Ch 8 Notes KEY

Tuesday, October 3, 2023 12:38 PM

8.1 Notes: Adding and Subtracting Polynomials

Lesson Objectives

1) Define terms used for polynomials
2) Add and subtract polynomials

Key Vocabulary

Monomial
one term $3x^2y$

Binomial
two terms $x^2 + 7x$

Trinomial
three terms $8b^2 - 4d + 17$

Polynomial
"many terms" $5 + 7x^1 - 3y + 18a$

Note: All the exponents must be whole (positive) numbers!

Degree of a polynomial
the highest power
degree 5
$8x^2 - 4x^5 + 2x^3 + 17$

Leading Coefficient
the constant (#) multiplying the variable with the heighest power → -4

Descending order
highest order first
$-4x^5 + 2x^3 + 8x^2 + 17$

Like Terms
Same variables taken to same power

Adding polynomials

* combine like terms (powers stay the same)

For Examples #1 – 2: Find each sum (simplify):

1) $4x^3 + 1x^2 - 5 + 7x + 1x^3 - 3x^2$

$5x^3 - 2x^2 + 7x - 5$

You Try! 2) $(x^2 + x + 8) + (x^2 - x - 1)$

$2x^2 + 7$

Subtracting polynomials

* Distribute -1 into the ( ) after the $-$ sign
* Combine like terms

Remember to multiply each term in the polynomial by – 1 when you write the subtraction as addition.

For Examples 3 – 4: Find the difference (simplify).

3) $4z^2 - 3 - (-2z^2 + 5z - 1)$

$4z^2 - 3 + 2z^2 - 5z + 1$

$6z^2 - 5z - 2$

You Try! 4) $3x^2 + 6x - 4 - (x^2 - x - 7)$

$3x^2 + 6x - 4 - 1x^2 + 1x + 7$

$2x^2 + 7x + 3$

You try! Example 5) Simplify the expression: $3x^2 + 5 - (x^2 + 2) + (-3x + 1)$

$3x^2 + 5 - x^2 - 2 - 3x + 1$

$2x^2 - 3x + 4$

Reminder: Using the Distributive Property

* multiply a monomial into each term in the parenthesis
* exponents change! (add + same base)

For #6 – 8: Simplify each expression. **You Try #7 and #8!**

6) $3x^3(2x^3 - x^2 - 7x - 3)$

$6x^6 - 3x^5 - 21x^4 - 9x^3$

7) $-x^2(x - 6)$

$-x^3 + 6x^2$

8) $\frac{1}{2}y^3(6xy^2 + 8xy - 4)$

$3xy^5 + 4xy^4 - 2y^3$

Example 9) Find the perimeter of the rectangle shown.

Perimeter = sum of all four sides

+ combine all like terms

Perimeter: $8x$

Example 10) Find $h(x) = f(x) + g(x)$ if $f(x) = (7x^2 - 3x + 2)$ and $g(x) = (5x - 2)$.

"h of x"

$h(x) = 7x^2 - 3x + 2 + 5x - 2$

$h(x) = 7x^2 + 2x$

Example 11) Find $h(x) = f(x) - g(x)$ if $f(x) = (-2x^3 - 4x + 2)$ and $g(x) = (5x^3 + 5x^2 - 2x)$

$h(x) = -2x^3 - 4x + 2 - 1(5x^3 + 5x^2 - 2x)$

$= -2x^3 - 4x + 2 - 5x^3 - 5x^2 + 2x$

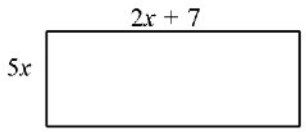
$h(x) = -7x^3 - 5x^2 - 2x + 2$

Example 12) Write a polynomial expression to represent the area of the rectangle shown.

Area = (length)(width)

$Area = 5x(2x + 7)$

$Area = 10x^2 + 35x$



8.2 Notes: Multiplying Polynomials

Lesson Objectives

1) Multiply binomials
2) Square binomials
3) Multiply a binomial and a polynomial

expand Box Method **Multiplying binomials** Distribute 2x FOIL	① Distribute 1st term ② Distribute 2nd term ③ combine like terms

$(a+b)(c+d)$

$ac + ad + bc + bd$

First ac
Outside ad
Inside bc
Last bd
} combine terms

Examples 1 – 4: Multiply the binomials. You Try #3 – 4!

1) $(x+3)(x+4)$

$x^2 + 4x + 3x + 12$

$x^2 + 7x + 12$

2) $(x+3)(x-2)$

$x^2 - 2x + 3x - 6$

$x^2 + x - 6$

3) Multiply: $(3x+7)(x-8)$

$3x^2 - 24x + 7x - 56$

$3x^2 - 17x - 56$

4) Multiply: $(x^2-4)(x-x^2)$

$x^3 - x^4 - 4x + 4x^2$

$= -x^4 + x^3 + 4x^2 - 4x$

Brain Challenge! **Can you multiply these binomials without showing work? Look for patterns!**

5) $(x+5)(x+2)$

$x^2 + 7x + 10$

6) $(a+3)(a+4)$

$a^2 + 7a + 12$

7) $(b+9)(b+3)$

$b^2 + 12b + 27$

8) $(y-2)(y+6)$

$y^2 + 4y - 12$

9) $(d-7)(d-1)$

$d^2 - 8d + 7$

10) $(x-3)(x+3)$

$x^2 - 9$

Example 11: Find $h(x) = f(x) \cdot g(x)$ if $f(x) = (2x+7)$ and $g(x) = (x-9)$.

$(2x+7)(x-9)$

$2x^2 - 18x + 7x - 63$

$2x^2 - 11x - 63$

Squaring binomials	square → multiply by itself $(a+b)^2 = (a+b)(a+b)$ $a^2+ab+ab+b^2 = a^2+2ab+b^2$

Examples 12 – 13: Simplify each expression. **You Try #13!**

12) $(x+4)^2$

$(x+4)(x+4)$

$x^2+8x+16$

13) $(x-7)^2$

$(x-7)(x-7)$

$x^2-7x-7x+49$

$x^2-14x+49$

Examples 14 – 15: Simplify each expression. **You Try #13!**

14) $(3x+4)^2$

$(3x+4)(3x+4)$

$9x^2+12x+12x+16$

$9x^2+24x+16$

15) $(5x-1)^2$

$(5x-1)(5x-1)$

$25x^2-5x-5x+1$

$25x^2-10x+1$

Challenge! #16: Simplify the expression: $(m+7)(m-3)+(m-4)(m+5)$

$1m^2+4m-21+1m^2+1m-20$

$2m^2+5m-41$

Example 17 – 19: Find each product. Challenge: See if you can do these without work 😊

17) $(x-5)(x+5)$

$x^2+5x-5x-25$

x^2-25

18) $(y-3)(y+3)$

y^2-9

19) $(2a-7)(2a+7)$

$4a^2-49$

Conjugates:

binomials that have the same exact expression but with OPPOSITE signs

$(a+b)(a-b)$

What happens when you multiply two conjugates (see examples 17 – 19)?

* middle terms cancel out
* binomial

a^2-b^2

Example 20) Write two binomials that are conjugates and whose product equals $x^2 - 4$.

$(x + 2)(x - 2)$

Check: $x^2 - 4$ ✓

Examples 21 – 24: Multiply the polynomials.

21) $(2a - 5)(a^2 - 6a - 3)$

$2a^3 - 12a^2 - 6a$
$- 5a^2 + 30a + 15$

$2a^3 - 17a^2 + 24a + 15$

22) $(5p - 2)(3p^2 - 2p + 1)$

$15p^3 - 10p^2 + 5p$
$+ \quad -6p^2 + 4p - 2$

$15p^3 - 16p^2 + 9p - 2$

You Try #23 – 24!

23) $(3x - 2)(4x^2 - 5x + 1)$

$12x^3 - 15x^2 + 3x$
$+ \quad -8x^2 + 10x - 2$

$12x^2 - 23x^2 + 13x - 2$

24) $(2a^2 + 3a - 2)(a - 4)$

$(a - 4)(2a^2 + 3a - 2)$

$2a^3 + 3a^2 - 2a$
$+ \quad -8a^2 - 12a + 8$

$2a^3 - 5a^2 - 14a + 8$

Challenge! **#25:** Simplify $2(-4a + 9)^2 + 5$

$2(-4a + 9)(-4a + 9) + 5$

$2(16a^2 - 72a + 81) + 5$

$32a^2 - 144a + 162 + 5$

$32a^2 - 144a + 167$

8.3: Factoring Out the Greatest Common Factor (GCF)

Lesson Objectives

1) Find the GCF for a polynomial expression
2) Factor the GCF out of a polynomial

Exploration: What are the **factors** of each number below?

6: 2 • 3, 1 • 6

15: 3 • 5, 1 • 15

18: 2 • 9, 3 • 6, 1 • 18

What is the greatest common factor of all three numbers?

3 ← GCF

Expand each expression to show all factors. Then find the greatest common factor for all three of the expressions.

$8x^3$: 2 • 4 • 2 • 2, x x x

$-4x^2$: −1 • 2 • 2, x x

$6x^3$: 2 • 3, x x x

$2x^2$ ← GCF

Greatest Common Factor (GCF)	Great expression (numbers & variables) that all terms have as common factors
Factoring out the GCF	a + b + c → GCF(☆ + ☆ + ☆) opposite of distributing!

Examples 1 – 6: Factor each expression by taking out the GCF.

1) $5x + 20$ GCF = 5

$5(x + 4)$

2) $8x - 4x^2$ GCF: $4x$

$4x(2 - x)$

3) $16x^2y + 40xy + 8xy^2$ GCF: $8xy$

$8xy(2x + 5 + y)$

You Try #4 – 6!

4) $6m^2 - 30m^3$ GCF: $6m^2$

$6m^2(1 - 5m)$

5) $12ab + 32b$ GCF: $4b$

$4b(3a + 8)$

6) $8ax^3 + ax^2 - 3ax$ GCF: ax

$ax(8x^2 + x - 3)$

Factoring out the GCF if the first term is negative	* use a negative GCF *all the signs inside ( ) will change

Examples 7 – 12: Factor each expression by taking out the GCF.

7) $-4nm - 2n^2$ GCF: $-2n$

$-2n(2m + n)$

8) $-5wx^3 + 10wx^2$ GCF: $-5wx^2$

$-5wx^2(x - 2)$

9) $-a^3 + 4a^2 - 8a$ GCF: $-a$

$-a(a^2 - 4a + 8)$

You Try #10 – 12!

10) $-6y + 15y^3$ GCF: $-3y$

$-3y(2y - 5y^2)$

11) $-9dm^3 + 1dm^2$ GCF: $-dm^2$

$-dm^2(9m - 1)$

12) $-b^2 - 4b + 1$ GCF: -1

$-1(b^2 + 4b - 1)$

13) One factor of $-7x^3y - 21x^2y^2$ is $(-7x^2y)$. What is the other factor? → other ( )?

GCF

$-7x^2y(x + 3y)$

$(x + 3y)$

14) Factor: $15x^3 - 7y^4 - 2z$ GCF: 1

13 = 1 · 13

PRIME

When a polynomial expression does not have any factors besides 1 and itself, we say that the expression is PRIME.

REVIEW #15 – 17: Find each product. Try to do these in your head.

15) $(x + 2)(x + 3)$ → $x^2 + 5x + 6$

16) $(y + 4)(y + 7)$ → $y^2 + 11y + 28$

17) $(h - 3)(h + 5)$ → $h^2 + 2h - 15$

Challenge! What are the factors of each trinomial? (Try to work backwards to figure this out!)

18) $x^2 + 6x + 8$ → $(x + 2)(x + 4)$; $+ + = 6$, $\cdot = 8$ (2, 4 / 1, 8)

19) $x^2 + 7x + 10$ → $(x + 2)(x + 5)$; $2 + 5 = 7$, $2 \cdot 5 = 10$ (2, 5 / 1, 10)

8.4 Notes: Intro to Factoring Trinomials and Binomials

Lesson Objectives

1) Factor a trinomial into two binomials
2) Factor a difference of two squares
3) Determine if a polynomial is prime (unable to be factored).

Work in groups to multiply (expand) the following expressions: Challenge: Try this without work!

$(x+5)(x-3)$ $(x+2)(x+8)$ $(x+4)(x-4)$

$x^2+2x-15$ $x^2+10x+16$ x^2-16

Factoring a Trinomial in the form x^2+bx+c (no GCF)	$\_ + \_ = b$, $\_ \cdot \_ = c$, $(x+\_)(x+\_)$ — Look for factor of c that combine (+ or −) to b

Example 1: Factor $x^2+10x+16$ (add, mult: 1,16 / 2,8 / 4,4)

$(x+2)(x+8)$

Check by multiplying your answer:

$x^2+8x+2x+16$ ✓

Examples 2 – 7: Factor each expression.

2) $x^2+10x+9$ (add, mult: 1,9 / 3,3)

$(x+9)(x+1)$

3) a^2+6a+9 (add, mult: 1,9 / 3,3)

$(a+3)(a+3)$

$(a+3)^2$

4) $x^2+7x+12$ (add, mult: 1,12 / 2,6 / 3,4)

$(x+3)(x+4)$

You try #5 – 7!

5) x^2+5x+6 (1,6 / 2,3)

$(x+2)(x+3)$

6) y^2+5y+4 (1,4 / 2,2)

$(y+1)(y+4)$

7) x^2+2x+1 (1,1)

$(x+1)(x+1)$ or $(x+1)^2$

Examples 9 – 14: Factor each trinomial. You might have to use some negative values at times.

9) $x^2+4x-12$ (add) 1, 12 / −2, 6 / 3, 4

$(x-2)(x+6)$

10) $w^2-10w+25$ 1, 25 / −5, −5

$(w-5)(w-5)$ or $(w-5)^2$

11) a^2-a-30 1, 30 / 2, 15 / 3, 10 / 5, −6

$(a+5)(a-6)$

You try #12 – 14! Factor each trinomial.

12) $x^2 - 10x - 24$ — $(x+2)(x-12)$

13) $b^2 + 7b - 18$ — $(b-2)(b+9)$

14) $y^2 - 3y + 2$ — $(y-1)(y-2)$

Reminder: What are conjugates? $(a+b)(a-b)$

Multiply each expression below. (Try to do this without work!)

$(x-5)(x+5)$ — $x^2 - 25$

$(x+11)(x-11)$ — $x^2 - 121$

Factoring a Difference of Two Perfect Squares $a^2 - b^2$	$a^2 - b^2 = (a+b)(a-b)$ conjugates * binomial * subtract * perfect square

For #15 – 22: Factor each expression.

15) $x^2 - 25$ — $(x+5)(x-5)$

16) $a^2 - 49b^2$ — $(a+7b)(a-7b)$

17) $36 - y^{10}$ — $(6+y^5)(6-y^5)$

18) $x^6 - 100$ — $(x^3+10)(x^3-10)$

You try #19 – 22!

17) $g^2 - 4$ — $(g+2)(g-2)$

18) $1 - b^2$ — $(1+b)(1-b)$

19) $k^8 - 81j^2$ — $(k^4+9j)(k^4-9j)$

20) $n^{10} - 9$ — $(n^5+3)(n^5-3)$

Consider: $a^2 + 25$. Try to factor this expression. Multiply out any answers you get to check your ideas.

PRIME

$(a+5)(a-5)$

$a^2 - 25$

$(a+5)(a+5)$

$a^2 + 10a + 25$

If an expression does not factor at all, then it is PRIME.

8.5: More Factoring Trinomials and Binomials

Lesson Objectives

1) **Factor a trinomial of the form $ax^2 + bx + c$**
2) **Review other factoring techniques.**

Exploration: Simplify each expression. Try to do these without showing work!

a) $(3x - 1)(x + 4)$

$3x^2 + 11x - 4$

b) $(5x + 2)(3x - 7)$ -end

$15x^2 - 29x - 14$

Start

Factoring a Trinomial in the form $ax^2 + bx + c$	① check for GCF ② examine factors of a and c ← use together to combine to b factors of a factors of c

10

1,2 mult — add 1 — 1,5 mult

Example 1: Factor $2x^2 + 11x + 5$

$(1x + 5)(2x + 1)$

10x

1x

Check your solution by using multiplication:

$x^2 + 1x + 10x + 5$

$x^2 + 11x + 5$

For Examples 2 – 7: Factor each expression.

1,3 1,1

2) $3n^2 + 4n + 1$

$(n + 1)(3n + 1)$

1,9 3,3

3) $9y^2 + 6y + 1$ +1 +1

$(3y + 1)(3y + 1)$

$(3y+1)^2$

19

1,2 1 1,9 3,3

4) $2x^2 + 19x + 9$

$(x + 9)(2x + 1)$

1,2 14 1,7

You Try # 5 – 7!

5) $2y^2 + 15y + 7$

$(y + 7)(2y + 1)$

3,1 1,4 2,2

6) $3a^2 + 8a + 4$

$(3a + 2)(a + 2)$

1,15 3,5 1,2

7) $15x^2 + 13x + 2$

$(5x + 1)(3x + 2)$

Examples 8 – 13: Factor each expression. You might have to use some negative values or extra variables!

8) $3x^2 - 1x - 2$ (1,3) (1,2)

$(x-1)(3x+2)$

9) $6d^2 - 23d + 7$ (2,3) (1,7) (1,6)

$(3d-1)(2d-7)$

10) $8b^2 + 14ab - 15a^2$ (1,8) (2,4) (1,15) (3,5)

$(2b+5a)(4b-3a)$

You Try #11 – 13!

11) $3a^2 - 10a - 8$ (1,3) (2,4) (1,8)

$(a-2)(3a-4)$

12) $15y^2 - 13y + 2$ (1,15) (3,5) (1,2)

$(5y-1)(3y-2)$

13) $2m^2 + 1mn - 21n^2$ (1,21) (3,7) (1,2)

$(m-3n)(2m+7n)$

Reminder: Factoring Difference of Perfect Squares

$a^2 - b^2$ $(a+b)(a-b)$ conjugates

Examples 14 – 19: Factor each expression.

14) $25x^2 - 4$

$(5x+2)(5x-2)$

15) $49b^4 - 9d^2$

$(7b^2+3d)(7b^2-3d)$

16) $36a^2 - b^6$

$(6a+b^3)(6a-b^3)$

You try #17 – 19!

17) $121h^2 - 4g^8$

$(11h+2g^4)(11h-2g^4)$

18) $25 - 16k^2$

$(5+4k)(5-4k)$

19) $49x^2 - 1$

$(7x+1)(7x-1)$

Reminder! Factoring out the GCF:

GCF(+ +)

opposite of distrib. prop

Examples 20 – 22: Factor out the GCF for each expression.

20) $6x^2 - 8x$ GCF: $2x$

$2x(3x-4)$

21) $-a^5 - 9a^6$ GCF: $-a^5$

$-a^5(1+9a)$

22) $-4x^3y + 4x^2y$ GCF: $-4x^2y$

$-4x^2y(x-1)$

Challenge! Factor the trinomial below in 2 steps. First factor out the GCF and then factor the remaining trinomial: $5x^2 + 15x + 10$ GCF: 5

$5(x^2+3x+2)$ (2,1)

$5(x+2)(x+1)$

8.6 Notes: Factoring Completely

Lesson Objective

- Completely factor all polynomials (or state they are prime).

Factoring COMPLETELY	① Check for GCF first! Binomial: $a^2 - b^2$ → GCF $(a+b)(a-b)$ Trinomial: $ax^2 + bx + c$ → GCF $(\quad)(\quad)$

Examples 1 – 6: Factor each polynomial completely. Always look for a GCF first!

1) $5a^2 - 405$ GCF: 5

$5(a^2 - 81)$

$5(a+9)(a-9)$

2) $2x^2 - 8x - 10$ GCF: 2

$2(x^2 - 4x - 5)$ (−5, 1)

$2(x-5)(x+1)$

3) $-3r^3 + 21r^2 - 30r$ GCF: $-3r$

$-3r(r^2 - 7r + 10)$ (−5, −2; 1, 10)

$-3r(r-5)(r-2)$

You Try #4 – 6!

4) $6x^6 - 24$ GCF = 6

$6(x^6 - 4)$

$6(x^3+2)(x^3-2)$

5) $5x^2 + 10x - 15$ GCF = 5

$5(x^2 + 2x - 3)$ (3, −1)

$5(x+3)(x-1)$

6) $-x^3 - x^2 + 12x$ GCF = $-x$

$-x(x^2 + 1x - 12)$ (1, 12; 2, 6; −3, 4)

$-x(x-3)(x+4)$

Example 7 – 10: Factor completely. Look for a GCF first!

7) $6x^2 + 26x + 8$ GCF = 2

$2(3x^2 + 13x + 4)$ (1, 3; 1, 4; 2, 2)

$2(x+4)(3x+1)$

8) $-2x^3 - 5x^2y - 2xy^2$ GCF = $-x$

$-x(2x^2 + 5xy + 2y^2)$ (1, 2; 1, 2)

$-x(x+2y)(2x+y)$

You try!

9) $-20x^2 + 10x + 10$ GCF = −10

$-10(2x^2 - 1x - 1)$ (1x, 2x; 1, −1)

$-10(x-1)(2x+1)$

10) $30a^3 + 21a^2b + 3ab^2$ GCF: $3a$

$3a(10a^2 + 7ab + 1b^2)$ (5a, 2a; 1a, 10a; 1b, 1b)

$3a(5a+1b)(2a+1b)$

Examples 11 – 19: Factor each expression completely. Write "prime" if no factoring can be done. Not all problems will have a GCF, but some might.

11) $5x^3 - 20x$ GCF: $5x$

$5x(x^2 - 4)$

$5x(x+2)(x-2)$

12) $3x^2 - 13x + 12$

$1x, 3x$; $1,12$; $2,6$; $3,4$

$(x-3)(3x-4)$

13) $g^2 + 16$

PRIME

You try #14 – 19!

14) $-x^5 + 9x^3$ GCF = $-x^3$

$-x^3(x^2 - 9)$

$-x^3(x+3)(x-3)$

15) $-2x^3 - 20x^2 - 42x$ GCF = $-2x$

$-2x(x^2 + 10x + 21)$

$7,3$; $1,21$

$-2x(x+7)(x+3)$

16) $8x^2 - 63x - 81$ GCF = none

~~$2x, 4x$~~ ; $1x, 8x$; $1,81$; $3,27$; $9,9$

$(x-9)(8x+9)$

17) $-a^5 - 3a^4$ GCF = $-a^4$

$-a^4(a+3)$

degree of 1 will not factor further

18) $25x^2 + 1$

PRIME

19) $-15x^2 + x + 2$ GCF = -1

$-1(15x^2 - 1x - 2)$

$1x, 15x$; $3x, 5x$; $1,2$

$-1(3x+1)(5x-2)$