 2 $\dfrac{£1.19 \mid 1\ \$}{0.63\ £} = \$1.89$

option 3 $\dfrac{1.99\ AD \mid 1\ \$}{0.97\ AD} = \$2.05$