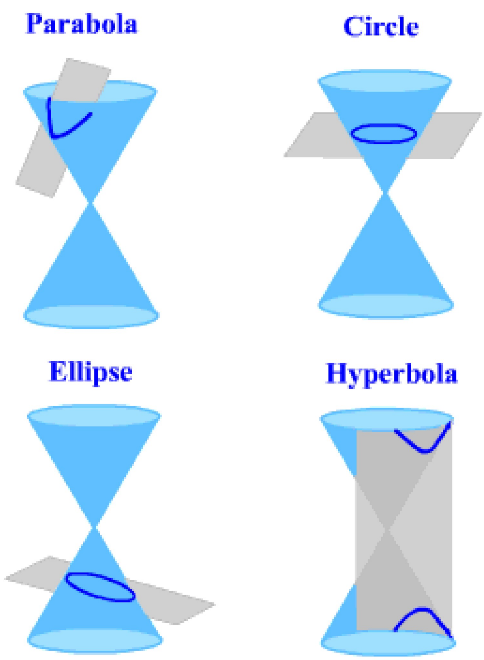
Conics Notes

Tuesday, March 1, 2022 8:59 AM

Conic Sections in standard form



main strategy: complete the square

Parabola (vertical) : One variable is squared (x) and one variable is linear (y).

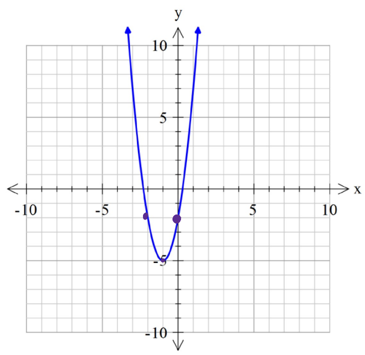
a>0

$$\boxed{y = a(x-h)^2 + k}$$

a<0

The vertex is found at (h, k).

Example:
$y = 3(x+1)^2 - 5$

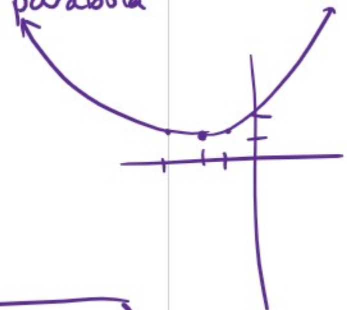


Parabola

1) isolate the linear term (degree 1)
2) complete the square on other side
3) as needed, divide by coeff of linear term.

Write the following function in standard form and identify the conic section:

parabola



linear term

$$x^2 + 4x - \boxed{6y} = -10$$

$$x^2 + 4x + 10 = 6y$$

$$(x^2 + 4x + 4) - 4 + 10 = 6y$$

$$\left(\frac{4}{2}\right)^2$$

$$(2)^2$$

$$\frac{1(x+2)^2}{6} + \frac{6}{6} = \frac{6y}{6}$$

$$\boxed{y = \frac{1}{6}(x+2)^2 + 1}$$

Write the following function in standard form and identify the conic section: parabola

$$\boxed{3y} - 2x^2 = 12x - 9$$

$$3y = 2x^2 + 12x - 9$$

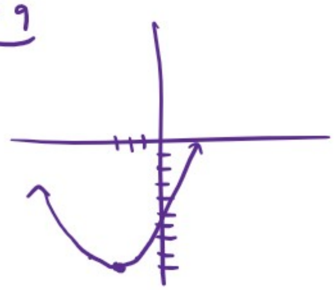
$$3y = 2(x^2 + 6x + 9) \underbrace{- 18 - 9}$$

$$\left(\frac{6}{2}\right)^2$$

$$(3)^2$$

$$\frac{3y}{3} = \frac{2(x+3)^2}{3} - \frac{27}{3}$$

$$y = \frac{2}{3}(x+3)^2 - 9$$



Parabola (horizontal) : One variable is squared (y) and one variable is linear (x).

a>0

a<0

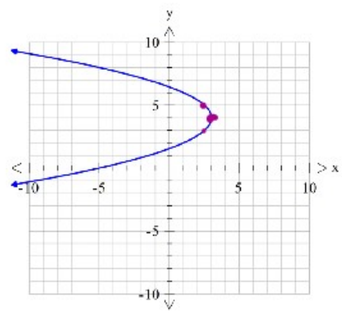
$$x = a(y-k)^2 + h$$

The vertex is (h, k).

Example:

$$x = -\frac{1}{2}(y-4)^2 + 3$$

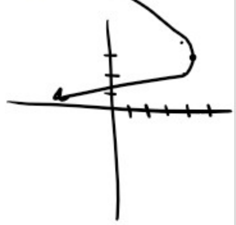
$a = \dfrac{-\frac{1}{2}}{1}$ ← ↑↓



Write the following function in standard form and identify the conic section:

parabola

$$\boxed{4x} + y^2 - 6y = 9$$



$y^2 - 6y - 9 = -4x$

$(y^2 - 6y + \underline{9}) - \underline{9} - 9 = -4x$

$\left(\frac{-6}{2}\right)^2$

$(-3)^2$

$\frac{1(y-3)^2}{-4} - \frac{18}{-4} = \frac{-4x}{-4}$

$x = -\frac{1}{4}(y-3)^2 + \frac{9}{2}$

$\left(\frac{9}{2}, 3\right)$

$\frac{-\frac{1}{4}}{1}$ ← ↑↓

Write the following function in standard form and identify the conic section:

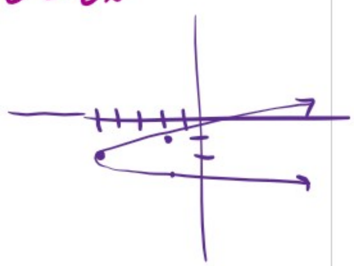
parabola

$$8y^2 + 32y + 22 = \boxed{2x}$$

$8(y^2 + 4y + \underline{4}) - \underline{32} + 22 = 2x$

$\left(\frac{4}{2}\right)^2$

$(2)^2$



$\frac{8(y+2)^2}{2} - \frac{10}{2} = \frac{2x}{2}$

$x = 4(y+2)^2 - 5$ vertex $(-5, -2)$ $a = \frac{4}{1}$ → ↑↓

Circles: Both variables are squared with the same coefficient.

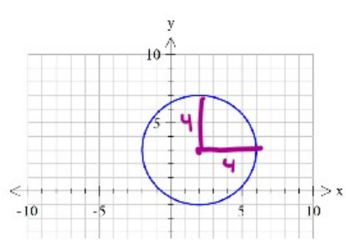
$$(x-h)^2 + (y-k)^2 = r^2$$

The center is (h, k). The radius is r.

Example:

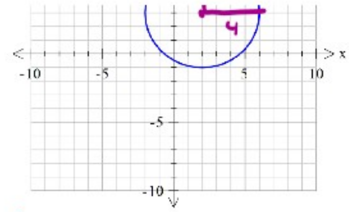
$(x-2)^2 + (y-3)^2 = 16$

$(2, 3)$



(2,3)

r = 4



Circle in standard form

1) isolate the constant (on the right)
2) complete the square w/ x + y terms

+ ____ = + ____

3) If needed, ÷ by leading coeff of $x^2 + y^2$ terms

Write the following function in standard form and identify the conic section: circle

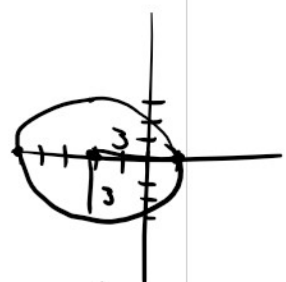
$$x^2 + 4x + y^2 = 5$$

$(x^2 + 4x + \underline{4}) + y^2 = 5 + \underline{4}$

$\left(\frac{4}{2}\right)^2$

2^2

$$\boxed{(x+2)^2 + y^2 = 9}$$



c: (-2,0)

r = 3

Write the following function in standard form and identify the conic section: $\boxed{\text{Circle}}$

$$x^2 - 6x + y^2 + 2y = 0$$

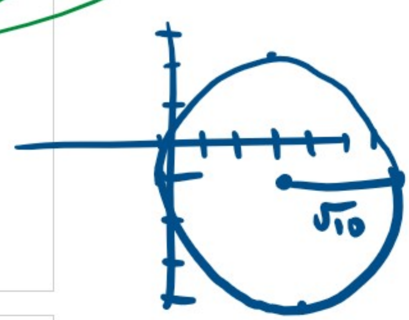
$(x^2 - 6x + \underline{9}) + (y^2 + 2y + \underline{1}) = 0 + \underline{9} + \underline{1}$

$\left(-\frac{6}{2}\right)^2$ $\left(\frac{2}{2}\right)^2$

$(-3)^2$ $(1)^2$

$$\boxed{(x-3)^2 + (y+1)^2 = 10}$$

c: (3,-1) $r = \sqrt{10} \approx 3.1$



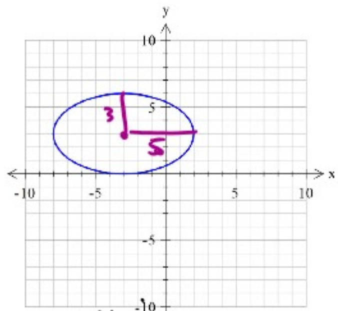
Ellipse: Both variables are squared and positive but with different coefficients.

$$\frac{(x-h)^2}{} + \frac{(y-k)^2}{} = 1$$

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$

The center is (h, k).

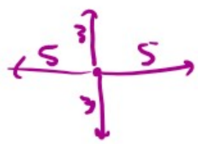
The vertical axis has a length of $2b$ and the horizontal axis has a length of $2a$.



Example:

$$\frac{(x+3)^2}{25}+\frac{(y-3)^2}{9}=1$$

C: (-3, 3)



ellipse
- constants on right
- complete the square w/ both x & y, as needed + ___ = + ___
- ÷ to create a 1 on the right hand side

Write the following function in standard form and identify the conic section:

$$8x^2+4y^2-8y-60=0$$

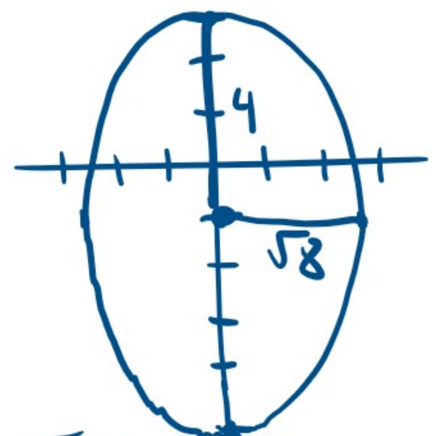
$8x^2 + 4y^2 - 8y = 60$

$8x^2 + 4(y^2 - 2y + \underline{1}) = 60 + \underline{4}$

$\left(-\frac{2}{2}\right)^2$

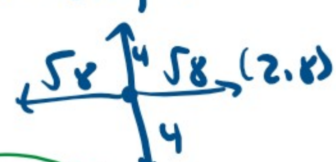
$(-1)^2$

$\frac{8x^2}{64} + \frac{4(y-1)^2}{64} = \frac{64}{64}$



$$\boxed{\frac{x^2}{8}+\frac{(y-1)^2}{16}=1}$$

C: (0, 1)



Write the following function in standard form and identify the conic section: $\boxed{\text{ellipse}}$

$$2x^2+3y^2-16x+18y=-11$$

$2x^2 - 16x + 3y^2 + 18y = -11$

$2(\quad + 16) + 3(\quad + 9) = -11 + 32 + 27$

$2x^2 - 16x + 3y^2 + 18y = -11$

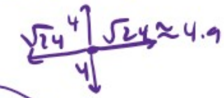
$2(x^2 - 8x + 16) + 3(y^2 + 6y + 9) = -11 + 32 + 27$

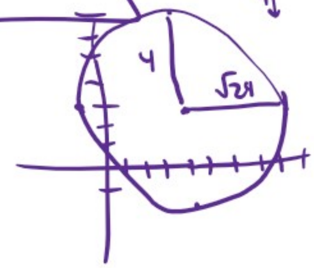
$\left(-\frac{8}{2}\right)^2 \quad (-4)^2 \qquad \left(\frac{6}{2}\right)^2 \quad (3)^2$

$\frac{2(x-4)^2}{48} + \frac{3(y+3)^2}{48} = \frac{48}{48}$

$$\boxed{\frac{(x-4)^2}{24} + \frac{(y-3)^2}{16} = 1}$$

c: (4, 3)





Hyperbola (horizontal): Both variables are squared and have different coefficients. One variable has a negative coefficient (y).

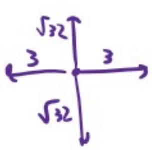
$$\frac{(x-h)^2}{b^2} - \frac{(y-k)^2}{a^2} = 1$$

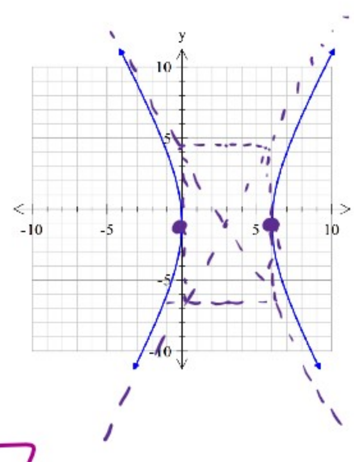
The center is (h, k).

Example:

$$\frac{(x-3)^2}{9} - \frac{(y+1)^2}{32} = 1$$

c: (3, -1)





$\boxed{\text{same directions as ellipse}}$

Write the following function in standard form and identify the conic section: hyperbola

$$x^2 - y^2 - 6x - 10y - 20 = 0$$

$x^2 - 6x - y^2 - 10y = 20$

$(x^2 - 6x + 9) - (y^2 + 10y + 25) = 20 + 9 + -25$

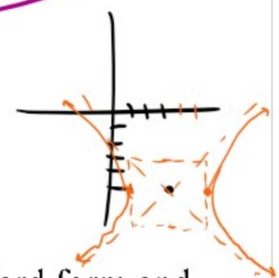
$\left(-\frac{6}{2}\right)^2 \qquad \left(\frac{10}{2}\right)^2$

$$(x^2 - 6x + 9) - (y^2 + 10y + 25) = 20 + 9 + -25$$

$(\frac{-6}{2})^2$ $(-3)^2$ $(\frac{10}{2})^2$ $(5)^2$

$$\frac{(x-3)^2}{4} - \frac{(y+5)^2}{4} = \frac{4}{4}$$

$$\boxed{\frac{(x-3)^2}{4} - \frac{(y+5)^2}{4} = 1}$$



Write the following function in standard form and identify the conic section: hyperbola

$$4x^2 - 3y^2 = -12y + 24$$

$$4x^2 - 3y^2 + 12y = 24$$

$$4x^2 - 3(y^2 - 4y + 4) = 24 + -12$$

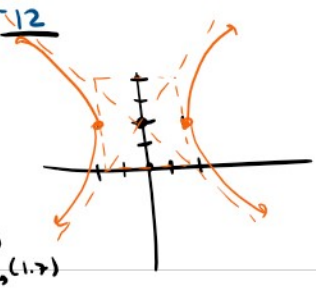
$(\frac{-4}{2})^2$ $(-2)^2$

$$\frac{4x^2}{12} - \frac{3(y-2)^2}{12} = \frac{12}{12}$$

$$\boxed{\frac{x^2}{3} - \frac{(y-2)^2}{4} = 1}$$

C: (0, 2)

$\sqrt{3}$ $\sqrt{3}$ 2 (1.7)



Hyperbola (vertical): Both variables are squared and have different coefficients. One variable has a negative coefficient (x).

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

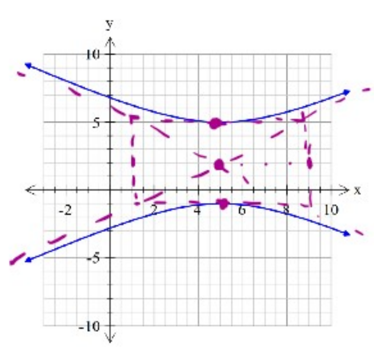
The center is (h, k).

Example:

$$\frac{(y-2)^2}{9} - \frac{(x-5)^2}{16} = 1$$

C: (5, 2)

4 4 3 3



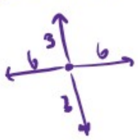
Write the following function in standard form and identify the conic section:

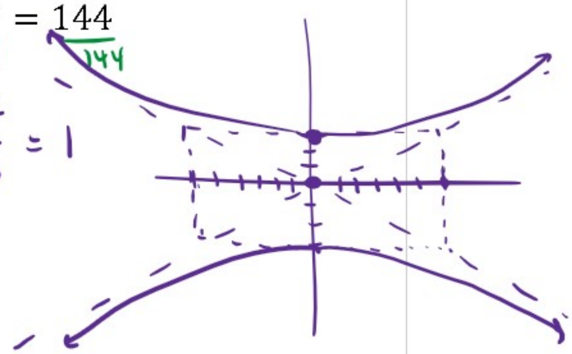
$$\frac{36y^2}{144} - \frac{4x^2}{144} = \frac{144}{144}$$

$\boxed{\text{hyp}}$

$$\frac{y^2}{9} - \frac{x^2}{36} = 1$$

C: (0,0)





Write the following function in standard form and identify the conic section:

hyperbola

$$9y^2 - 4x^2 - 16x - 18y - 43 = 0$$

$$9y^2 - 18y - 4x^2 - 16x = 43$$

$$9(y^2 - 2y + 1) - 4(x^2 + 4x + 4) = 43 + 9 + {}^{-}16$$

$\left(\frac{-2}{2}\right)^2$

$(-1)^2$

$\left(\frac{4}{2}\right)^2$

$(2)^2$

$$\frac{9(y-1)^2}{36} - \frac{4(x+2)^2}{36} = \frac{36}{36}$$

$$\boxed{\frac{(y-1)^2}{4} - \frac{(x+2)^2}{6} = 1}$$

C: (-2,1)