Alg 1 Unit 6 Notes

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6.1 Notes: Multiplying with Exponents

Solutions

Key Terms

Base	**Exponent**	**Coefficient**	*Sample*
the value raised to a power Sample: 2	the power Sample: 3	the value multiplying the base Sample: 5	$5(2)^3$ coeff base exp

Exploration A:

* Expand: x^3

$x \cdot x \cdot x$

* Expand and simplify: $x^3 \cdot x^4$

$x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x = x^7$

*Expand: x^4

$x \cdot x \cdot x \cdot x$

*What do you notice?

you can add the power with the same base

Exploration B:

* Expand: $2a^5$

$2 \cdot a \cdot a \cdot a \cdot a \cdot a$

* Expand and simplify: $2a^5 \cdot 7a^3$

$2 \cdot a \cdot a \cdot a \cdot a \cdot a \cdot 7 \cdot a \cdot a \cdot a$

$= 14a^8$

*Expand: $7a^3$

$7 \cdot a \cdot a \cdot a$

*What do you notice?

- multiply coeff
- add exponents with same base

Exploration C:

* Expand: $-4h^2g$

$-4 \cdot h \cdot h \cdot g$

* Expand and simplify: $-4h^2g \cdot 3hg^4$

$-4 \cdot h \cdot h \cdot g \cdot 3 \cdot h \cdot g \cdot g \cdot g \cdot g$

$= -12h^3g^5$

*Expand: $3hg^4$

$3 \cdot h \cdot g \cdot g \cdot g \cdot g$

*What do you notice?

- multiply coeff
- add exponents with same base

Multiplying Expressions with the Same Base:

- multiply coefficients
- add exponents with the same base

Examples #1 – 6: Simplify each expression.

1) $b^7 \cdot b^5 \cdot b^1$

$b^{7+5+1} = b^{13}$

2) $3w^4 \cdot -7w^{21}$

$3 \cdot -7 \cdot w^{4+21}$

$-21w^{24}$

3) $(-10a^3b^{14})(-7db^2)$

$-10 \cdot -7 \cdot a^{3+1} \cdot b^{14+2}$

$70a^4b^{16}$

You try #4 – 6!

4) $5p \cdot p^8 \cdot 2p^3$

$5 \cdot 2 \cdot p^{1+8+3}$

$10p^{12}$

5) $(-x^5y)(3xy^4)$

$-1 \cdot 3 \cdot x^{5+1} \cdot y^{1+4}$

$-3x^6y^5$

6) $d^5 \cdot d^{13} \cdot d$

d^{5+13+1}

d^{19}

Exploration D: Expand each expression, and then evaluate. Verify on a calculator.

* $(-2)^4$

$(-2)(-2)(-2)(-2)$

16

*-2^4

$-2 \cdot 2 \cdot 2 \cdot 2$

$= -16$

*$(-2)^3$

$(-2)(-2)(-2)$

-8

*-2^3

$-2 \cdot 2 \cdot 2 = -8$

Draw a conclusion from your observations.

parenthesis matter with negatives and even powers!

Examples #7 – 12: Simplify each expression. Evaluate numerical bases. calculator okay!

7) $(-3)^3 \cdot (-3)$

$(-3)^{3+1}$

$(-3)^4$

81

8) $-5^3 \cdot 5$

$-5^{3+1} = -5^4$

$= -625$

9) $(-2)^2(x^3yw^4)(-2xy^5w^3)$

$(-2)^3 \cdot x^{3+1} \cdot y^{1+5} \cdot w^{4+3}$

$-8x^4y^6w^7$

You try!

10) $(-5)^2 \cdot (-5)$

$(-5)^3$

-125

11) $4^2 \cdot -4^2$

-4^4

-256

12) $(-6)^3(a^5b^4)(-6ab)$

$(-6)^4 \cdot a^{5+1} \cdot b^{4+1}$

$1296a^6b^5$

6.2 Notes: Dividing with Exponents

Objectives:

- Students will be able to simplify division with expressions with the same base taken to a power.

Exploration A:

* Expand: x^6

$xxxxxx$

* Expand and simplify: $\frac{x^6}{x^4}$

$\frac{xxxxxx}{xxxx} = x^2$

*Expand: x^4

$xxxx$

*What do you notice?

Dividing with same base... subtract the exponents

Exploration B:

* Expand: $12a^5$

$12aaaaa$

* Expand and simplify: $\frac{12a^5}{4a^8}$

$\frac{12aaaaa}{4aaaaaaaa} = \frac{3}{a^3}$

*Expand: $4a^8$

$4aaaaaaaa$

*What do you notice?

- Divide coeff
- subtract exponents w/ same base
- can have more on denom

Exploration C:

* Expand: $-4h^2gk$

$-4hhgk$

* Expand and simplify: $\frac{-4h^2gk}{-6hg^4k}$

$\frac{-4hhgk}{-6hgggk} = \frac{2h}{3g^3}$

*Expand: $-6hg^4k$

$-6hggggk$

*What do you notice?

- Divide/reduce coeff
- subtract powers same base
- extras on num or denom

Dividing Expressions with the Same Base:

- Divide/reduce coeff
- Subtract powers with same base
  → answer on num or denom (whichever had more)

Examples #1 – 6: Simplify each expression.

1) $\frac{b^7}{b^5}$ b^{7-5} b^2

2) $\frac{18w^4}{-9w^{21}}$ $\frac{-2}{w^{21-4}} = \frac{-2}{w^{17}}$

3) $-\frac{2x^{10}}{4x^{10}} = -\frac{1}{2}$

You try #4 – 6!

4) $\frac{50p^8}{2p^3}$ $25p^{8-3} = 25p^5$

5) $\frac{3a^{14}}{9a^{14}}$ $\frac{1}{3}$

6) $\frac{-4w^{10}}{-2w^{12}}$ $\frac{2}{w^{12-10}} = \frac{2}{w^2}$

Examples #7 – 10: Simplify each expression.

7) $\frac{x^4 \cdot x^3}{x^1}$ $4+3-1$ x^6

8) $\frac{-10a^3b^{14}}{-15a^3b^2}$ $\frac{2b^{14-2}}{3} = \frac{2b^{12}}{3}$

You try #9 – 10!

9) $-\frac{2x^5y}{10xy^4}$ $-\frac{1x^4}{5y^3}$

10) $\frac{d^5 \cdot d^{13}}{d^6}$ $d^{5+13-6} = d^{12}$

11) Find and correct the error(s) in the solution shown. **Simplify:** $-\frac{3x^5y^3z^4}{12x^7yz^3}$

Solution:

Coefficients: $-12 \div 3 = -4$ reduce $\frac{-3}{12} = \frac{-1}{4}$

Powers for bases: xs $7-5=2$ on denom; ys $3-1=2$; zs $4-3=1$

$= -4x^2y^2z$

$-\frac{1y^2z}{4x^2}$ or $\frac{-y^2z}{4x^2}$

6.3 Notes: More Rules with Exponents

Objective:

- **Students will simplify exponential expressions with more than one base raised to the same power.**
- **Students will simplify exponential expressions with more than one power.**

Exploration A: Expand out the following expressions and then simplify.

- $(3xy)^2$

$3xy \cdot 3xy = 9x^2y^2$

- $\left(\frac{5a}{bc}\right)^3$

$\frac{5a}{bc} \cdot \frac{5a}{bc} \cdot \frac{5a}{bc}$

$\frac{125a^3}{b^3c^3}$

What do you notice?

all items in ( ) are taken to the power

Power to a Product Rule	**Power to Quotient Rule**
all items in ( ) are raised to the power	all items are taken to the power

Examples #1 – 6: Simplify each expression. Evaluate numerical bases.

1) $(4bd)^3$

$4^3b^3d^3$

$64b^3d^3$

You try #4 - 6!

2) $\left(\frac{3x}{y}\right)^4$

$\frac{3^4x^4}{y^4}$

$\frac{81x^4}{y^4}$

3) $\left(\frac{-5g}{2h}\right)^2$

$\frac{(-5)^2g^2}{2^2h^2}$

$\frac{25g^2}{4h^2}$

4) $(-4fh)^3$

$(-4)^3f^3h^3$

$-64f^3h^3$

5) $\left(\frac{8w}{z}\right)^2$

$\frac{8^2w^2}{z^2} = \frac{64w^2}{z^2}$

6) $\left(\frac{9x}{5y}\right)^2$

$\frac{9^2x^2}{5^2y^2} = \frac{81x^2}{25y^2}$

Exploration B: Expand out the following expressions and then simplify.

- $(5x^4)^2$

$5x^4 \cdot 5x^4 = 25x^8$

- $(a^5b^2)^3$

$a^5b^2 \cdot a^5b^2 \cdot a^5b^2$

$= a^{15}b^6$

What do you notice?

- raise coeff to the power
- multiply exponents (keep the base the same)

Power to a Power Rule

- raise coeff to the power
- multiply exponents
  → keep the base the same

Examples #7 – 12: Simplify each expression. Evaluate numerical bases.

7) $(4b^5)^3$

$(4)^3 b^{5\cdot3}$

$64b^{15}$

8) $(-3x^5)^4$

$(-3)^4 \cdot x^{5\cdot4}$

$81x^{20}$

9) $-(5a^{10})^2$

$-(5)^2 a^{10\cdot2}$

$-25a^{20}$

You try #10 – 12!

10) $(-10y^3)^3$

$(-10)^3 y^{3\cdot3}$

$-1000y^9$

11) $-(8g^6)^2$

$-(8)^2 g^{6\cdot2}$

$-64g^{12}$

12) $(-7h^{11})^2$

$(-7)^2 h^{11\cdot2}$

$49h^{22}$

What if there is more than one rule in a single problem?

- do extra powers first
- then do mult/division

} use order of operations PEMDAS

Examples 13 – 16: Simplify each expression. Evaluate numerical bases.

13) $(2x^8y)^3 \cdot 7x^2$

$(2)^3 x^{8\cdot3} y^3 \cdot 7x^2$

$8\cdot7x^{24+2}y^{3+2}$

$56x^{26}y^5$

14) $\frac{(x^7y^3)^5}{(xy^4)^6}$

$\frac{x^{35}y^{15}}{x^6y^{24}} = x^{35-6}y^{15-24}$

$= x^{29}y^{-9}$

$= \frac{x^{29}}{y^9}$

You try #15 – 16!

15) $(-4ab^2)^2 \cdot 8a^5b$

$(-4)^2 a^2 b^4 \cdot 8a^5 b$

$16\cdot8a^{2+5}b^{4+1}$

$128a^7b^5$

16) $\left(\frac{-3y^7}{xy^5}\right)^2$

$\frac{(-3)^2y^{14}}{x^2y^{10}} = 9y^{14-10}x^{-2}$

$\frac{9y^4}{x^2}$

6.4 Notes: Zero and Negative Powers

Objectives: Students will be able to simplify expressions with zero and negative powers

Exploration A: Consider the expanded terms. Simplify using exponential notation.

a) $-3 \cdot x \cdot x \cdot x \cdot x \cdot x$ = $-3x^5$

b) $-3 \cdot x \cdot x \cdot x$ = $-3x^3$

c) $-3 \cdot x$ = $-3x^1$ = $-3x$

d) -3 = $-3x^0$ = -3

For part d), how many times does x appear in this expression?

Taking a Base to the Power of Zero

$base^0 = 1$

$base \neq 0$

$1 \cdot (base)^0 = 1 \cdot base$ 0 times = 1

Note: 0^0 is undefined. Why do you think this is so? having 0, 0 times, doesn't make sense

Examples 1 – 6: Simplify each expression.

1. a^0 → 1
2. $-2x^0$ → $-2 \cdot 1 = -2$
3. $-6 \cdot (392{,}568{,}132.873)^0$ → $-6 \cdot 1 = -6$

You try #4 – 6!

4. $(-24x^3 + 10x^4y^{10})^0$ → 1
5. $-16b^0$ → $-16 \cdot 1 = -16$
6. $2(7x^3)^0$ → $= 2 \cdot 1 = 2$

Exploration B: Consider the expanded terms. Simplify using exponential notation. Look for a pattern in your answers!

a) $\frac{x \cdot x \cdot x}{x} = x^2$ (x^{2-1})

b) $\frac{x \cdot x \cdot x}{x \cdot x} = x$ (x^{3-2})

c) $\frac{x \cdot x \cdot x}{x \cdot x \cdot x} = 1$ ($x^{3-3} = x^0 = 1$)

d) $\frac{x \cdot x \cdot x}{x \cdot x \cdot x \cdot x} = \frac{1}{x}$ or x^{-1} (x^{3-4})

e) $\frac{x \cdot x \cdot x}{x \cdot x \cdot x \cdot x \cdot x} = \frac{1}{x^2}$ or x^{-2} (x^{3-5})

f) $\frac{x \cdot x \cdot x}{x \cdot x \cdot x \cdot x \cdot x \cdot x} = \frac{1}{x^3}$ or x^{-3} (x^{3-6})

Negative Exponents

negative exponents tell us we have that base^{power} in the other position (numerator or denom) of a fraction. $\frac{1}{x^{-a}} = x^a$ { $x^{-a} = \frac{1}{x^a}$

* a reciprocal!

Examples 7 – 12: Simplify each expression. Do not write negative or 0 exponents in your final answer.

7) 5^{-1} $= \frac{1}{5^1} = \frac{1}{5}$

8) $\left(\frac{1}{b}\right)^{-2} = (b)^2 = b^2$

9) $\frac{1}{7^{-2}} = \frac{7^2}{1} = \frac{49}{1} = 49$

You try #10 – 12!

10) 14^{-1} $\frac{1}{14}$

11) $\frac{1}{2^{-6}} = \frac{2^6}{1} = \frac{64}{1} = 64$

12) b^{-3} $= \frac{1}{b^3}$

Examples 13 – 18: Simplify each expression. Do not write negative or 0 exponents in your final answer.

13) $\frac{a^3}{a^{-4}}$ $a^3 \cdot a^4$ a^7

14) $\frac{x^3y^{10}}{x^7y^{-3}}$ $\frac{y^{10} \cdot y^3}{x^{7-3}}$ $\frac{y^{13}}{x^4}$

15) $\frac{5^{-1}b^3d^{-2}}{b^3d^4}$ $\frac{1}{5d^4 \cdot d^2} = \frac{1}{5d^6}$

You try #16 – 18!

16) $\frac{y^{-4}}{y^6}$ $\frac{1}{y^6 \cdot y^4} = \frac{1}{y^{10}}$

17) $\frac{x^{-4}y^{-2}}{x^7y^{-5}}$ $\frac{y^5}{x^7 \cdot x^4 y^2} = \frac{y^{5-2}}{x^{7+4}}$ $\frac{y^3}{x^{11}}$

18) $\frac{x^{-3}y^3}{xy^{-4}} = \frac{y^3 \cdot y^4}{x^3 \cdot x}$ $\frac{y^{3+4}}{x^{3+1}}$ $\frac{y^7}{x^4}$

Challenge Problems: Simplify each expression.

19) $\frac{5a^{-2}}{a^5b} \cdot \frac{b^3}{25b^{-8}}$ $\frac{5b^3b^8}{25a^2a^5b} = \frac{1b^{3+8-1}}{5a^{2+5}}$ $\frac{b^{10}}{5a^7}$

20) $\left(\frac{3x^2y}{x^{-2}y^4}\right)^{-2}$ $\frac{3^{-2}x^{-4}y^{-2}}{x^4y^{-8}} = \frac{y^8}{3^2x^4x^4y^2}$ $\frac{y^{8-2}}{9x^{4+4}}$ $= \frac{y^6}{9x^8}$

6.5 Notes: Solving Exponential Equations

Objectives: Students will be able to solve equations with exponents.

Exploration #1: Which of the following expressions below are equivalent to 9^2? Choose all that apply.

$9^2 = 81$

A) 3^4 B) 81^1 C) $\left(\frac{1}{9}\right)^{-2}$ D) $\left(\frac{1}{3}\right)^{-4}$ E) $\left(\frac{1}{81}\right)^{-1}$

$3^4 = 81$ $81^1 = 81$

Exploration #2: Rewrite the expression $\mathbf{64^5}$ in as many ways as you can thinking of by changing the base and the power. Hint: Use bases that go into 64 like 2, 4, ?

$(8^2)^5 = 8^{10}$ $(4^3)^5 = 4^{15}$ $(2^4)^5 = 2^{20}$

$\left(\left(\frac{1}{8}\right)^{-2}\right)^5 = \frac{1}{8}^{-10} = \frac{1}{4}^{-15} = \frac{1}{2}^{-20}$

Solving Exponential Equations using the same base.

- write both sides as the same base
- set exponents equal & solve

Examples #1 – 3: Solve the following exponential equations.

1) $6^x = 36$

$6^x = 6^2$

$x = 2$

2) $2^{x+5} = 8$

$2^{x+5} = 2^3$

$x + 5 = 3$

$x = -2$

3) $2^x = 2^{3x-7}$

$x = 3x - 7$

$0 = 2x - 7$

$7 = 2x$

$\frac{7}{2} = x$

You try #4 – 6!

4) $5^x = 25$

$5^x = 5^2$

$x = 2$

5) $11^{2x-4} = 121$

$11^{2x-4} = 11^2$

$2x - 4 = 2$

$2x = 6$

$x = 3$

6) $6^{2x-9} = 216$

$6^{2x-9} = 6^3$

$2x - 9 = 3$

$2x = 12$

$x = 6$

Solving Multi-Step Exponential Equations:

① use inverse operations to undo values that are NOT a base or exponent.

② write both sides as same base

③ set exponents equal & solve.

Examples 7 – 12: Solve the following exponential equations.

7) $3^{x-7}+1=4$

$3^{x-7}=3^{1}$

$x-7=1$

$x=8$

8) $5(3)^{x}=405$

$(3)^{x}=81$

$3^{x}=3^{4}$

$x=4$

9) $\left(\frac{1}{4}\right)^{5x}=4^{x+8}$

$4^{-5x}=4^{x+8}$

$-5x=x+8$

$-6x=8$

$x=\frac{-4}{3}$

You try #10 – 12!

10) $4\left(\frac{1}{3}\right)^{x}=108$

$\left(\frac{1}{3}\right)^{x}=27$

$3^{-x}=3^{3}$

$-x=3$

$x=-3$

11) $4^{5x+1}+3=19$

$4^{5x+1}=16$

$4^{5x+1}=4^{2}$

$5x+1=2$

$5x=1$

$x=\frac{1}{5}$

12) $\left(\frac{2}{3}\right)^{4}=\left(\frac{3}{2}\right)^{4x+11}$

$\left(\frac{3}{2}\right)^{-4}=\left(\frac{3}{2}\right)^{4x+11}$

$-4=4x+11$

$-15=4x$

$\frac{-15}{4}=x$