3.1 Factor and Solving Quadratics

Summary of factoring quadratics:

Trinomials: $x^2 + bx + c$, $ax^2 + bx + c$

Difference of Two Perfect Squares: $x^2 - y^2$

$(x+y)(x-y)$

GCF

Examples: Factor each polynomial.

1) $x^2 - 9x + 20$

$-5 + -4 = -9$

$-5 \cdot -4 = 20$

$\boxed{(x-5)(x-4)}$

2) $m^2 - 9$

3^2

$\boxed{(m+3)(m-3)}$

3) $\frac{6x^2}{3x} - \frac{9x}{3x}$

$\boxed{3x(2x-3)}$

4) $s^2 + 100$ (use i)

$s^2 - -100$

$(10i)^2$

$\boxed{(s+10i)(s-10i)}$

5) $5x^2 - 17x + 6$

$-15 + -2 = -17$

$-15 \cdot -2 = 30$

$\frac{-15}{5} = -3$

$\frac{-2}{5}$

$\boxed{(5x-2)(x-3)}$

6) $\frac{3x^3}{3x} - \frac{6x^2}{3x} - \frac{9x}{3x}$ (GCF first)

$3x(x^2 - 2x - 3)$

$-3 + 1 = -2$

$-3 \cdot 1 = -3$

$\boxed{3x(x-3)(x+1)}$

Solving Quadratic Equations

The solutions of a quadratic equation (or any equation) are called the zeros, roots or x-int.

Solve each polynomial by factoring:

7) $9s^2 - 64 = 0$

$(3s)^2 - 8^2$

$(3s+8)(3s-8) = 0$

$3s + 8 = 0 \qquad 3s - 8 = 0$

$3s = -8 \qquad 3s = 8$

$\boxed{s = -8/3 \qquad s = 8/3}$

8) $4r^3 - 9r^2 = -2r$

$+2r \quad +2r$

$4r^3 - 9r^2 + 2r = 0$

$r(4r^2 - 9r + 2) = 0$

$r(4r-1)(r-2) = 0$

$-8 + -1 = -9$

$-8 \cdot -1 = 8$

$\frac{-8}{4} = -2$

$\frac{-1}{4}$

$\boxed{r = 0}$

$4r - 1 = 0$

$\boxed{r = 1/4}$

$r - 2 = 0$

$\boxed{r = 2}$

9) $5p^2 - 16p + 15 = 4p - 5$

$-4p + 5$

$\frac{5p^2}{5} - \frac{20p}{5} + \frac{20}{5} = \frac{0}{5}$

$p^2 - 4p + 4 = 0$

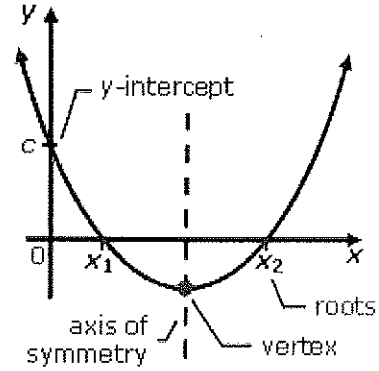
$-2 + -2 = -4$

$-2 \cdot -2 = 4$

$(p-2)(p-2) = 0$

$\boxed{p = 2}$

3.2: Solve by Square Rooting



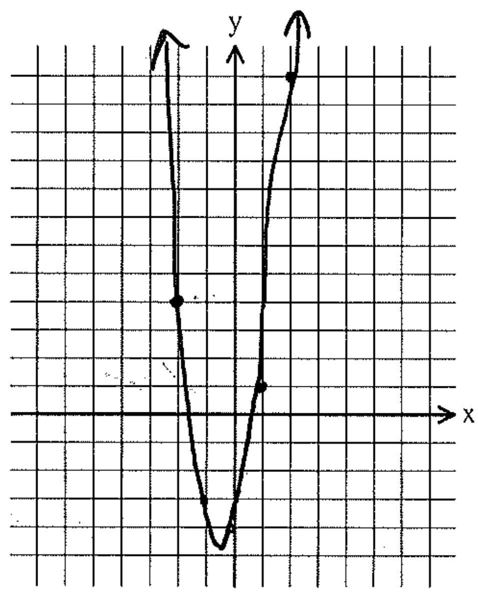
What is a QUADRATIC FUNCTION? A function with degree 2; shaped like a parabola.

Exploration: Make a table of values to graph $y = 3x^2 + 2x - 4$. Use -2, -1, 0, 1, 2 as the inputs. Identify as many key features as you can.

x	$y = 3x^2 + 2x - 4$
-2	$3(-2)^2 + 2(-2) - 4 = 4$
-1	-3
0	-4
1	1
2	12

without a calculator I can see the y-int.

Use calc to find x-int, vertex, and axis of sym.



Then use the graphing calculator to find the vertex, y-intercept, x-intercepts. *(Show them how to do these!)*

X-int: -1.54, 0.87

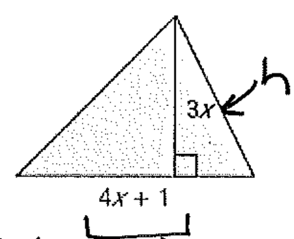
Vertex: $(-0.33, -4.33)$ AoS: $x = -0.33$

Review word problems:

1) The area of the triangle shown is 27 units squared. Find the value of x.

$\underbrace{27}_{A}$

$\frac{1}{2}bh = A$



$2 \cdot \frac{1}{2}(4x+1)(3x) = 27 \cdot 2$

$(4x+1)(3x) = 54$

$12x^2 + 3x = 54$

$\frac{12x^2}{3} + \frac{3x}{3} - \frac{54}{3} = \frac{0}{3}$

$4x^2 + x - 18 = 0$

$9 + -8 = 1$

$9 \cdot -8 = -72$

$\frac{9}{4} \quad -\frac{8}{4} = -2$

$(4x+9)(x-2) = 0$

$\cancel{-9/4}$ $\quad$ 2

$\boxed{2 = x}$

*This will make neg. height + base

$l \cdot w = A$
$bh = A$

2) A rectangle has an area of 36 u^2. The length is 5 units longer than the width. Find the dimensions of the rectangle.

$w+5=l$

$$\frac{9}{9} + \frac{-4}{\cdot -4} = 5 \quad \text{and} \quad = -36$$

$l \cdot w = A$
$(w+5)w = 36$
$w^2 + 5w = 36$
$w^2 + 5w - 36 = 0$

$(w+9)(w-4) = 0$
$w = \cancel{-9}, 4$

$$\boxed{\begin{aligned} w &= 4 \\ l &= 9 \end{aligned}}$$

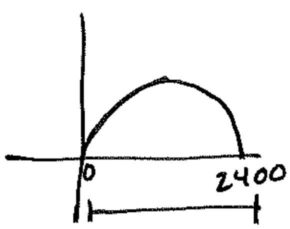
3) **Storage Building** The storage building shown can be modeled by the graph of the function $y = -10x^2 + 24{,}000x$ where x is the horizontal distance and y is the height (in cm). What is the width of the building at the base?

Find x-int

$y = \frac{-10x^2}{-10x} + \frac{24000x}{-10x}$

$0 = -10x(x - 2400)$

$x = 0, 2400$



$\boxed{\text{width is } 2400 \text{ cm}}$

Other methods of solving:

***Quadratic Formula** $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

***Solving Equations by Square Rooting (no *b* term)**

1) Isolate the squared portion of the equation
2) Square root both sides (2 solutions)
3) Solve for x.

Examples:

4) $3x^2 - 7x = 5$

can't factor

$3x^2 - 7x - 5 = 0$

$$\frac{-(-7) \pm \sqrt{(-7)^2 - 4(3)(-5)}}{2(3)}$$

$$\boxed{\frac{7 \pm \sqrt{109}}{6}}$$

5) $3 \cdot \frac{1}{3}x^2 = 12 \cdot 3$

$\sqrt{x^2} = \sqrt{36}$

$\boxed{x = \pm 6}$

6) $x^2 + 3x - 6 = 0$

$$\frac{-3 \pm \sqrt{3^2 - 4(1)(-6)}}{2(1)}$$

$$\boxed{x = \frac{-3 \pm \sqrt{33}}{2}}$$

7) $(x-2)^2 - 9 = 0$

$\sqrt{(x-2)^2} = \sqrt{9}$

$x - 2 = \pm 3$

$x - 2 = 3 \qquad x - 2 = -3$

$\boxed{x = 5 \qquad x = -1}$

8) $4 \cdot \frac{1}{4}(y-6)^2 = 8 \cdot 4$

$\sqrt{(y-6)^2} = \sqrt{32}$

$y - 6 = \pm\sqrt{32}$

$y = 6 \pm \sqrt{32} <^{2}_{16}$ (4)(4)

3

$\boxed{y = 6 \pm 4\sqrt{2}}$

9) $x^2 + 10 = 2x$

$x^2 - 2x + 10 = 0$

$$\frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(10)}}{2(1)}$$

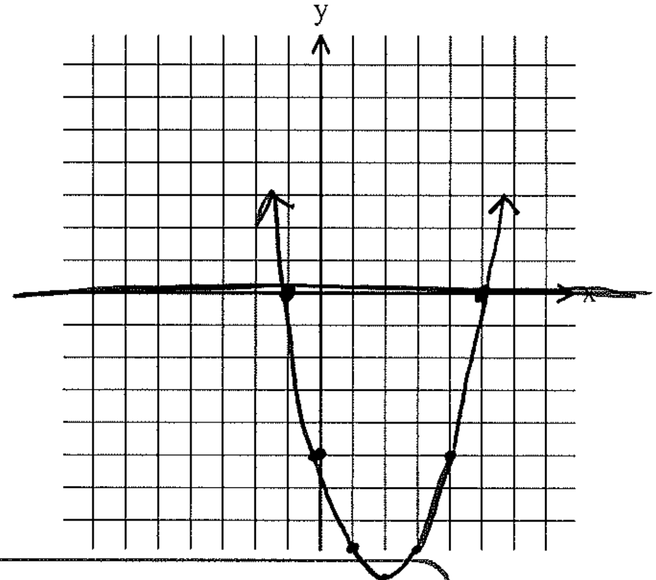
$$\frac{2 \pm \sqrt{-36}}{2} = \frac{2 \pm 6i}{2} = \boxed{1 \pm 3i}$$

Solve #7 above by graphing the quadratic and the line $y = 0$ (find the intersection points.) Why does this work?

$(x-2)^2-9=0$

-1	0
0	-5
1	-8
2	-9
3	-8
4	-5
5	0

The x-int. are when $y=0$



Vertical Motion Formulas : Memorize these!

In feet: $h = -16t^2 + v_0t + h_0$

In meters: $h = -4.9t^2 + v_0t + h_0$

Note: v_0 is the initial velocity; h_0 is the initial height; h is the height at time t.

10) An object is launched at $\overbrace{\text{19.6 meters per second (m/s)}}^{V_0}$ from a $\overbrace{\text{58.8-meter tall platform}}^{h_0}$. When does the object strike the ground?

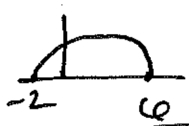
ground: $h=0$

find x-int.

$h=-4.9t^2+V_0t+h_0$

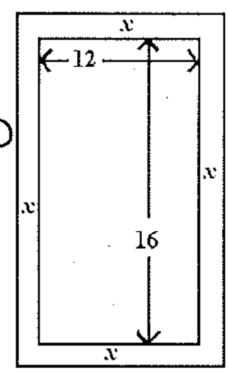
$0=-4.9t^2+19.6t+58.8$

$$\frac{-19.6 \pm \sqrt{19.6^2-4(-4.9)(58.8)}}{2(-4.9)} = -2, 6$$



$\boxed{\text{6 seconds}}$

11) A rectangular garden measuring 12 meters by 16 meters is to have a pedestrian pathway installed all around it, increasing the total area to 285 square meters. What will be the width of the pathway?



$l \cdot w = A$

$(16+2x)(12+2x)=285$

$192+32x+24x+4x^2=285$

$4x^2+56x-93=0$

$(2x+31)(2x-3)=0$

$\cancel{-\frac{31}{2}}$ $x=3/2$

$62+-6=56$

$62 \cdot -6=-372$

$\frac{62}{4}=\frac{31}{2}$

$\frac{-6}{4}=\frac{-3}{2}$

<u>3.3 Notes: Intro to Graphing Quadratics</u>

Have student's complete page 1 of the foldable. Go over domain and range in interval notation, end behavior and increasing/decreasing in interval notation.

The Parent Function

Use a table of values to graph the parent function
$y = x^2$ and identify as many key features as possible.

Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

Vertex: $(0,0)$

Max/min: Min @ 0

End Behavior: as $x \to \pm\infty$ $y \to \infty$

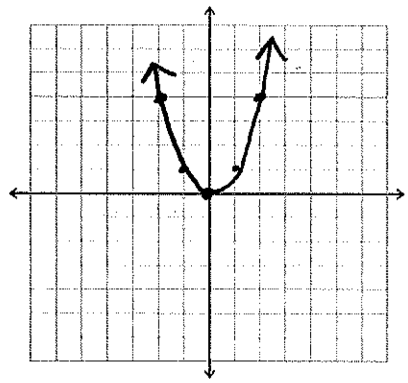
Increasing or Decreasing? I: $(0, \infty)$
D: $(-\infty, 0)$

y-intercept: $(0,0)$

x-intercept: $(0,0)$

Axis of symmetry: $x = 0$

x	$y=x^2$
-2	4
-1	1
0	0
1	1
2	4



Review the transformations on page 2 of the foldable.

Transformations:

The transformations for a quadratic equation in vertex form work the same as the transformations for absolute value functions.

$-a$
Refletion

stretch/compress

$$y = a(x - h)^2 + k$$

up/down

left/right
(opp. sign)

Complete pages 3 and 4 of the foldable

Foldable for Graphing Parabolas:

Vertex Form:

$y = a(x-h)^2 + k$

The vertex is at (h, k)

The max/min is at k

The axis of symmetry is $x = h$

To find roots: $y=0$, solve for x

The graph opens up if $a > 0$

The graph opens down if $a < 0$

The graph is stretched if $|a| > 1$

The graph is compress if $|a| < 1$

To find the y-intercept:

$x = 0$
solve for y

Example: $y = -2(x-3)^2 + 8$

Vertex: $(3, 8)$

AoS: $x = 3$

x-int: $x = 5, 1$

y-int: $y = -10$

Max/min: Max @ 8

Increasing: $(-\infty, 3)$

Decreasing: $(3, \infty)$

End Behavior:
as $x \to \pm\infty$ $y \to -\infty$

Domain: $(-\infty, \infty)$

Range: $(-\infty, 8)$

x-int

$0 = -2(x-3)^2 + 8$

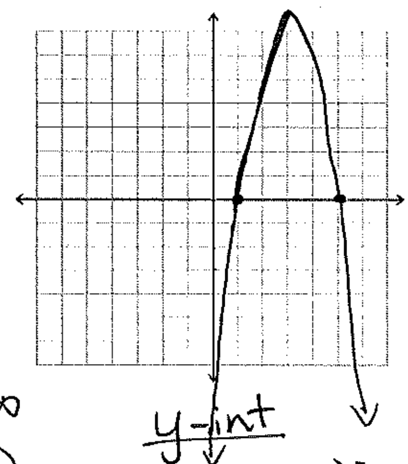
$-8 = -2(x-3)^2$

$4 = (x-3)^2$

$\pm 2 = x - 3$

$x - 3 = 2 \qquad x - 3 = -2$

$x = 5 \qquad x = 1$



y-int

$y = -2(0-3)^2 + 8$

$y = -10$

Extra Examples for notes (in notes packet):

1) Find the vertex and max or min value: $y = -4(x-3)^2 + 5$.

$(3, 5)$ max @ 5

2) A football is kicked in the air, and its path can be modeled by the equation $f(x) = -16(x-5)^2 + 21$, where x is the horizontal distance, in feet, and $f(x)$ is the height, in feet. What is the maximum height of the football?

find vertex!

Vertex: $(5, 21)$

$\boxed{\text{Max @ } 21}$

Complete pages 7 and 8 of the foldable

Intercept Form

$y = a(x-p)(x-q)$

x-intercepts are: p , q

The axis of symmetry has an equation of
$x = \frac{p+q}{2}$ → half way between int.

The vertex is halfway between $(p, 0)$ and $(q, 0)$. Find y!

The vertex is at: $\left(\frac{p+q}{2}, f\left(\frac{p+q}{2}\right)\right)$

The graph opens up if $a > 0$

The graph opens down if $a < 0$

The graph is stretched if $|a| > 1$

The graph is compressed if $|a| < 1$

To find the y-intercept:

x=0, solve for y

Example: $y = -2(x-4)(x+1)$

$\frac{4-1}{2} = \frac{3}{2} = 1.5$

$-2(1.5-4)(1.5+1)$

$y = -2(0-4)(0+1)$

$y = 8$

Vertex: $(1.5, 12.5)$

Axis of Symmetry: $x = 1.5$

x-int: $4, -1$

y-int: 8

Max/min: Max @ 12.5

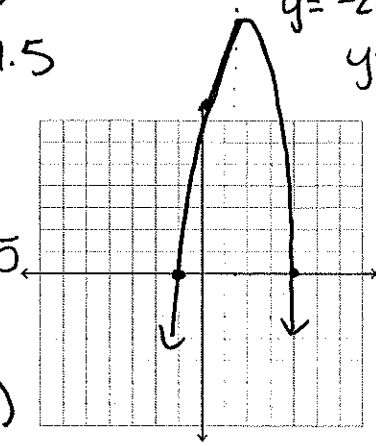
Increasing: $(-\infty, 1.5)$

Decreasing: $(1.5, \infty)$

End Behavior: as $x \to \pm\infty$ $y \to -\infty$

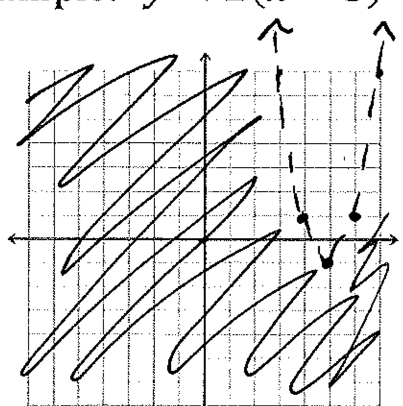
Domain: $(-\infty, \infty)$

Range: $(-\infty, 12.5)$



Quadratics with Inequalities (in foldable)

Example: $y < 2(x-5)^2 - 1$



$0 < 2(0-5)^2 - 1$

$0 < 49$

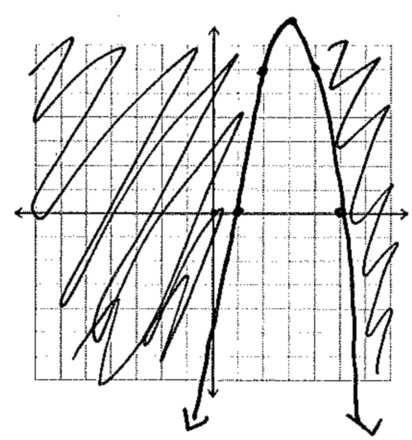
True

< or >

dotted

1) Plot points
2) Decided Dotted or solid lines
3) Shade
Test a point

$y \geq -2(x-3)^2 + 8$



$0 \geq -2(0-3)^2 + 8$

$0 \geq -10$

True

≤ or ≥

solid

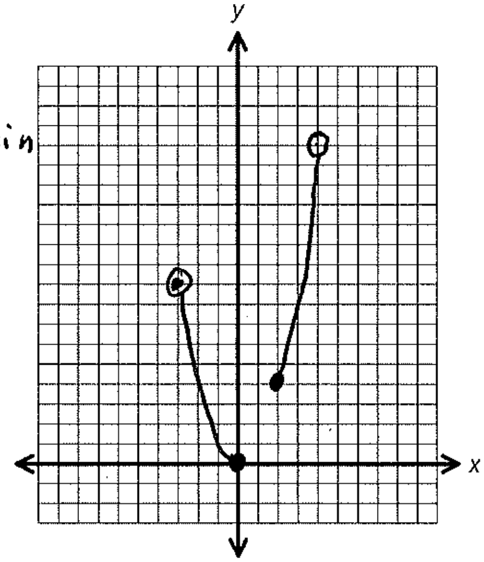
( ) [ ]
open close

3) Graph $y = x^2$ over the domain (-3, 0] ∪ [2, 4)

x	y
-3	9
0	0
2	4
4	16

- Find endpoints of restricted Domain
- Decide open/close
- Connect where stated

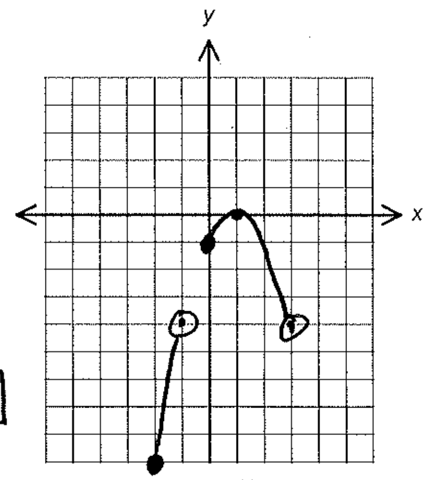
★ Always check where vertex will be.



4) Graph $f(x) = -(x-1)^2$ over the domain [-2, -1) ∪ [0, 3)

x	y
-2	-9
-1	-4
0	-1
3	-4

Vertex: (1, 0)



5) Graph y = - (x + 3)(x – 1) over the domain [-3,-1) ∪ (1 ,3]

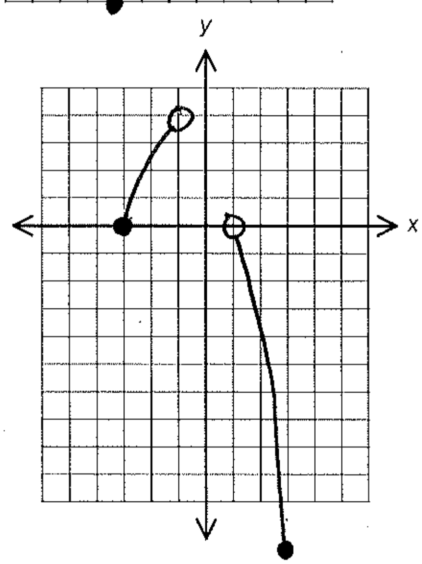
x	y
-3	0
-1	4
1	0
3	-12

Vertex:

$\frac{-3+1}{2} = \frac{-2}{2} = -1$

$-(-1+3)(-1-1) = 4$

$(-1, 4)$



3.4 Notes: Completing the Square

Completing the Square: We can change a quadratic in standard form into vertex form by completing the square.

In order to complete the square we need to create a perfect square trinomial that can be factored into a binomial squared. → Ex: $(x-4)(x-4) = (x-4)^2$

perfect trinomial → $x^2 - 8x + 16$

Steps to completing the square:

→ Separate the problem

→ Use $\left(\frac{b}{2}\right)^2$, Add AND Sub. the num.

→ Simplify

Examples: Write each quadratic in vertex form.

1) $y = x^2 - 4x + 3$

$(x^2 - 4x + \underline{4}) + 3 - \underline{4}$

$\left(\frac{-4}{2}\right)^2 = (-2)^2 = 4$

$\boxed{y = (x-2)^2 - 1}$

2) $y = -3x^2 - 18x - 10$

Take out GCF when possible

$(-3x^2 - 18x + \underline{\quad}) - 10 - \underline{\quad}$

$-3(x^2 + 6x + \underline{9}) - 10 - \underline{-27}$

Because of GCF

$\left(\frac{6}{2}\right)^2 = (3)^2 = 9$

$\boxed{y = -3(x+3)^2 + 17}$

Examples: Write each function in vertex form. Draw a sketch that includes the vertex, x-intercepts, and domain and range in interval notation.

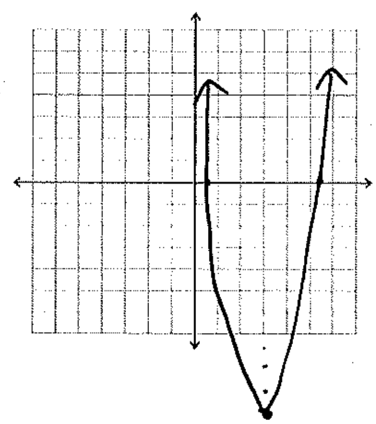
3) $y = 2x^2 - 12x + 7$

$(2x^2 - 12x + \underline{\quad}) + 7 - \underline{\quad}$

$2(x^2 - 6x + \underline{9}) + 7 - \underline{18}$

$\left(\frac{-6}{2}\right)^2 = (-3)^2 = 9$

$\boxed{y = 2(x-3)^2 - 11}$



x-int

$0 = 2(x-3)^2 - 11$

$11 = 2(x-3)^2$

$5.5 = (x-3)^2$

$\pm\sqrt{5.5} = x - 3$

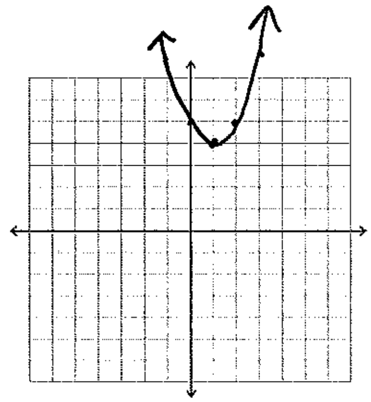
$3 \pm \sqrt{5.5} = x$

D: $(-\infty, \infty)$

R: $(-11, \infty)$

$x = 5.3, 0.65$

4) $y = x^2 - 2x + 5$ (note: why no x-intercepts?)



D: $(-\infty, \infty)$

R: $[4, \infty)$

$(x^2 - 2x + \underline{1}) + 5 - \underline{1}$

$\left(\frac{-2}{2}\right)^2 = (-1)^2 = 1$

$\underline{y = (x-1)^2 + 4}$

$0 = (x-1)^2 + 4$

$-4 = (x-1)^2$

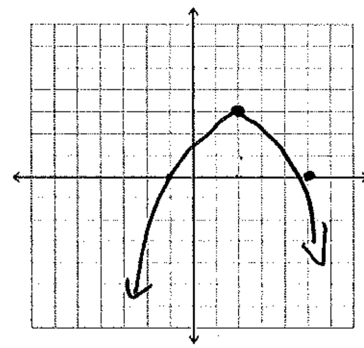
$\pm 2i = x - 1$

$1 \pm 2i = x$

No real x-int!

Matches graph

5) $y = -\frac{1}{3}x^2 + \frac{4}{3}x + \frac{5}{3}$



D: $(-\infty, \infty)$

R: $(-\infty, 3]$

$\left(-\frac{1}{3}x^2 + \frac{4}{3}x + \underline{\quad}\right) + \frac{5}{3} - \underline{\quad}$

$-\frac{1}{3}(x^2 - 4x + \underline{4}) + \frac{5}{3} - \underline{-\frac{4}{3}}$

$\left(\frac{-4}{2}\right)^2 = (-2)^2 = 4$

$-\frac{1}{3}(x-2)^2 + \frac{9}{3} = -\frac{1}{3}(x-2)^2 + 3$

$0 = -\frac{1}{3}(x-2)^2 + 3$

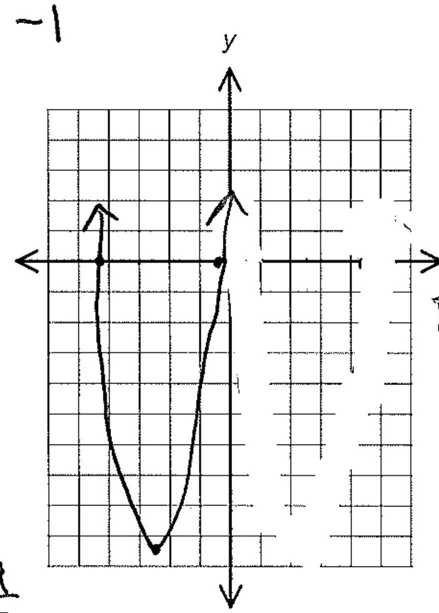
$-3 = -\frac{1}{3}(x-2)^2$

$9 = (x-2)^2$

$\pm 3 = x - 2$

$x = 5, -1$

6) $y = 2x^2 + 10x + 3$



D: $(-\infty, \infty)$

R: $\left[-\frac{19}{2}, \infty\right)$

$(2x^2 + 10x + \underline{\quad}) + 3 - \underline{\quad}$

$2\left(x^2 + 5x + \underline{\frac{25}{4}}\right) + 3 - \underline{\frac{25}{2}}$

$\left(\frac{5}{2}\right)^2 = \frac{25}{4}$ $\quad \frac{6}{2} - \frac{25}{2}$

$2\left(x + \frac{5}{2}\right)^2 - \frac{19}{2}$

$0 = 2\left(x + \frac{5}{2}\right)^2 - \frac{19}{2}$

$\frac{19}{2} = 2\left(x + \frac{5}{2}\right)^2$

$\frac{19}{4} = \left(x + \frac{5}{2}\right)^2$

$\frac{\pm\sqrt{19}}{2} = x + \frac{5}{2}$

$x = \frac{-5}{2} \pm \frac{\sqrt{19}}{2}$

$x = -4.68, -0.32$

3.5 Notes: Standard Form

Complete pages 5 and 6 of the foldable

Foldable for Graphing Parabolas:

Standard Form

$y = ax^2 + bx + c$

y-intercept is: c

To find the vertex:

complete the square
or
$(-b/2a, f(-b/2a))$

To find the x-intercepts:

factor
or
Quad formula

The graph opens up if $a > 0$

The graph opens down if $a < 0$

The graph is stretched if $|a| > 1$

The graph is compressed if $|a| < 1$

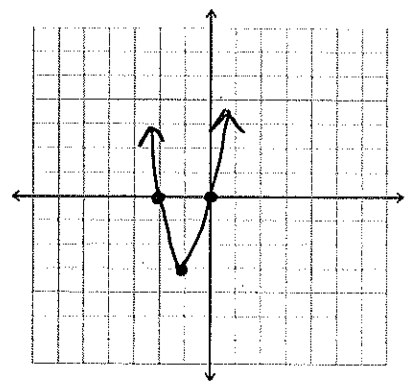
Example: $y = 3x^2 + 6x$

$3x(x+2)$

$3(-1)(-1+2)$
$-3(1)$
-3

Vertex: $(-1, -3)$

Axis of Symmetry: $x = -1$



x-int: $0, -2$

y-int: 0

Max/min: @ -3

Increasing: $[-1, \infty)$

Decreasing: $(-\infty, -1)$

End Behavior: as $x \to \pm\infty$ $y \to \infty$

Domain: $(-\infty, \infty)$

Range: $[-3, \infty)$

Complete Page 9 of the foldable

Word problems with Quadratics

- **Asking for max or min? Find vertex**
- **Asking for horizontal distance? Find the x-intercepts**
- **Note: A graphing calculator can find both of these.**

max height : 1445 ft
horizontal : 20 ft

Example) A rocket is shot into the air, and its path can be modeled by $y = -5x^2 + 30x + 1400$, where x is the horizontal distance, in feet, and h is the vertical distance, in feet. Find the max height reached and the total horizontal distance traveled by the rocket when it returns to the ground.

vertex

$\frac{-b}{2a} = \frac{-30}{2(-5)} = 3$

$-5(3)^2 + 30(3) + 1400$

1445

x-int

vertex

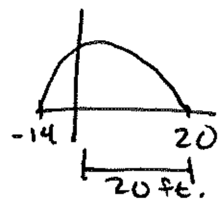
$(3, 1445)$

$0 = -5x^2 + 30x + 1400$

$0 = x^2 - 6x - 280$

$0 = (x-20)(x+14)$

1) An object in launched directly upward at 64 feet per second (ft/s) from a platform 80 feet high. What will be the object's maximum height? When will it attain this height?



$y = -16t^2 + v_0 t + h_0$

vertex

$y = -16t^2 + 64t + 80$

$\frac{-64}{2(-16)} = 2$

$-16(2)^2 + 64(2) + 80 = 144$

$(2, 144)$

At 2 sec.
will be a max @ 144 ft

2) The path of a placekicked football can be modeled by the function y = -0.026x(x – 46) where x is the horizontal distance (in yards) and the y is the corresponding height (in yards). How far is the football kicked? What is the football's maximum height?

x-int

$y = -0.026x(x-46)$

x-int: 0, 46

46 yds

vertex

$\frac{46}{2} = 23$

$-0.026(23)(23-46) = 13.754$

$(23, 13.754)$

13.754 yds

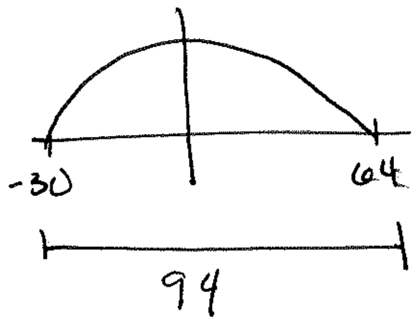
Some review, more word problems:

1) A rainbow's path follows the quadratic $r(x) = -\frac{1}{43}(x+30)(x-64)$, where x is the horizontal distance in miles, and $r(x)$ is the height of the rainbow, in miles. What is the distance between the two places where the rainbow appears to hit the ground?

x-int

$$-\frac{1}{43}(x+30)(x-64)$$

$$x = -30, 64$$



$\boxed{94 \text{ miles}}$

2) Your factory produces lemon-scented widgets. You know that each unit is cheaper, the more you produce. But you also know that costs will eventually go up if you make too many widgets, due to the costs of storage of the overstock. The guy in accounting says that your cost for producing x thousands of units a day can be approximated by the formula $C = 0.04x^2 - 8.504x + 25302$. Find the daily production level that will minimize your costs and state what the cost would be.

$$\frac{--8.504}{2(0.04)} = 106.3$$

vertex

$(106.3, 24850.0124)$

$$0.04(106.3)^2 - 8.504(106.3) + 25302 =$$

106,300 widgets a day

3.6 : Modeling Quadratics

Example 1: Write a quadratic function in vertex form with a vertex at (8, 2) that passes through (-4, -5).

h k

x y

Find a

$y = a(x-h)^2 + k$

$-5 = a(-4-8)^2 + 2$

$-5 = a(144) + 2$

$-7 = a(144)$

$\frac{-7}{144} = a$

$\boxed{y = \frac{-7}{144}(x-8) + 2}$

Example 2: Write a quadratic function in intercept form whose graph has *x*-intercepts at -7 and 2 and passes through the point (-6, -2).

p q x y

$y = a(x-p)(x-q)$

$-2 = a(-6+7)(-6-2)$

$-2 = a(-8)$

$\frac{2}{8} = a$

$\frac{1}{4} = a$

$\boxed{y = \frac{1}{4}(x+7)(x-2)}$

Example 3: Write a quadratic function in vertex form whose graph has vertex at (0 , 2) and passes through the point (1, -3).

h k

x y

$y = a(x-h)^2 + k$

$-3 = a(1-0)^2 + 2$

$-5 = a$

$\boxed{y = -5x^2 + 2}$

Example 4: Write a quadratic function in intercept form with roots at (-2, 0) and (5, 0) passing through (-1, -6). *Which of the following points would be on the parabola?* *(0, -10); (7, 12); (-3, 8) (Choose all that apply)*

p q x y

(0,10) and (-3,8)

$-6 = a(-1+2)(-1-5)$

$-6 = a(-6)$

$1 = a$

$\boxed{y = (x+2)(x-5)}$

(0, -10) ✓

$-10 = (0+2)(0-5)$

$-10 = -10$

(7, 12) ✗

$12 = (7+2)(7-5)$

$12 = 18$

(-3, 8) ✓

$8 = (-3+2)(-3-5)$

$8 = 8$

Example 5: Choose all of the following functions that represent a parabola opening downward with a stretch factor of 2 and x-intercepts at -3 and 2.

A. $y = -2\left(x + \frac{1}{2}\right)^2 + \frac{25}{2}$
~~B.~~ $y = 2(x+3)(x+2)$
C. $y = -2x^2 - 2x + 12$
D. $y = -2(x+3)(x-2)$
~~E.~~ $y = -2\left(x + \frac{1}{2}\right)^2 + 12$

$0 = -2(x+\frac{1}{2})^2 + \frac{25}{2}$
$\frac{-25}{2} = -2(x+\frac{1}{2})^2$
$\frac{25}{4} = (x+\frac{1}{2})^2$
$\pm\frac{5}{2} = x + \frac{1}{2}$
$-\frac{1}{2} \pm \frac{5}{2} = x$
$2, -3 = x$

C)
$0 = -2x^2 - 2x + 12$
$0 = x^2 + x - 6$
$(x+3)(x-2)$
$-3, 2$

$0 = -2(x+\frac{1}{2})^2 + 12$
$-12 = -2(x+\frac{1}{2})^2$
$6 = (x+\frac{1}{2})^2 \rightarrow \pm\sqrt{6} = x + \frac{1}{2}$

Example 6: Write a quadratic function in standard form for the parabola passing through the points (-4, -8), (1, -3) and (2, 10). Hint: create a system of equations.

x y x y x y

$y = ax^2 + bx + c$

$-8 = a(-4)^2 + b(-4) + c$ $\quad -3 = a(1)^2 + b(1) + c$ $\quad 10 = a(2)^2 + b(2) + c$

(a) $-8 = 16a - 4b + c$ $\quad$ (b) $-3 = a + b + c$ $\quad$ (c) $10 = 4a + 2b + c$

(a) $-8 = 16a - 4b + \cancel{c}$
$-$(b) $3 = -a - b - \cancel{c}$
(D) $-5 = 15a - 5b$

(c) $10 = 4a + 2b + \cancel{c}$
$-$(b) $3 = -a - b - \cancel{c}$
(E) $13 = 3a + b$

(D) $-5 = 15a - \cancel{5}b$
5 (E) $65 = 15a + \cancel{5}b$
$60 = 30a$
$2 = a$

$13 = 3(2) + b$
$7 = b$

$-3 = 2 + 7 + c$
$-12 = c$

$\boxed{y = 2x^2 + 7x - 12}$

Challenge:
Given the quadratic $0 = ax^2 + bx + c$, solve for x. Write your answer as a simplified radical.

(Hint, start by completing the square, and then use square roots to solve for x.)

$0 = a(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}) + c - \frac{b^2}{4a}$

$\left(\frac{\frac{b}{a}}{2}\right)^2 = \left(\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2}$

$0 = a\left(x + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a}$

$\frac{-4ac + b^2}{4a} = a\left(x + \frac{b}{2a}\right)^2$

$\pm\sqrt{\frac{-4ac + b^2}{4a^2}} = \sqrt{\left(x + \frac{b}{2a}\right)^2}$

$\pm\frac{\sqrt{b^2 - 4ac}}{2a} = x + \frac{b}{2a}$

$\frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} = x$

$\boxed{\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = x}$

Idea to review:

2 Four Corners Activities! ☺ Divide paper into four quadrants.

Activity 1:
Given the quadratic $y = x^2 - 6x - 7$, do the following in each quadrant.

1) In standard form, find the y-intercept.
2) Complete the square to write the quadratic in vertex form. What is the vertex?
3) Write the quadratic in intercept form. What are the x-intercepts.
4) Graph the quadratic, showing the vertex, axis of symmetry, and all intercepts.

Activity 2:
In groups, students will divide a piece of paper into 4 quadrants.

*For the quadratic $y = -3x^2 + 27$, they will find the x-intercepts by the following methods:
Factoring, square rooting, quadratic formula, and by graphing (okay to use graphing calculator).
*On the back, with the quadratic $y = 3x^2 - 18x - 21$, they will find the vertex using the following methods:
(-b/2a, completing the square, and by using intercept form.) In the fourth quadrant, they will write a paragraph about how to decide when to use each method.

Front

Factor	Square rooting
Quadratic formula	graphing

Back

(-b/2a, y)	Complete the square
Intercept form	paragraph