Ch 9 Notes KEY

Monday, October 2, 2023 1:42 PM

9.1 Notes: Simplifying Radicals

square root $\sqrt{}$

Lesson Objectives

- Simplify square root expressions with numbers and variables

WARM UP
Complete the table with a calculator for as many values as you can. Use a calculator to find the ones you don't already know.

Put a star next to the ones you need to memorize.

n	n^2 (Perfect Squares)	n	n^2 (Perfect Squares)	n	n^2 (Perfect Squares)
1	1	6	36	11	121
2	4	7	49	12	144
3	9	8	64	13	169
4	16	9	81	14	196
5	25	10	100	15	225

Exploration: Work with your group or a partner.

$\sqrt{49}$ what #, times itself equals 49?

a) Simplify: $\sqrt{49} = 7$

b) Simplify: $\sqrt{64} = 8$

c) A square television set has an area of 144 square inches. Find the length of one side.

$\sqrt{144} = \boxed{12 \text{ inches}}$

Square Roots and Radicals	$\sqrt{\text{radicand}}$ what #, times itself, is equal to the radicand?

Examples #1 – 8: Simplify each expression.

1) $\sqrt{49} = 7$

2) $\sqrt{64} = 8$

3) $\sqrt{81} = 9$

4) $\sqrt{1} = 1$

5) $6\sqrt{4} = 6 \cdot 2 = \boxed{12}$

6) $3\sqrt{16}$ $3 \cdot 4 = \boxed{12}$

7) $-7\sqrt{25}$ $-7 \cdot 5 = \boxed{-35}$

8) $5\sqrt{36}$ $5 \cdot 6 = \boxed{30}$

Simplifying Square Roots	* no decimal answers ① make a factor tree ② Look for factors that repeat "pairs" ③ pairs $\sqrt{}$ ~~left-over factors~~ Goal: find any perfect squares & square root those factors

Example 9) Simplify $\sqrt{20}$. Also, use a calculator to find the decimal approximation.

2 · 10

2 · 5

$\boxed{2\sqrt{5}}$

≈ 4.5

Examples #10 – 15: Simplify each of the following radical expressions.

10. $\sqrt{12}$

$\boxed{2\sqrt{3}}$

11. $\sqrt{360}$

$2\cdot 3\sqrt{2\cdot 5}$

$\boxed{6\sqrt{10}}$

12. $-5\sqrt{24}$

$-5\cdot 2\sqrt{2\cdot 3}$

$\boxed{-10\sqrt{6}}$

You Try #13 – 15!

13. $\sqrt{90}$

$3\sqrt{2\cdot 5}$

$\boxed{3\sqrt{10}}$

14. $\sqrt{600}$

$2\cdot 5\sqrt{3\cdot 2} = \boxed{10\sqrt{6}}$

15. $4\sqrt{8}$

$4\cdot 2\sqrt{2}$

$\boxed{8\sqrt{2}}$

Simplifying Square Roots with Variables:

pairs $\sqrt{\text{left-overs}}$

* looking for pairs
* power tells us how many of each variables we have

Examples 16 – 21: Simplify each radical expression. Assume all variables are positive.

16) $\sqrt{x^5}$

$\sqrt{xxxxx}$

$\boxed{x^2\sqrt{x}}$

17) $\sqrt{40x^{11}y^4}$

$2\cdot 2x^5y^2\sqrt{5x}$

$\boxed{4x^5y^2\sqrt{5x}}$

18) $-3\sqrt{50b^7}$

$-3\cdot 5b^3\cdot\sqrt{2b} = \boxed{-15b^3\sqrt{2b}}$

You Try #19 – 21!

19) $\sqrt{a^9b^{14}}$

$\boxed{a^4b^7\sqrt{a}}$

20) $2\sqrt{18x^3y^5}$

$2\cdot 3xy^2\sqrt{2xy}$

$= \boxed{6xy^2\sqrt{2xy}}$

21) $\sqrt{36x^4y^{10}}$

$\boxed{6x^2y^5}$

9.2 Notes: Multiplying and Dividing Radicals

Lesson Objectives

- Multiply square root expressions
- Square radical expressions
- Divide simply square root expressions

Multiplying Square Roots	coefficient $\sqrt{\text{radicand}}$ • coefficient $\sqrt{\text{radicand}}$ mult coeff $\sqrt{\text{mult radicand}}$ ← factor tree

For Examples #1 – 6, simplify each expression.

1) $\sqrt{3}\cdot\sqrt{6}$ $=\sqrt{18}$ $\boxed{3\sqrt{2}}$

2) $\sqrt{8\cdot 2}$ $=\sqrt{16}$ $\boxed{4}$

3) $\sqrt{15}\cdot\sqrt{10}$ $=\sqrt{150}$ $5\sqrt{2\cdot 3}$ $\boxed{5\sqrt{6}}$

You Try #4 – 6!

4) $\sqrt{21}\cdot\sqrt{3}$ $=\sqrt{63}$ $\boxed{3\sqrt{7}}$

5) $\sqrt{2\cdot 6}$ $=\sqrt{12}$ $\boxed{2\sqrt{3}}$

6) $\sqrt{3}\cdot\sqrt{12}$ $=\sqrt{36}$ ← perfect square! $=6$

For Examples #7 – 12: Simplify each expression.

7) $1\sqrt{3}(2\sqrt{3})$ $=2\sqrt{9}$ $=2\cdot 3$ $=\boxed{6}$

8) $5\sqrt{8\cdot 20}$ $=5\sqrt{160}$ $5\cdot 2\cdot 2\sqrt{2\cdot 5}$ $\boxed{20\sqrt{10}}$

9) $-2\sqrt{10}\cdot 5\sqrt{14}$ $=-10\sqrt{140}$ $-10\cdot 2\sqrt{5\cdot 7}$ $\boxed{-20\sqrt{35}}$

You try!

10) $-4\sqrt{35\cdot 21}$ $=-4\sqrt{735}$ $-4\cdot 7\cdot\sqrt{5\cdot 3}$ $\boxed{-28\sqrt{15}}$

11) $1\sqrt{7}(3\sqrt{21})$ $=3\sqrt{147}$ $3\cdot 7\sqrt{3}$ $\boxed{21\sqrt{3}}$

12) $3\sqrt{6}\cdot 4\sqrt{2}$ $=12\sqrt{12}$ $12\cdot 2\sqrt{3}$ $=\boxed{24\sqrt{3}}$

Squaring Radical Expressions	Square means → multiply by itself $(\sqrt{a})^2 \to \sqrt{a} \cdot \sqrt{a} = \cancel{\sqrt{a^2}} = a$ ↳ assume a is positive a a

For Examples #13 – 18, simplify each expression:

13) $(\cancel{\sqrt{5}})^2 = \boxed{5}$

$\sqrt{5} \cdot \sqrt{5} = \sqrt{25}$

14) $(3\sqrt{2})^2 = 3\sqrt{2} \cdot 3\sqrt{2}$

$3^2 \cdot \sqrt{2}^2$ $\quad = 9\sqrt{4}$

$9 \cdot 2$ $\quad 9 \cdot 2 = \boxed{18}$

15) $(-8\sqrt{5})^2 = -8\sqrt{5} \cdot -8\sqrt{5}$

$64\sqrt{25}$

$64 \cdot 5$

$\boxed{320}$

You Try #16 – 18!

16) $(\cancel{\sqrt{11}})^2$

$= \boxed{11}$

17) $(-6\sqrt{3})^2$

$= -6\sqrt{3} \cdot -6\sqrt{3}$

$= 36\sqrt{9} = 36 \cdot 3$

$= 108$

18) $(4\sqrt{7})^2$

$= 4\sqrt{7} \cdot 4\sqrt{7}$

$= 16\sqrt{49} = 16 \cdot 7 = \boxed{112}$

Dividing with Square Roots	① Look for perfect squares ② Fraction reducing ③ Use factor trees for $\sqrt{}$

For #19 – 26, simplify each expression.

19) $\sqrt{\frac{49}{25}} = \frac{\sqrt{49}}{\sqrt{25}} = \boxed{\frac{7}{5}}$

20) $\frac{\sqrt{25}}{\sqrt{9}} = \boxed{\frac{5}{3}}$

21) $\frac{\sqrt{48}}{\sqrt{12}} = \sqrt{4}$

$= \boxed{2}$

22) $\frac{9\sqrt{40}}{3\sqrt{2}}$ $\quad 3\sqrt{20}$ (2, 10; 2, 5)

$3 \cdot 2\sqrt{5}$

$\boxed{6\sqrt{5}}$

You Try #23 – 26!

23) $\sqrt{\frac{4}{121}} = \frac{\sqrt{4}}{\sqrt{121}} = \boxed{\frac{2}{11}}$

24) $\frac{\sqrt{49}}{\sqrt{36}} = \boxed{\frac{7}{6}}$

25) $\frac{\sqrt{27}}{\sqrt{3}}$ divide

$= \sqrt{9}$

$= \boxed{3}$

26) $\frac{10\sqrt{56}}{2\sqrt{7}}$

$= 5\sqrt{8}$ (2, 4; 2, 2)

$5 \cdot 2\sqrt{2} = \boxed{10\sqrt{2}}$

index → $\sqrt[2]{25} = 5$

index → $\sqrt[2]{25} = 5$

5 5

9.3 Notes: Cube Roots and Rational Exponents

Lesson Objectives

- Simplify cube root expressions
- Simplify expressions with rational exponents (fractional)

	n	n^2 (Perfect Squares)	n	n^2 (Perfect Squares)	n	n^2 (Perfect Squares)
WARM UP Complete table without a calculator.	1	1	6	36	11	121
	2	4	7	49	12	144
	3	9	8	64	13	169
	4	16	9	81	14	196
	5	25	10	100	15	225
	n	n^3 (Perfect Cubes)	n	n^3 (Perfect Cubes)		
	1	1	4	64		
	2	8	5	125		
	3	27	6	216		

Simplifying Cube Roots	index $\sqrt[3]{}$ * look for sets of 3 of a kind → make a factor tree outside $\sqrt[3]{\text{left-over}}$

Examples #1 – 6: Simplify each expression. Assume all variables are positive.

1) $\sqrt[3]{54}$

2 27

3 9

3 3

$\boxed{3\sqrt[3]{2}}$

2) $-10\sqrt[3]{40}$

2 20

2 10

2 5

$-10 \cdot 2\sqrt[3]{5}$

$\boxed{-20\sqrt[3]{5}}$

3) $\sqrt[3]{27x^6y^8}$

3 9

3 3

xxxxxx

yyyyyyyy

$\boxed{3x^2y^2 \cdot \sqrt[3]{y^2}}$

You try #4 – 6!

4) $\sqrt[3]{80}$

2 40

2 20

2 10

2 5

$2\sqrt[3]{2 \cdot 5}$

$= \boxed{2\sqrt[3]{10}}$

5) $15\sqrt[3]{270}$

3 90

3 30

3 10

2 5

$15 \cdot 3\sqrt[3]{2 \cdot 5}$

$\boxed{45\sqrt[3]{10}}$

6) $\sqrt[3]{128a^5b^{15}}$

2 64

2 32

2 16

2 8

2 4

2 2

aaaaa

bbbbbbbbbbbbbbb

$2 \cdot 2 \cdot 2ab^5\sqrt[3]{2a^2} = \boxed{8ab^5\sqrt[3]{2a^2}}$

Rational Powers	$x^{1/2} = \sqrt{x}$	$x^{1/3} = \sqrt[3]{x}$

For Examples #7 – 12, simplify each expression.

7) $98^{1/2}$ $\sqrt{98}$ 2 49 7 7 $\boxed{7\sqrt{2}}$

8) $45^{1/2}$ $\sqrt{45}$ 5 9 3 3 $\boxed{3\sqrt{5}}$

9) $250^{1/3}$ $\sqrt[3]{250}$ 2 125 5 25 5 5 $\boxed{5\sqrt[3]{2}}$

You Try #10 – 12!

10) $216^{1/2}$ $\sqrt{216}$ 2 108 2 54 2 27 3 9 3 3 $2\cdot3\sqrt{2\cdot3}$ $\boxed{6\sqrt{6}}$

11) $216^{1/3}$ $\sqrt[3]{216}$ perfect cube $= \boxed{6}$

12) $48^{1/3}$ $\sqrt[3]{48}$ 2 24 2 12 2 6 2 3 $2\sqrt[3]{2\cdot3}$ $\boxed{2\sqrt[3]{6}}$

We can also multiply (and divide) square roots with variables. Below are some examples with multiplication with variables and radicals.

For Examples #13 – 18: Simplify each expression. Assume all variables are positive.

13) $(\sqrt{x^5})(\sqrt{x^6})$ $\sqrt{x^{11}}$ $\boxed{x^5\sqrt{x}}$

14) $5a\sqrt{6a \cdot 3a^4}$ $= 5a\sqrt{18a^5}$ 2 9 3 3 aa aa a $5a\cdot3a^2 \sqrt{2a}$ $\boxed{15a^3\sqrt{2a}}$

15) $6\sqrt{x^3y^2} \cdot 4\sqrt{xy^5}$ $24\sqrt{x^4y^7}$ yy yy yy y $\boxed{24x^2y^3\sqrt{y}}$

You try #16 – 18!

16) $-4\sqrt{3a^3 \cdot 12a}$ $-4\sqrt{36a^4}$ $-4\cdot6a^2$ $= \boxed{-24a^2}$

17) $4b\sqrt{b^3} \cdot \sqrt{b^7}$ $4b\sqrt{b^{10}}$ $4b\cdot b^5$ $\boxed{4b^6}$

18) $(3\sqrt{xy^2})(4\sqrt{x^5y^6})$ $12\sqrt{x^6y^8}$ $\boxed{12x^3y^4}$

Challenge! Assume all variables are positive. Simplify: $-10a^2b \cdot \sqrt[3]{24a^3b^6}$

2 12 2 6 2 3 aaa bbb bbb $\sqrt[3]{3}$

$-10a^2b\cdot2ab^2$

$\boxed{-20a^3b^3\cdot\sqrt[3]{3}}$

9.4 Notes: Rationalizing Expressions

$\frac{3}{\sqrt{5}}$ ← difficult to type

Lesson Objectives

- Rationalize numerical expressions with square roots
- Rationalize variable expressions with square roots

Rationalizing with Square Roots	process of multiplying by a form of 1 so that no fraction has a $\sqrt{\ }$ on denom. $\cdot \frac{\sqrt{\text{denom}}}{\sqrt{\text{denom}}}$

Examples #1 – 8: Simplify each expression. Rationalize as needed. Hint: look for a pattern!

1) $\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \boxed{\frac{\sqrt{3}}{3}}$ 2) $\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \boxed{\frac{\sqrt{2}}{2}}$ 3) $\frac{1}{\sqrt{15}} \cdot \frac{\sqrt{15}}{\sqrt{15}} = \boxed{\frac{\sqrt{15}}{15}}$ 4) $\frac{1}{\sqrt{11}} \cdot \frac{\sqrt{11}}{\sqrt{11}} = \frac{\sqrt{11}}{11}$

You Try #5 – 8!

5) $\frac{1}{\sqrt{23}} \cdot \frac{\sqrt{23}}{\sqrt{23}} = \boxed{\frac{\sqrt{23}}{23}}$ 6) $\frac{1}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \boxed{\frac{\sqrt{5}}{5}}$ 7) $\frac{1}{\sqrt{17}} \cdot \frac{\sqrt{17}}{\sqrt{17}} = \boxed{\frac{\sqrt{17}}{17}}$ 8) $\frac{1}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \boxed{\frac{\sqrt{7}}{7}}$

Examples #9 – 16: Simplify each expression. Rationalize as needed. Hint: look for a pattern!

9) $\frac{5}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \boxed{\frac{5\sqrt{3}}{3}}$ 10) $\frac{-7}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \boxed{\frac{-7\sqrt{2}}{2}}$ 11) $\frac{15}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$ $\frac{15\sqrt{3}}{3} = \boxed{5\sqrt{3}}$ 12) $\frac{14}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \frac{14\sqrt{7}}{7} = \boxed{2\sqrt{7}}$

You Try #13 – 16!

13) $\frac{-4}{\sqrt{23}} \cdot \frac{\sqrt{23}}{\sqrt{23}}$ $\boxed{\frac{-4\sqrt{23}}{23}}$ 14) $\frac{2}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \boxed{\frac{2\sqrt{5}}{5}}$ 15) $\frac{4}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{4\sqrt{2}}{2} = \boxed{2\sqrt{2}}$ 16) $\frac{10}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{10\sqrt{5}}{5} = \boxed{2\sqrt{5}}$

Simplifying Radical Expressions with Ratios (Fractions)	1) Look for Fraction Reducing 2) Simplify Radicals (factor tree) 3) Rationalize, if needed 4) Check for steps 1 and 2 one more time!

Examples #17 – 24: Simplify each expression. Rationalize as needed.

17) $\frac{\sqrt{15} \div \sqrt{15}}{\sqrt{30} \div \sqrt{15}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$ $\boxed{\frac{\sqrt{2}}{2}}$

18) $\frac{\sqrt{20}}{\sqrt{50}} = \frac{\sqrt{2}}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \boxed{\frac{\sqrt{10}}{5}}$

19) $\frac{2}{\sqrt{28}} = \frac{\cancel{2}}{\cancel{2}\sqrt{7}} = \frac{1}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}}$ $\frac{\sqrt{7}}{7}$

20) $\frac{6\sqrt{2} \div \sqrt{2}}{3\sqrt{10} \div \sqrt{2}} = \frac{2}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$ $\boxed{\frac{2\sqrt{5}}{5}}$

You Try #21 – 24!

21) $\frac{\sqrt{20} \div \sqrt{20}}{\sqrt{60} \div \sqrt{20}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$ $= \boxed{\frac{\sqrt{3}}{3}}$

22) $\frac{\sqrt{12} \div \sqrt{3}}{\sqrt{15} \div \sqrt{3}}$ $= \frac{\sqrt{4}}{\sqrt{5}} = \frac{2}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$ $= \boxed{\frac{2\sqrt{5}}{5}}$

23) $\frac{4}{\sqrt{20}} = \frac{4}{2\sqrt{5}} = \frac{2}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$ $= \boxed{\frac{2\sqrt{5}}{5}}$

24) $\frac{10\sqrt{7} \div \sqrt{7}}{2\sqrt{42} \div \sqrt{7}} = \frac{5}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}}$ $= \boxed{\frac{5\sqrt{6}}{6}}$

Rationalizing with Variables	$\cdot \frac{\sqrt{\text{denom}}}{\sqrt{\text{denom}}}$

Examples #25 – 28: Simplify each expression. Rationalize as needed. Assume all variables are positive.

25) $\frac{3}{\sqrt{x}} \cdot \frac{\sqrt{x}}{\sqrt{x}}$ $\boxed{\frac{3\sqrt{x}}{x}}$

26) $\frac{5}{2\sqrt{a}} \cdot \frac{\sqrt{a}}{\sqrt{a}}$ $\boxed{\frac{5\sqrt{a}}{2a}}$

27) $\frac{2}{\sqrt{b}} \cdot \frac{\sqrt{b}}{\sqrt{b}}$ $= \boxed{\frac{2\sqrt{b}}{b}}$

28) $\frac{7}{3\sqrt{y}} \cdot \frac{\sqrt{y}}{\sqrt{y}} = \boxed{\frac{7\sqrt{y}}{3y}}$