Ch 7 Notes KEY

Monday, October 2, 2023 1:42 PM

7.1 Notes: Modeling with Exponential Functions

Learning Objectives:

- Model exponential growth and decay from real life problems using a growth/decay rate and initial value.
- Explore graphs of exponential functions in the form $y = ab^x$ by using technology.

Exploration: David is offered a choice: Option A: receive $1000 or Option B: start $3, where the $3 is doubled every day for ten days.

A) Which offer do you think should David take? Why?

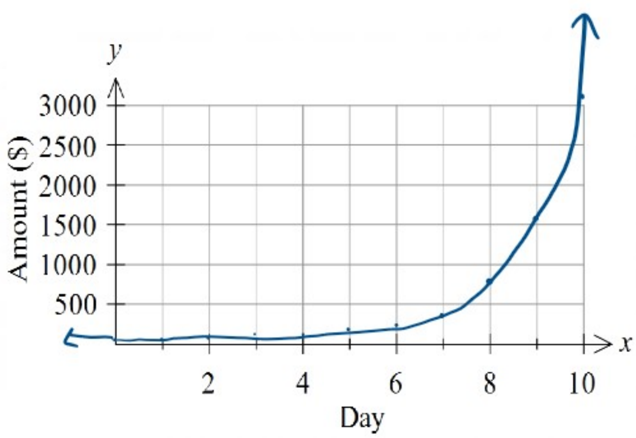
Option B → earns more $
$3072 versus $1000

Day (x)	Amount (y)
0	3
1	6
2	12
3	24
4	48
5	96
6	192
7	384
8	768
9	1536
10	3072

(×2 between consecutive amounts)

B) Fill out the table to the right for David for Option B, where Day 0 is the starting day (when he receives $3). Reminder: his money doubles every day for the next 10 days. **Was your decision in part A the best one?**

C) As a class, use the table of values you created in part B to sketch a graph of the function modeling David's money.



D) The equation that model's David's money with Option B is $y = 3(2)^x$. As a class, use a graphing calculator technology to graph this equation. Compare it to the sketch you created as a graph.

same graph

E) Knowing now that the equation for Option B is $y = 3(2)^x$, where does the 3 come from in the equation?

starting amount

Where does the 2 come from in the equation?

doubling the $ each day

Key Vocabulary for Exponential Growth in the form $y = a\,b^x$.

$y = a \cdot b^x$

Exponential Growth	where our values are multiplied by the same # (> 1) each time; x is a power
Initial Value	"a" starting amount
Growth Factor	# multiplying by each time "b" 100 + %

$y = a \cdot b^x$

Examples using Exponential Growth:

1) Andrea sets a reading goal for the summer. She has been spending 3 hours each week reading all semester, but during summer break, she wants to multiple the number of hours she is reading by 1.5 each week.

a) Write an equation to model the number of hours of reading ($\boldsymbol{y}$) Andrea will do for each week of summer break ($\boldsymbol{x}$).

Initial amount: 3 ← "a"

Growth factor: 1.5 ← "b"

$y = 3(1.5)^x$

b) Use your equation to determine how many hours Andrea will read during the 5th week of summer break. Also, use technology to graph your equation from part a) to verify your answer.

$y = 3(1.5)^5$

$y \approx 22.78$ hours

$y = a \cdot b^x$

2) You put \$250 into a savings account that earns 4% annual interest each year.

a) Write an exponential growth function that could be used to find the value ($\boldsymbol{A}$) of your savings account after $\boldsymbol{t}$ years. Assume you do not make any deposits or withdrawals.

Initial amount: 250

Growth factor: 100 + 4 = 104% = 1.04

$A = a \cdot b^t$

$A = 250(1.04)^t$

b) Use your equation to find the value in the account after 6 years. Use technology to graph your equation from part a) to verify your answer.

$A = 250(1.04)^6$

\$316.33

3) Eduardo buys a rare baseball card for \$150. The value of the card increases by 30% each year.

a) Write an exponential growth function that could be used to find the value ($\boldsymbol{y}$) of the card $\boldsymbol{t}$ years after he bought it.

Initial amount: 150

Growth factor: 100 + 30 = 130% → 1.30

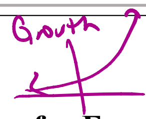
$y = a \cdot b^x$

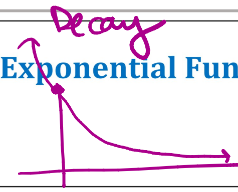
$y = a \cdot b^t$ → $y = 150(1.30)^t$

b) Find the value of the card after 4 years. Graph your equation from part a) to verify your answer.

$y = 150(1.30)^4$

\$428.42





Key Vocabulary for Exponential Decay in the form $y = a \cdot b^x$.

Exponential Decay	$y = a \cdot b^x$ multiplier is 0 < multiplier < 1
Initial Value	"a" starting amount
Decay Factor	multiplier "b" 100 − %

Examples using Exponential Decay:

1) A new car is purchased for a cost of $42,000. Each year, it loses half of its value.

a) Write an exponential decay equation to model the value y of the car after x years.

Initial amount: 42,000

Decay factor: $\frac{1}{2}$ or 0.5

$y = a \cdot b^x$

$y = 42{,}000\left(\frac{1}{2}\right)^x$

b) Use your equation to find the value of the car after 3 years. Use technology to graph your equation from part a) to verify your answer.

$y = 42{,}000\left(\frac{1}{2}\right)^3$

$5,250

2) Christina takes 500 grams of a certain medication. Every hour, the amount of medication in her bloodstream decays by 30%.

a) Write an exponential decay equation to model the number of grams of medication (y) left in Christina's bloodstream after t hours.

Initial amount: 500

Decay factor: 100 − 30 = 70% → .70

$y = a \cdot b^x$

$y = a \cdot b^t$

$y = 500(0.7)^t$

b) Use your equation to find the amount of medication left in her bloodstream after 4 hours. Use technology to graph your equation from part a) to verify your answer.

$y = 500(0.7)^4$

120.05 grams

3) You buy a computer for $1,500. It depreciates at the rate of 20% per year.

a) Write an exponential decay equation that could be used to find the value (A) of the computer after t years.

Initial amount: 1500

Decay **factor:** 100 − 20 = 80% = 0.80

$y = a \cdot b^x$

$A = a \cdot b^t$

$A = 1500(0.80)^t$

b) Find the value of the computer after 5 years. Graph your equation from part a) to verify your answer.

$A = 1500(0.8)^5$

$491.52

7.2 Notes: Graphing Exponential Functions, Day 1

Learning Objectives:

- Graph exponential functions in the form $\boldsymbol{y = a \cdot b^x}$
- Identify exponential functions as growth or decay.

Key Vocabulary

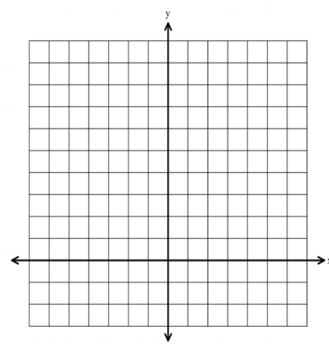
Exponential Function	
Horizontal Asymptote	

Exploration: Use a graphing calculator or other technology to graph the following exponential functions. Describe any patterns that you observe.

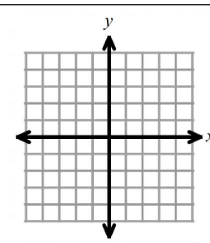
$y = 3^x$ $y = 2 \cdot 3^x$ $y = 5 \cdot 3^x$ $y = -3^x$

Example 1: Use a table of values to graph $y = 2^x$.

$\boldsymbol{x}$	$\boldsymbol{y = 2^x}$
-2	
-1	
0	
1	
2	



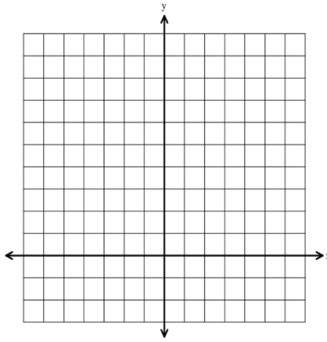
Key Vocabulary

Exponential Growth Function	$\boldsymbol{y = a \cdot b^x}$ where $\boldsymbol{b > 1}$ 
Graphing Exponential Growth Functions	1. Draw a horizontal asymptote at $\boldsymbol{y = 0}$ 2. Plot the initial value (or "anchor point") ______ units above (or below) the horizontal asymptote, on the $\boldsymbol{y}$-axis. 3. Evaluate the function at $\boldsymbol{x = 1}$ to get another point. 4. Draw a growth curve. Alternative method: Evaluate the function at 3 points and draw a growth curve.

For examples 2 – 8: Graph each exponential growth function.

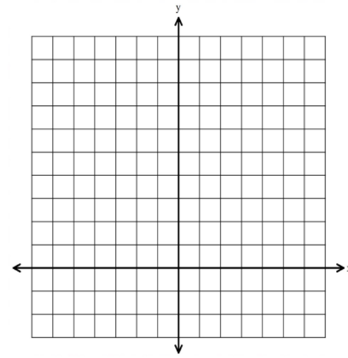
2) $f(x) = 4^x$

x	y



3) $y = 2 \cdot 4^x$

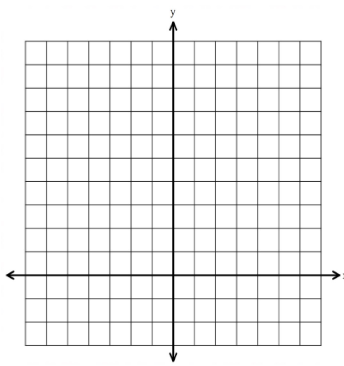
x	y



You Try #4 – 5!

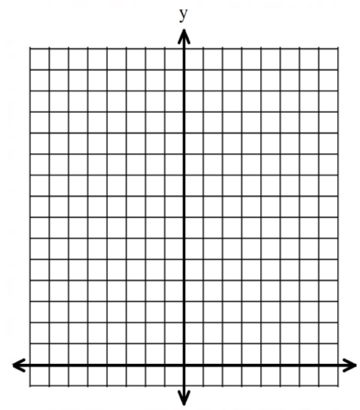
4) $y = 5^x$

x	y



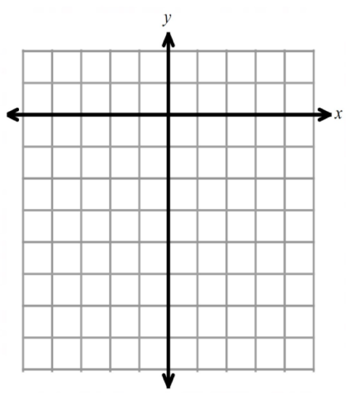
5) $h(x) = 3(5)^x$

x	y



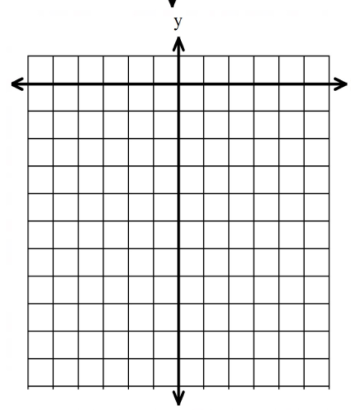
6) $y = -2^x$

x	y



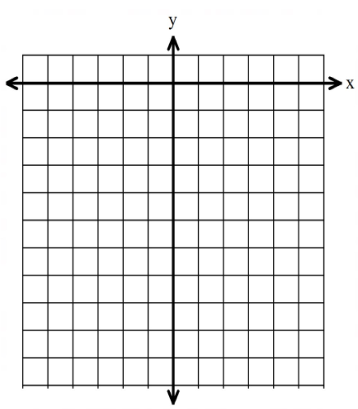
7) $f(x) = -3 \cdot 2^x$

x	y

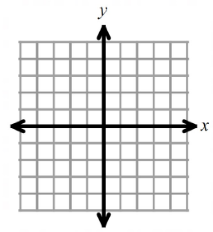


You Try #8!

8) $g(x) = -4(2)^x$



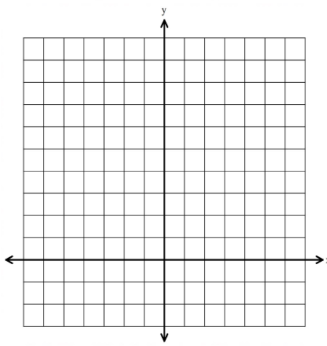
Key Vocabulary

Exponential Decay Function	$y = a \cdot b^x$ where $\mathbf{0 < b < 1}$ 
Graphing Exponential Decay Functions	1. Draw a horizontal asymptote at $y = 0$ 2. Plot the initial value (or "anchor point") _______ units above (or below) the horizontal asymptote, on the y-axis. 3. Evaluate the function at $x = -1$ to get another point. 4. Draw a decay curve. Alternative method: Evaluate the function at 3 points and draw a growth curve.

For examples 9 – 12: Graph each exponential decay function.

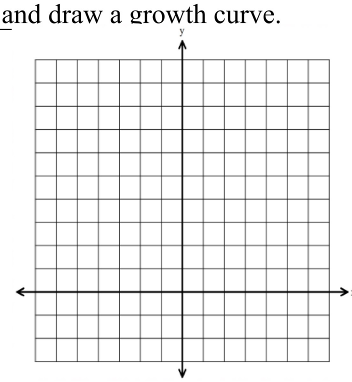
9) $g(x) = \left(\frac{1}{2}\right)^x$

x	*y*



9) $y = 3\left(\frac{1}{2}\right)^x$

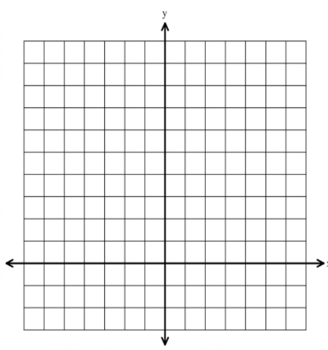
x	*y*



You Try #4 – 5!

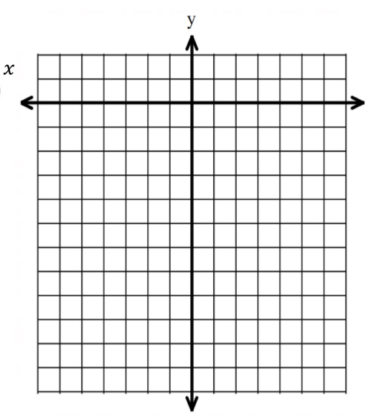
10) $y = \left(\frac{1}{5}\right)^x$

x	*y*



11) $f(x) = -2\left(\frac{1}{5}\right)^x$

x	*y*



For #12 – 17, identify each exponential function as GROWTH or DECAY.

12) $y = -2\left(\frac{3}{4}\right)^x$

13) $y = 5(4)^x$

14) $f(x) = 5(3)^{-x}$

You Try #15 – 17!

15) $h(x) = -3 \cdot 5^x$

16) $y = -2\left(\frac{1}{6}\right)^{-x}$

17) $g(x) = 4\left(\frac{2}{5}\right)^x$

7.3 Notes: Graphing Exponential Functions, Day 2

Learning Objectives:

- Graph exponential functions in the form $\boldsymbol{y = a \cdot b^{x-h} + k}$
- Identify exponential functions as growth or decay.
- Describe transformations from a parent function.

Exploration: Use a graphing calculator or technology to graph each set of exponential functions. Then describe any patterns that you see.

Set A) $y = 2^x$ $y = 2^x + 3$ $y = 2^x - 4$

Set B) $y = 2^x$ $y = 2^{x-3}$ $y = 2^{x+5}$

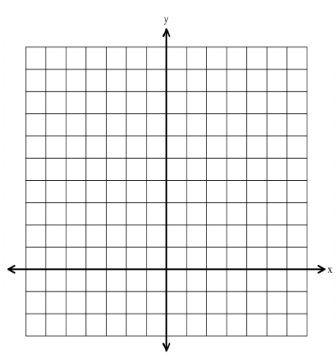
Key Vocabulary for $\boldsymbol{y = a \cdot b^{x-h} + k}$

Horizontal Asymptote		**Anchor Point**	
Graphing Exponential Functions in the form $\boldsymbol{y = a \cdot b^{x-h} + k}$	1. Draw a horizontal asymptote at $\boldsymbol{y =}$ _____ 2. Plot the initial value (or "anchor point") at $(\boldsymbol{h, a + k})$ 3. Draw a growth or decay curve, based on the value of $\boldsymbol{b}$. Alternative method: Evaluate the function at 3 points and draw a growth or decay curve.		

Examples #1 – 4: Graph each exponential function. Include the equation of the horizontal asymptote (HA), the coordinates of the anchor point, and identify the function as growth or decay.

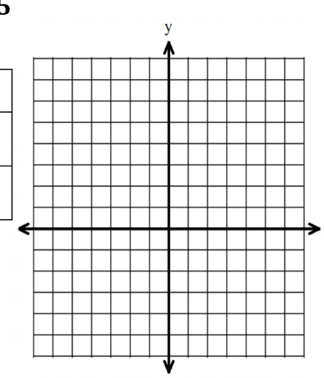
1) $\boldsymbol{y = 4 \cdot 3^{x-1} + 2}$

HA	
Anchor point	
Growth or decay?	



2) $\boldsymbol{y = \left(\frac{1}{2}\right)^{x+3} - 5}$

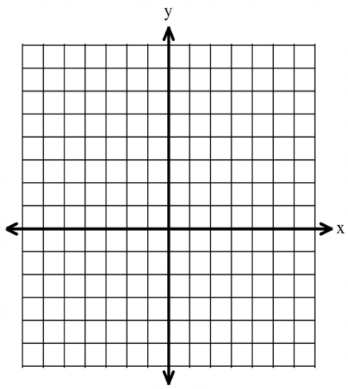
HA	
Anchor point	
Growth or decay?	



You Try #3 – 4!

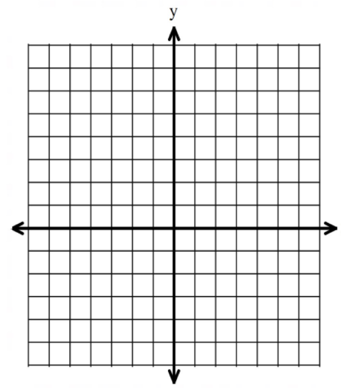
3) $y = 2 \cdot \left(\frac{1}{3}\right)^{x-4} - 3$

HA	
Anchor point	
Growth or decay?	



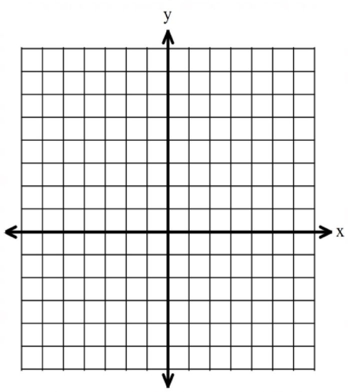
4) $y = (4)^{x+2} + 1$

HA	
Anchor point	
Growth or decay?	



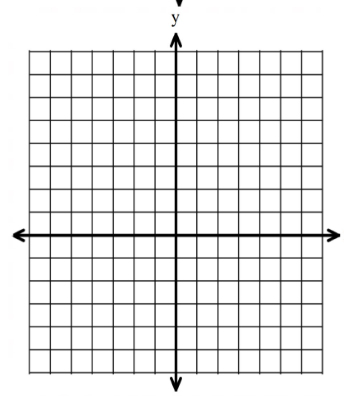
5) $y = \left(\frac{3}{4}\right)^{x+5}$

HA	
Anchor point	
Growth or decay?	



6) $y = 3(2)^{x} - 1$

HA	
Anchor point	
Growth or decay?	



Transformations from $y = b^x$ to $y = a \cdot b^{x-h} + k$

Transformations		**Vertical Reflection**	
Horizontal Shifts		**Vertical Shifts**	
Vertical Stretch		**Reminder about *h*!**	

Example 7: Describe the transformations from $f(x)$ to $g(x)$ if $f(x) = 3^x$ and $g(x) = -2 \cdot 3^{x+1} - 4$. Then graph both on a graphing calculator or other technology to verify the transformations.

For #8 – 11: Describe the transformations from $\boldsymbol{f(x)}$ to $\boldsymbol{g(x)}$.

8) $f(x) = \left(\frac{1}{3}\right)^x$; $g(x) = -\left(\frac{1}{3}\right)^x - 4$

You Try #9!

9) $f(x) = 4^x$; $g(x) = 5(4)^{x+3}$

10) $g(x) = -3f(x-5) + 2$

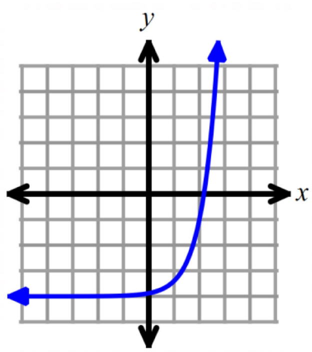
11) $g(x) = 2f(x+4) - 7$

Key Vocabulary for $y = a \cdot b^{x-h} + k$

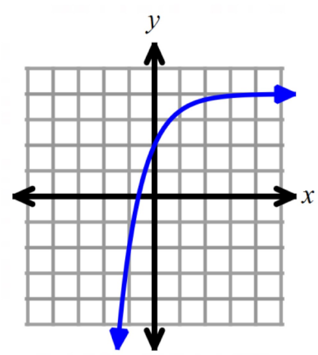
Domain		**Range**	

Examples 12 – 15: Find the domain and range of each exponential function. Also, graph #14 and #15.

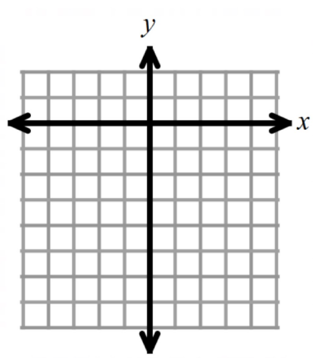
12) $y = 3(5)^{x-2} - 4$



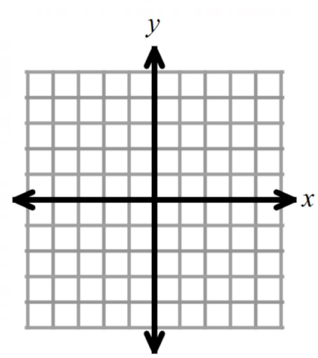
13) $y = -2\left(\frac{1}{3}\right)^x + 5$



14) $f(x) = -3(4)^{x+1} - 2$



15) $y = 5(2)^{x-4}$

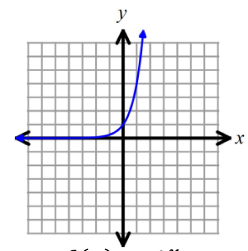


Graphs of Exponential Functions Summary

Characteristics of the graphs of Exponential Growth: $f(x) = a \cdot b^{x-h} + k$

When $b > 1$, $f(x)$ has exponential GROWTH

- There is a **horizontal asymptote** along the x-axis (at $y = k$.)
- For **exponential growth,** $f(x)$ moves **away** from the horizontal asymptote as the function moves to the right.
- There is an **anchor point** at $(h, a + k)$
- **Domain:** all real numbers
- **Range:** $y > k$ or $y < k$ if a is negative.
- The larger the base number, the faster the function grows away from the horizontal asymptote as the function moves to the right.
- **Transformations**
  - h will shift the graph left or right
  - k will shift the graph up and down
  - **vertical reflection** if a is negative
  - **vertical stretch** if $|a| > 1$



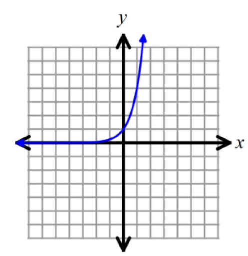
$f(x) = 4^x$

Exponential Growth

Characteristics of the graphs of Exponential Decay: $f(x) = a \cdot b^{x-h} + k$

When $0 < b < 1$, $f(x)$ has exponential DECAY

- There is a **horizontal asymptote** along the x-axis (at $y = k$.)
- For **exponential growth,** $f(x)$ moves **away** from the horizontal asymptote as the function moves to the LEFT.
- There is an **anchor point** at $(h, a + k)$
- **Domain:** all real numbers
- **Range:** $y > k$ or $y < k$ if a is negative.
- The smaller the base number, the faster the function grows away from the horizontal asymptote as the function moves to the LEFT
- **Transformations**
  - h will shift the graph left or right
  - k will shift the graph up and down
  - **vertical reflection** if a is negative
  - **vertical stretch** if $|a| > 1$



$f(x) = \left(\frac{1}{4}\right)^x$

Exponential Decay

7.4: Geometric Sequences and Forms of Functions

Learning Objectives:

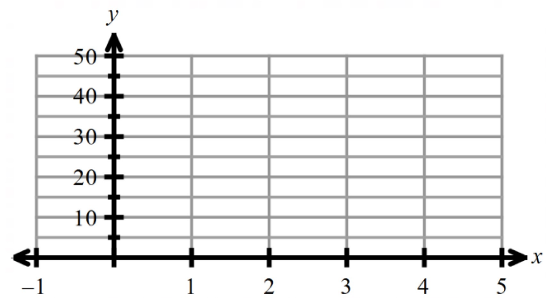
1) Write the explicit form of a geometric sequence.
2) Determine if a function is linear or exponential.

x	0	1	2	3
y	6	12	24	48

Exploration: Consider the exponential function described in the table:

A) Is this a growth or decay function? How do you know?

B) Write the equation that models this function in the form $y = a \cdot b^x$. If needed, reference the 7.1 Notes.



C) Plot the 4 points from this table on the graph:

D) Sketch the exponential function that passes through those points.

E) Take a guess! What do you think the height of the function will be when $x = 4$?

Key Vocabulary:

Geometric Sequence	Explicit Form of Geometric Sequence
	$a_n = a_1 \cdot r^{n-1}$

Terms of the Explicit Form			
a_n	a_1	r	n

For Examples 1 – 6: Write the explicit form for each geometric sequence.

1) 6, 12, 24, 48, …
2) 351, 117, 39, 13, …
3) 3, 12, 48, 192, …

You Try #4 – 6!

4) 27, 9, 3, 1, …
5) 10, 50, 250, 1250, …
6) 224, 112, 56, 28, …

7) For the geometric sequence given in #6, use the explicit form to find the following (use a calculator):
a) a_{10} b) a_{17} c) a_{22}

You Try!
8) For the geometric sequence given in #3, use the explicit form to find the following (use a calculator):
a) a_{12} b) a_{29} c) a_{40}

Key Vocabulary: Determine the Form of a Function

Linear Function	Exponential Function
From a table:	From a table:
From a graph:	From a graph:
From an equation:	From an equation:

For #9 – 16, determine whether the function is linear or exponential.

9)

x	-3	-2	-1	0	1	2
y	-7	-5	-3	-1	1	3

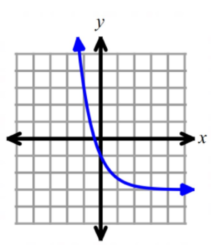
10)

x	-3	-2	-1	0	1	2
y	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4

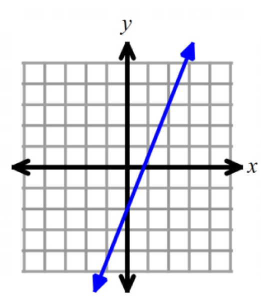
11) $y = -5^{x-2} + 3$

12) $2x + 3y = 9$

13)



14)



15)

x	0	1	2	3
y	6	10	14	18

16)

x	-3	-2	-1	0	1
y	4	12	36	108	324

You try #17 – 25! Linear or exponential?

17) $y = -4x + 5$

18) $y = \left(\frac{1}{3}\right)^x$

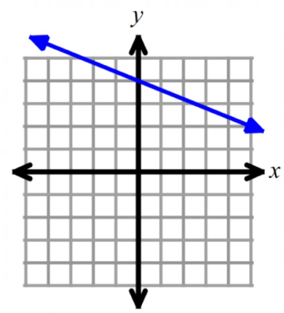
19)

x	0	1	2	3
y	351	117	39	13

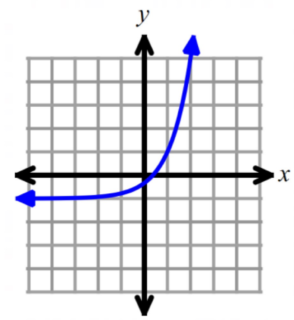
20)

x	0	1	2	3
y	3	-6	-15	-24

21)



22)



Rate of Change

- For a line, the rate of change (or ________________) stays the same for the entire line.

- For an exponential graph, the rate of change is not the same for the entire curve.

- For exponential growth, the rate of change __________________ as the function moves to the right. We can see this by looking at how the graph gets steeper as the function moves to the right.

- For exponential decay, the rate of change __________________ as the function moves to the right. We can see this by looking at how the graph gets less steep as the function moves to the right.