2.1: Introduction to Line Segments

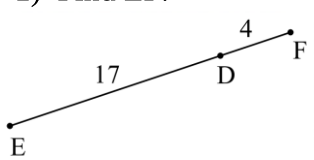
Essential Questions:
- Can you use the Segment Addition Postulate?
- Can you decide if points are collinear?

Exploration: Go to the following link to explore the Segment Addition Postulate: https://www.geogebra.org/m/NvChTa77
- Move the three named points around, and see what you notice about the segment lengths in the diagram.
- Make a conjecture about the lengths of the segments in the diagram:

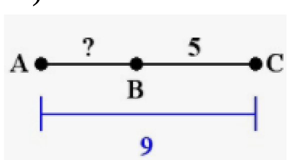
The Segment Addition Postulate	**If Point B is between A and C, then ______ + ________ = _______ is true by the Segment Addition Postulate.**	

Examples 1 – 5: Find the requested length(s) in each diagram.

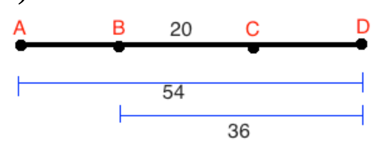
1) Find EF.



2) Find AB.

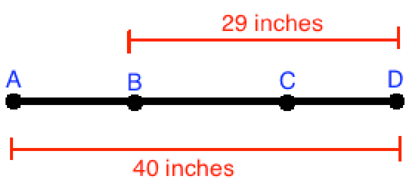


3) Find AB and CD.

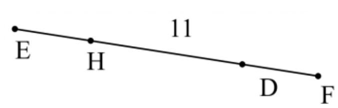


You Try #4 - 5!

4) Find AB.

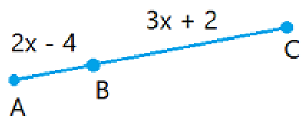


5) Find EH and DF if HF = 18 and EF = 21.



Examples 6 – 10: Find the value of the variable for each problem.

6) Find *x* if AC = 30

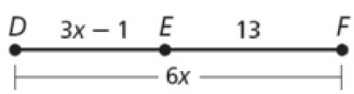


7) Given that X is between Y and Z, XY = $4a + 2$, XZ = $7a - 10$, and YZ = $9a + 19$, then find a.
Hint: draw and label a diagram.

Also, how long is segment XY?

You try #8 and 9!

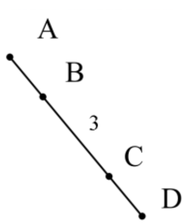
8) Find *x* and DE.



9) Given that B is between G and H, GB = $y - 5$, BH = $y + 2$, and GH = 8, then find y.
Hint: draw and label a diagram.

Congruent (≅) Segments	**If two segments are congruent,** then they have the ___________ ______________.	

10) Given the diagram below, where $AB \cong CD$ and AD = 15. Then find the lengths of AB and BD.



Point(s) of Intersection	**If two lines ____________, then the point they _________ at is called the point of intersection.**	
Collinear	**If two or more points are on the ____________ ____________ (even if the line is not drawn), then the points are collinear.**	

Exploration:

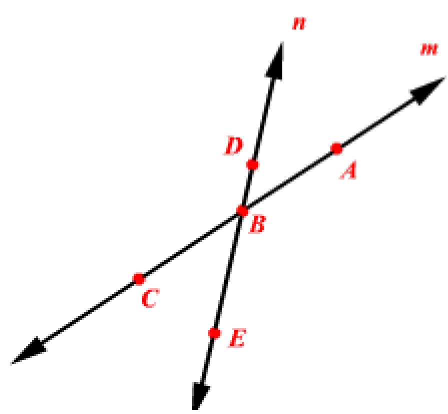
- Use the steps below to consider the conjecture: **Any two points are always collinear.**
  - Draw two points. Are they collinear?
  - Draw two different points. Are they collinear?
  - Do you agree with the conjecture?
- Now consider another conjecture: **Any set of three points are NOT always collinear.**
  - Try to draw three points that are not collinear.
  - Try to draw three points that ARE collinear.
  - Do you agree with the conjecture?

For Examples 11 – 15: Use the diagram shown.

11) What is the point of intersection for $\overleftrightarrow{AC}$ and $\overleftrightarrow{ED}$?

12) Name a point that is collinear with B and E.

13) Are points C, A, and B collinear? Explain.



14) Are points D and A collinear? Explain.

15) Name 3 points on the diagram that are *not* collinear.

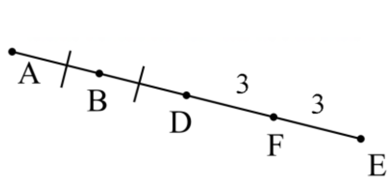
2.2: Using Midpoints

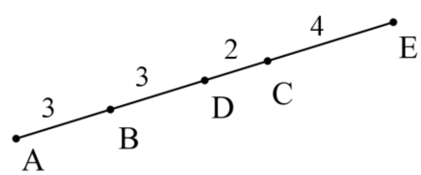
Essential Questions

- Can you use midpoints to solve a problem?
- Can you use the Midpoint Formula?

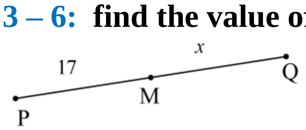
Midpoint of a Segment	**If a point is the midpoint of a segment, then it ______________the segment into two ____________ segments.**	

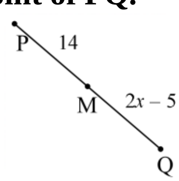
Examples 1 – 2: Which point(s) below are a midpoint? Explain. Hint: each problem has 2 answers!

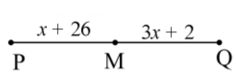
1) 

2) 

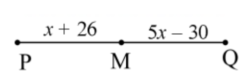
Examples 3 – 6: find the value of *x* if M is the midpoint of PQ.

3) 

4) 

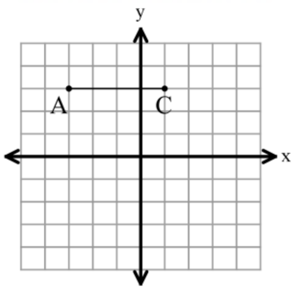
5) 

6) Find *x*, and the value of PQ.

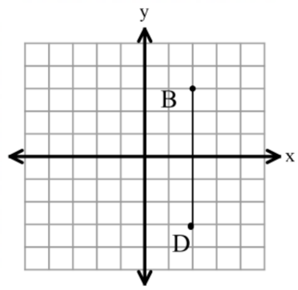


Exploration: For each graph below, plot the midpoint (point M) where you believe it should be for the given segment.

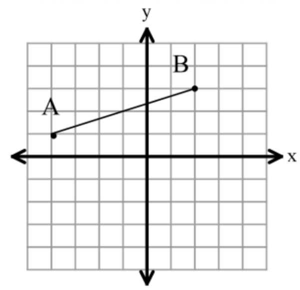
A)



B)



C)



D) Assume you have two test scores in Geometry: 80 and 90. What is your average test score? How did you find it?

The Midpoint Formula	**The midpoint of a segment, M, can be found by using:** $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$	

Examples 7 – 9: Find the midpoint for each set of ordered pairs, which are the endpoints of a segment.

7) $(5, 8)$ and $(2, 20)$

8) $(-3, 7)$ and $(-11, 7)$

9) You try! $(4, -1)$ and $(8, 9)$

Examples 10 – 11: Given that M is the midpoint of AB. Find the coordinates of the endpoint B.

10) $A(3, 2)$; $M(7, -3)$

11) $A(-2, 10)$; $M(4, 15)$

2.3: Pythagorean Theorem and Distance Formula

Essential Questions
- Can you use the Pythagorean Theorem to find distances in the coordinate plane?
- Can you use the Distance Formula to find the length of a segment?

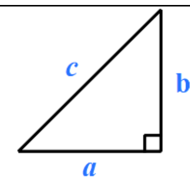
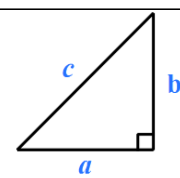
Exploration: Use the link below to explore the Pythagorean Theorem:

* Go to https://www.geogebra.org/m/jFFERBdd#material/HUbe242t

* Move the points and the slider to explore the diagram.

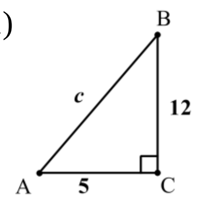
*Consider: How does this model the relationship from the Pythagorean Theorem? $a^2 + b^2 = c^2$

THE PYTHAGOREAN THEOREM

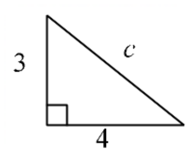
Hypotenuse of a Right Triangle		
Pythagorean Theorem		

Examples 1 – 3: Find the length of the missing side *c* in each right triangle. Simplify radical answers.

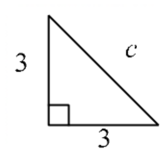
1)



2)



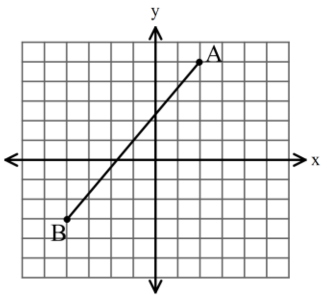
3)



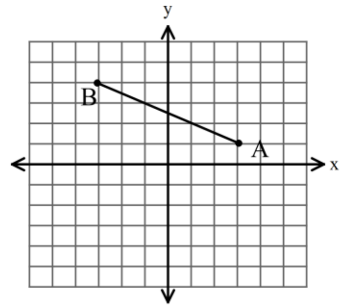
Finding Distance (or *length*) using the Pythagorean Theorem

Examples 4 – 5: Use the Pythagorean Theorem to find the length of segment AB in each diagram. If needed, write answers as *both* a simplified radical *and* a decimal rounded to the nearest tenth.

4)



5)



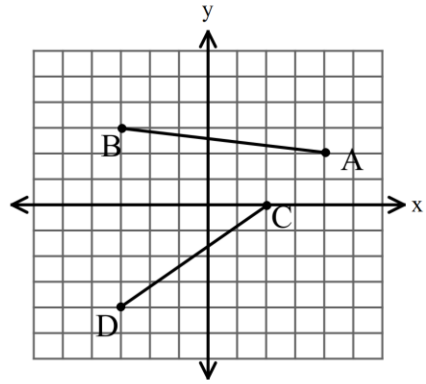
The Distance Formula	To find the distance d between two points (or the *length* of a segment), use: $$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$	

Examples 6 – 7: Find the length of segment AB with the given endpoints. If needed, round to 1 decimal.

6) $A\ (6, 7);\ \ B\ (14, -8)$

7) $A(-3, 2);\ \ B(4, 13)$

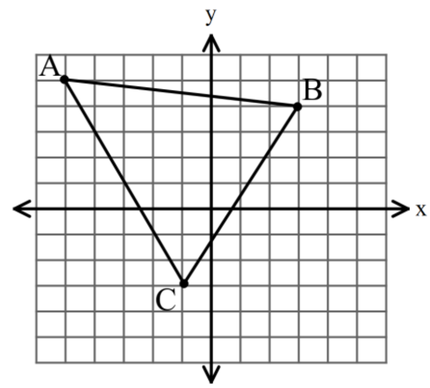
Example 8: Prove that $\overline{CD}$ is shorter than $\overline{AB}$. Use either the Pythagorean Theorem or the distance formula. Show your work!



Example 9:

An Equilateral triangle is a triangle where all sides are equal.
An Isosceles triangle is a triangle where two sides are equal.
A Scalene triangle is a triangle where no sides are equal.

Determine whether ΔABC is Equilateral, Isosceles, or Scalene.

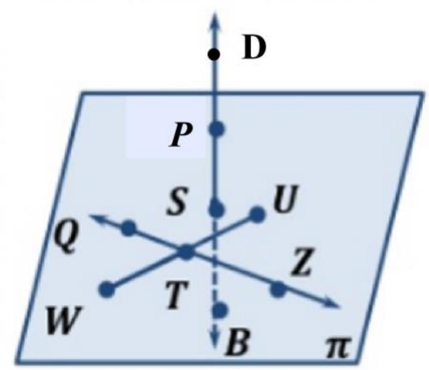


2.4: Planar Geometry

Essential Questions:
- Can you use planar geometry to solve problems?

Intersection	**When two or more shapes meet, the portions they have in ____________ are called the intersection.**	
Plane	**A plane is a ____________, two-dimensional surface that extends infinitely in all directions.**	
Coplanar	**If two or more points are on the __________ ______________ (even if the plane is not drawn), then the points are coplanar.**	

Consider the diagram. Make as many observations as you can. For example, how many lines are drawn? How many points? How many planes? Make as many true statements as you can.



Examples 1 – 5: Use the diagram above to answer the following questions.

1) What is the point of intersection for lines QZ and WU?

2) What is the point of intersection for line DB and plane π?

3) Are points S and D collinear? Are they coplanar?

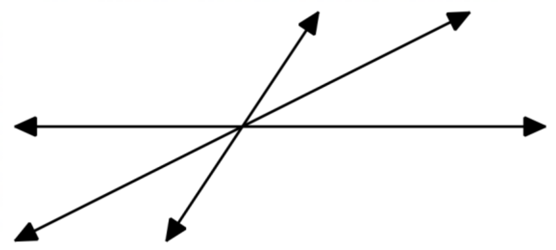
4) Are points Q and D collinear?

5) Points D and S are collinear. Name another point that is also collinear with D and S.

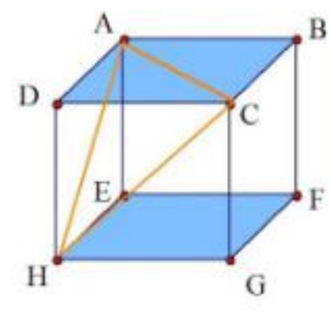
Example 6: Draw two lines that do *not* intersect.

Example 7: Peter drew a diagram with three coplanar lines that intersected at one point. He then concluded that three coplanar lines will always intersect at one point. Peter's statement is *not* always true. Draw two diagrams (each with 3 coplanar lines) that show specific examples that show how this statement is not always true. (These examples are called *counter-examples.)*

Peter's Diagram:



Consider the diagram shown. Make at least 4 true statements about this diagram.



Examples 8 – 11: Use the diagram above.

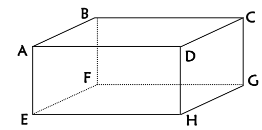
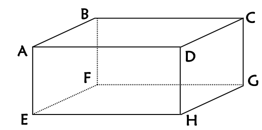
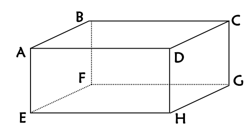
8) Name the intersection for planes EHG and CGF.

9) Points A, B, and C are on the same plane. Name another point that is coplanar with A, B, and C.

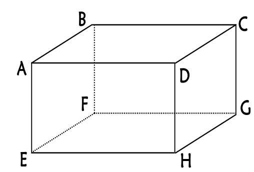
10) Name the intersection for planes GDC and EDH.

11) Name the intersection for planes ABC and EGH.

PARALLEL, PERPENDICULAR, & SKEW LINES

Parallel Lines	Two lines are parallel if and only if	
Perpendicular Lines	Two lines are perpendicular if and only if	
Skew Lines	Two lines are skew if and only if	

Example #12:
Does each pair of lines *appear* to be parallel, perpendicular, or skew?



a. $\overleftrightarrow{AE}$ & $\overleftrightarrow{GF}$

b. $\overleftrightarrow{BF}$ & $\overleftrightarrow{CG}$

c. $\overleftrightarrow{FG}$ & $\overleftrightarrow{EF}$

Example #13: Decide if each statement below is *sometimes*, *always*, or *never* true.

A) If two lines don't intersect, then they are parallel.

B) If two lines intersect to form right angles, then they are perpendicular.

C) If two lines are skew, then they are coplanar.

2.5: Conditional Statements and Syllogisms

Essential Questions:
- Can you identify the parts of a conditional statement?
- Can you write a counter-example for a false conditional statement?
- Can you use syllogisms to draw logical conclusions?

Watch this commercial, and notice how statements are *linked together* in order to draw a conclusion: https://www.youtube.com/watch?v=kIv3m2gMgUU

Conditional Statements

Conditional Statements	A **conditional statement** is written in _____ -______ form.	**If** you get all As and Bs on your report card, **then** your family will let you choose a restaurant for dinner.
Hypothesis	The **hypothesis** of a conditional statement is the ______ portion of the statement.	**If you get all As and Bs on your report card**, **then** your family will let you choose a restaurant for dinner.
Conclusion	The **conclusion** of a conditional statement is the ______ portion of the statement.	**If** you get all As and Bs on your report card, **then your family will let you choose a restaurant for dinner.**

Examples: For each conditional statement, identify the hypothesis and the conclusion.

1) If two angles are congruent, then they are vertical.

2) If water is cooled to below 32°F, then it will freeze.

You try #3 – 4!

3) If a ray bisects an angle, then it divides the angle into two congruent angles.

4) If Christie passes her driver's license test, then her parents will let her drive the family car.

Not all conditional statements are true. We can show that conditional statements are false by writing a counter-example. A **counter-example** is a *specific example* that shows that the statement is not true.

Examples: For each statement below, decide of the conditional statement is **true or false**. If the statement is false, then write a **counter-example.**

5) If I live in Nevada, then I live in Reno.

6) If two angles are complementary, then they are both acute angles.

You try #7 – 8!

7) If two angles are supplementary, then both angles are obtuse angles.

8) If a person drinks large quantities of salt water, then the person will get sick.

Syllogism	A **syllogism** is a collection of ______ or more ________________ statements, that follow a specific pattern to get a logical conclusion for the last statement.	**Syllogism pattern:** Statement 1: If ***a***, then ***b***. Statement 2: If ***b***, then ***c***. Conclusion: If ***a***, then ***c***. Start End
Example	If Amy makes the track team, then she will have practice every day after school. If Amy has practice every day after school, then she will have to walk home from school each day. Conclusion: If Amy makes the track them, then she will have to walk home from school each day.	

Examples: For each syllogism below, finish the last statement to complete the logical conclusion.

9) If Corey gets a job, then he will save up money.
If he saves up money, then he will buy a car.
Conclusion: If Corey ____________________, then he will ____________________.

You try #10!

10) If two angles are both right angles, then they each measure 90°.
If two angles each measure 90°, then they are congruent.
Conclusion: If two angles ________________________, then they___________________.

Watch one of the most famous syllogisms ever!
https://www.youtube.com/watch?v=QCDPkGjMBro

#11) Make your own syllogism with *at least* three statements.

2.6: One- and Two-Step Proofs

<table>
<tr><td>Proofs</td><td>A proof is a series of ____________ using ____________ to provide evidence for a conclusion.</td><td><table><tr><th>Statement</th><th>Reason</th></tr><tr><td>1.</td><td>1.</td></tr><tr><td>2.</td><td>2.</td></tr><tr><td>3.</td><td>3.</td></tr></table></td></tr>
<tr><td>Substitution Property of Equality</td><td>If $x = a$, then x can be __________ with a for any statement.</td><td></td></tr>
<tr><td>List of Possible Reasons</td><td colspan="2">
Given
Substitution Property of Equality.
If two angles are congruent, then they have the same measure.
If two segments are congruent, then they have the same measure.
If a point is a midpoint, then it divides a segment into two congruent segments.
If an angle is a right angle, then it has a measure of 90 degrees.
If two angles are complementary, then they have a sum of 90 degrees.
If two angles are supplementary, then they have a sum of 180 degrees.
If two angles form a linear pair, then they have a sum of 180 degrees.
If two angles are vertical, then they are congruent.
If a ray bisects and angle, then it divides the angle into two congruent angles.
If two lines are perpendicular, then they intersect at right angles.
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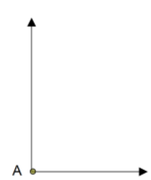
</table>

Example #1: Complete the proof below.

Given: $m\angle A = 90°$

Prove: $\angle A$ is a right angle.



Statement	Reason
1. $m\angle A = 90°$	1. Given
2. $\angle A$ is a right angle.	2.

Example #2: Complete the proof below.

Given: $\overline{XY} \cong \overline{YZ}$

Prove: Y is the midpoint of $\overline{XZ}$.

Statement	Reason
1. $XY \cong YZ$	1. Given
2. Y *is the midpoint of* XZ.	2.

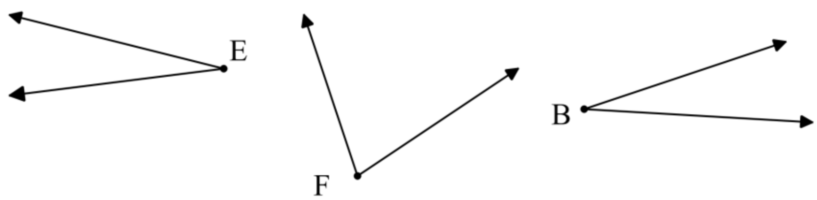
Example #3: Complete the proof below.

Given: $m\angle E + m\angle F = 180°$

Prove: $\angle E$ is supplementary to $\angle F$.

Statement	Reason
1. $m\angle E + m\angle F = 180°$	1. Given
2. $\angle E$ is supplementary to $\angle F$.	2.

Example #4: Complete the proof below.

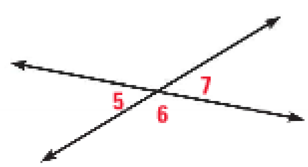


Given: $\angle E$ is complementary to $\angle F$ and
$\angle E = \angle B$

Prove: $\angle B$ is complementary to $\angle F$

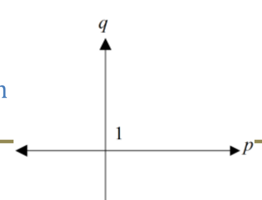
Statement	Reason
1. $\angle E$ is complementary to $\angle F$ and $\angle E = \angle B$	1. Given
2. $\angle B$ is complementary to $\angle F$	2.

Example #5: Write in the reason for the second step.



Given: $\angle 5$ and $\angle 7$ are vertical angles.
Prove: $\angle 5 \cong \angle 7$

Statement	Reason
1. $\angle 5$ and $\angle 7$ are vertical angles.	1. Given
2. $\angle 5 \cong \angle 7$	2.



Example #6:

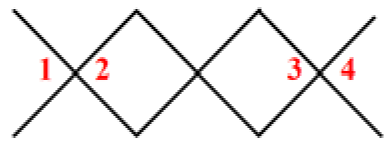
Given: $p \perp q$
Prove: $\angle 1$ is a right angle.

Statement	Reason
1. $p \perp q$	1. Given
2. $\angle 1$ is a right angle.	2.

Example #7: Finish the proof below.

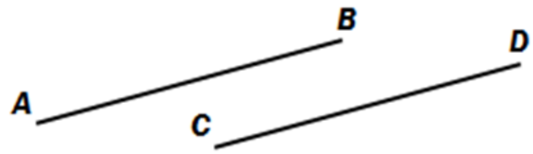
Given: $\angle 2 \cong \angle 3$ and $\angle 1$ and $\angle 2$ are vertical angles

Prove: $\angle 1 \cong \angle 3$



Statement	Reason
1. $\angle 2 \cong \angle 3$ *and* $\angle 1$ *and* $\angle 2$ *are vertical angles*	1. Given
2. $\angle 1 \cong \angle 2$	2.
3. $\angle 1 \cong \angle 3$	3.

Example #8:



Given: $\overline{AB} \cong \overline{CD}, AB = 8\ cm$
Prove: $CD = 8\ cm$

Statement	Reason
1. $AB \cong CD, AB = 8\ cm$	1. Given
2. $AB = CD$	2.
3. $CD = 8\ cm$	3.

Ch 2 Study Guide

- Collinear: Points are collinear if they are on the same line (even if the line is not drawn)
- Point of Intersection: If two lines cross, the point of intersection is the point where they cross.
- Segment Addition Postulate: If B is between A and C, then $AB + BC = AC$
- Congruent Segments: If two segments are congruent, then they have the same length.
- Midpoint of a Segment: If a point is the midpoint of a segment, then it divides the segment into two congruent segments.
- Midpoint Formula: $M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$
- Pythagorean Theorem: $a^2 + b^2 = c^2$
- Distance Formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
- Plane: A plane is a two-dimensional flat surface that extends to infinite in all directions.
- Coplanar: Shapes are coplanar if they are on the same plane.
- Parallel: Parallel lines are coplanar lines that do not intersect.
- Perpendicular: Perpendicular lines are lines that intersect to form right angles.
- Skew: Skew lines are non-coplanar lines that do not intersect.
- Conditional Statement: A statement written in "if-then" form.
- Counter-example: A specific example that shows a statement to be false.
- Syllogism: A collection of at least three statements that follow a pattern to a logical conclusion.

Reasons for Proofs:

- Given
- Substitution Property of Equality.
- If two angles are congruent, then they have the same measure.
- If two segments are congruent, then they have the same measure.
- If a point is a midpoint, then it divides a segment into two congruent segments.
- If an angle is a right angle, then it has a measure of 90 degrees.
- If two angles are complementary, then they have a sum of 90 degrees.
- If two angles are supplementary, then they have a sum of 180 degrees.
- If two angles form a linear pair, then they have a sum of 180 degrees.
- If two angles are vertical, then they are congruent.
- If a ray bisects and angle, then it divides the angle into two congruent angles.
- If two lines are perpendicular, then they intersect at right angles.