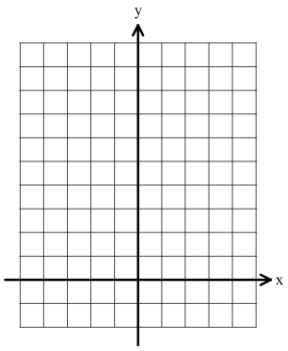
11.1 Notes: Graphing Quadratics in Vertex Form

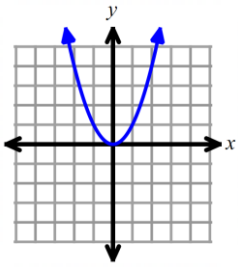
Lesson Objectives

- Create a table of values for the parent function $y = x^2$
- Graph quadratic functions in vertex form: $y = a(x - h)^2 + k$
- Identify the vertex, domain, range and transformations of quadratic functions.

Exploration: Work with a partner or in a group to create a table of values and sketch the graph of the function $\boldsymbol{y = x^2}$.



x	$y = x^2$
-3	
-2	
-1	
0	
1	
2	
3	

Quadratic Function	Note: the graph of a quadratic function is commonly called a "parabola."
Quadratic Parent Function $y = x^2$	 Vertex: Domain: Range: Max or Min? at what point?
Vertex Form of a Quadratic Function	Opening up Opening down Vertex:

For Examples #1 – 6, identify the vertex of each quadratic function and whether it opens up or down.

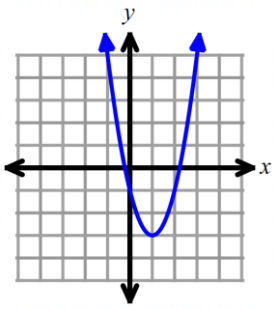
1) $y = (x - 3)^2 + 4$ 2) $f(x) = -2(x + 1)^2 + 5$ 3) $y = 4x^2 - 3$

You Try #4 – 6!

4) $h(x) = -(x + 4)^2 - 7$ 5) $y = 5(x - 3)^2$ 6) $y = \frac{1}{4}x^2 + 9$

Examples 7 – 8: For each graph below, find the requested information. **You Try #8!**

7) $y = 2(x-1)^2 - 3$

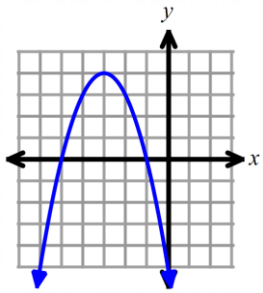


Vertex:

Max or Min?

at what point?

8) $y = -(x+3)^2 + 4$



Vertex:

Max or Min?

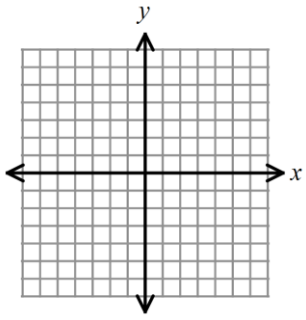
at what point?

How to Graph a Quadratic Function in Vertex Form	$y = a(x-h)^2 + k$ 1) Plot the vertex at _____. 2) Use _____ to find one point on each side of the vertex. Note: Why can this only be used to find one point? 3) Draw a parabola by connecting these three points.

Examples 9 – 11: Sketch each quadratic function. Identify the vertex and include two other points.

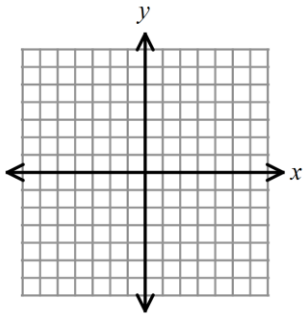
9) $y = 2(x-1)^2$

Vertex:



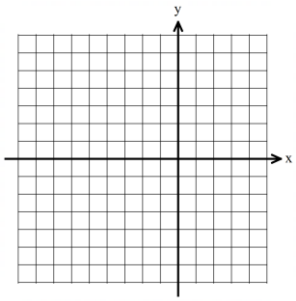
10) $f(x) = -5x^2 + 3$

Vertex:



11) $g(x) = \frac{1}{3}(x+4)^2 - 3$

Vertex:



Examples 12 – 14: Find the domain and range of the identified quadratic function. **You Try #14!**

12) From #9

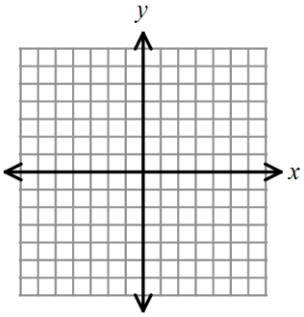
13) From #10

14) From #11

You Try #15 – 17! Sketch each quadratic function. Identify the vertex and include two other points.

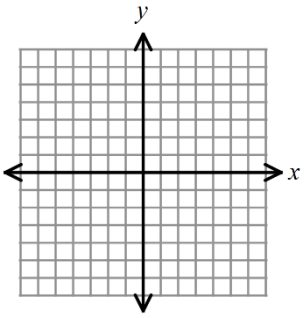
15) $y = 2x^2 - 1$

Vertex:



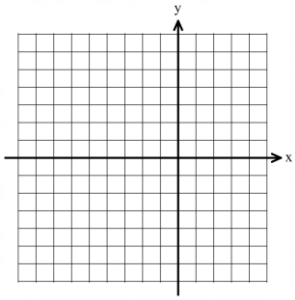
16) $f(x) = -(x-2)^2$

Vertex:



17) $g(x) = -\frac{1}{2}(x+1)^2 + 3$

Vertex:

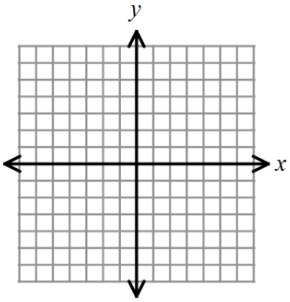


18) For #16: What is the domain and range? Does this function have a max or a min, and at what point?

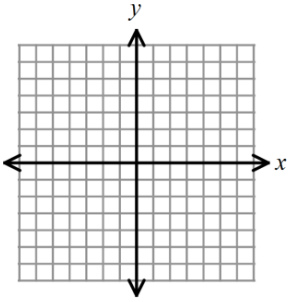
Transformations of Quadratic Functions in Vertex Form	$y = a(x-h)^2 + k$			
	Vertical stretch	Vertical compression	Vertical reflection	Shifts *(or translations)*

Examples 19 – 20: For each quadratic function, describe the transformations from $y = x^2$, and sketch the function.

19) $f(x) = -4(x-3)^2 + 5$

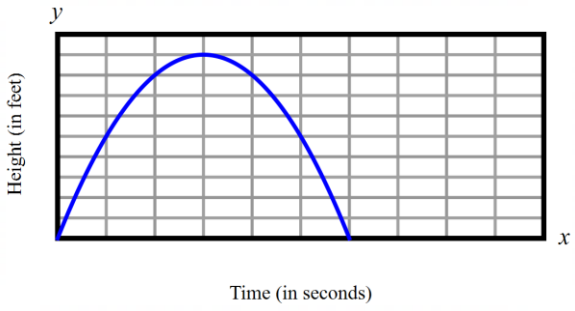


20) $g(x) = \frac{1}{2}x^2 - 4$



21) The function $f(x)$ describes the height of a football x seconds after it is kicked. $f(x)$ is graphed to the right. What is the max height of the football? _____

How many seconds after the football is kicked is the max height achieved? _____



11.2 Notes: Completing the Square

Lesson Objectives

- Complete the square in order to write a quadratic function in vertex form.
- Review how to graph a quadratic in vertex form.

Perfect Square Trinomials	

Examples 1 – 6 : Find the missing value that would make the trinomial a perfect square. Then factor each trinomial.

1) $x^2 + 6x + \_\_\_\_\_$ 2) $x^2 - 10x + \_\_\_\_\_$ 3) $x^2 + 8x + \_\_\_\_\_$

You Try #4 – 6!

4) $x^2 + 10x + \_\_\_\_\_$ 5) $x^2 - 2x + \_\_\_\_\_$ 6) $x^2 - 20x + \_\_\_\_\_$

Completing the Square	When to use completing the square: Standard form: $y = ax^2 + bx + c$	Steps for completing the square:

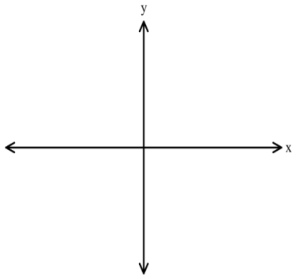
Examples 6 – 7: Complete the square to rewrite the equation in vertex form, and then identify the vertex.

6) $y = x^2 + 4x + 10$ **You try!** 7) $y = x^2 - 6x - 2$

Vertex Form of a Quadratic Function:

For Examples 8 – 12: Write each function in vertex form by completing the square, and then sketch the function. Include the vertex. Identify the domain and range of each.

8) $y = x^2 - 18x + 4$

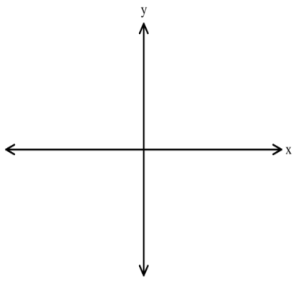


Vertex:

Domain:

Range:

9) **You try!** $y = x^2 + 8x + 5$



Vertex:

Domain:

Range:

For #10 – 11: Complete the square to write each quadratic function in vertex form. Then find the vertex.

10) $y = 3x^2 - 24x + 10$

You try! 11) $y = -4x^2 - 8x + 13$

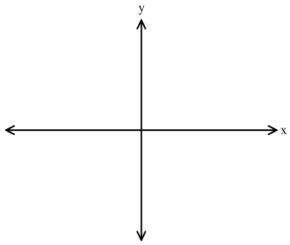
For #12 – 13: Write each quadratic in vertex form, sketch the function, and find the requested info.

12) $y = -2x^2 + 20x + 6$

Vertex:

Domain:

Range:

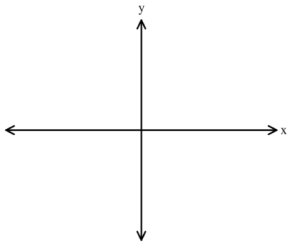


13) **You try!** $y = 3x^2 - 18x - 2$

Vertex:

Domain:

Range:



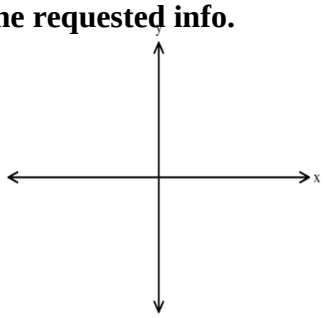
For #14: Write each quadratic in vertex form, sketch the function, and find the requested info.

14) $y = -x^2 + 10x + 2$

Vertex:

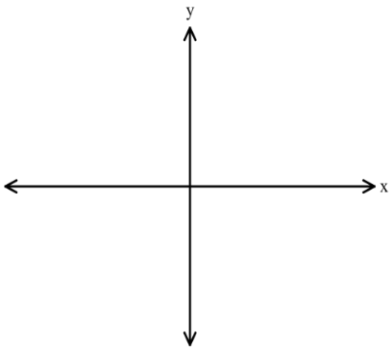
Domain:

Range:



Examples #15 – 16: A football is kicked in the air, and the height of the football can be modeled by the equation $y = -x^2 + 2x + 4$, where *x* is the number of seconds after the ball is kicked.

15) Find the maximum height of the football. Hint: Be sure to factor out the negative to start!



16) After how many seconds does the football reach its maximum height?

11.3 Notes: Graphing Quadratics in Intercept Form

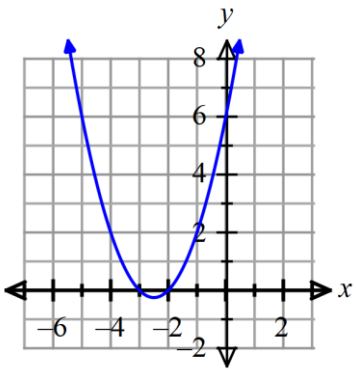
Lesson Objectives

- Write quadratic functions in intercept form.
- Graph quadratics in intercept form.

Exploration 1: Given the quadratic equation: $x^2 + 5x + 6 = 0$

a) Use factoring to solve for *x*.

b) Consider the graph of $f(x) = x^2 + 5x + 6$, as shown to the right. Where is the *y*-intercept for $f(x)$? Compare this to the equation for $f(x)$. What do you notice?

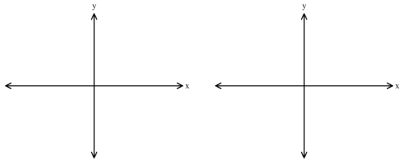


c) Where are the *x*-intercepts for $f(x)$? Compare this to your solutions from part **a).** What do you notice?

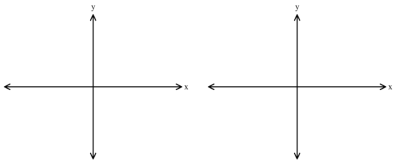
Quadratics in Standard Form	y-intercept:
Quadratics in Intercept Form	*x*-intercept(s):

Intercept Form of a Quadratic Function: $y = a(x - p)(x - q)$

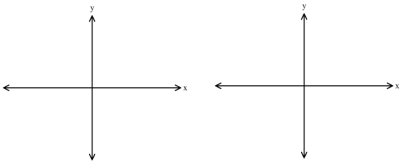
Quadratic with two *x*-intercepts



Quadratic with one *x*-intercept

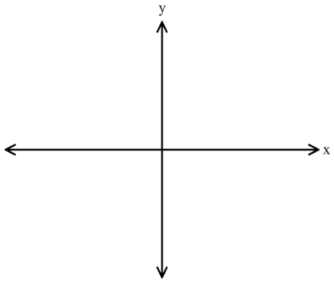


Quadratic with no *x*-intercepts

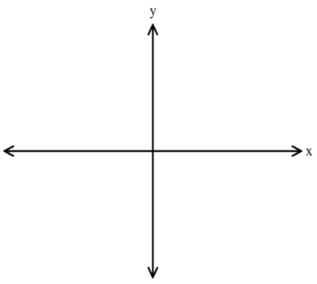


Examples #1 – 6: Sketch each quadratic function. Include the *y*-intercept and any *x*-intercepts.

1) $y = x^2 + 7x - 18$

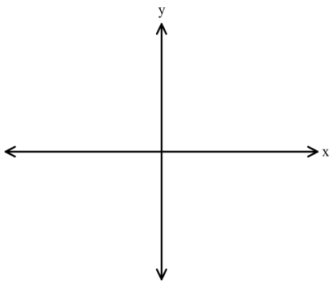


2) $f(x) = -2x^2 - 8x$

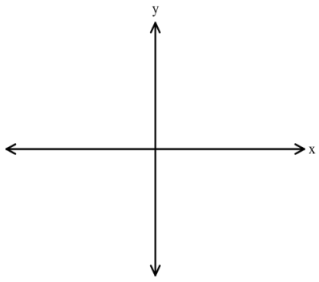


You try #3 – 4!

3) $h(x) = 2x^2 - 10x$

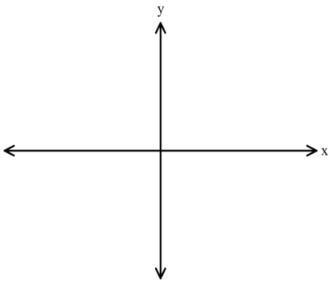


4) $y = x^2 + 8x + 16$

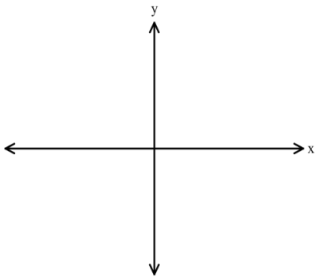


You try #6!

5) $g(x) = -3x^2 + 12$

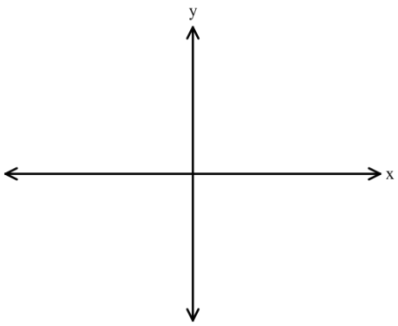


6) $a(x) = 5x^2 - 45$

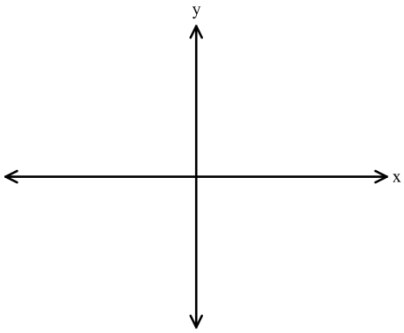


Finding the y-intercept from Intercept Form	

Example 7: Sketch the graph of $y = -(x + 3)(x - 1)$. Include the *x*-intercepts and the *y*-intercept.



You Try! 8) Sketch the graph of $y = 2x(x - 6)$. Include the *x*-intercepts and the *y*-intercept.



You Try! 9) Which of the following are terms for *x*-intercepts?

A) zeros B) roots C) solutions D) all of these are correct

11.4 Notes: Converting Quadratic Functions

Lesson Objectives

- Convert forms of quadratic functions to other forms.
- Analyze forms of quadratic functions to solve problems.

How are the various forms of quadratic functions useful?

Form	This form is useful to find…	To convert to another form…
Vertex Form		**Vertex Form → Standard Form** Expand by multiplying and combining like terms.
Standard Form		**Standard Form → Vertex Form** Complete the Square (see 11.2 Notes) **Standard Form → Intercept Form** Factor the expression, if possible
Intercept Form		**Intercept Form → Standard Form** Expand by multiplying and combining like terms.

Examples 1 – 8: Convert each quadratic function to the requested form.

1) $y = 3(x-2)(x+3)$; Standard Form

2) $y = -5(x-1)^2 + 4$; Standard Form

3) $f(x) = x^2 - 6x - 1$; Vertex Form

4) $g(x) = x^2 - 3x - 4$; Intercept Form

You Try # 5 – 8! Convert each quadratic function to the requested form.

5) $y = -4(x+1)(x-6)$; Standard Form

6) $y = 2(x+3)^2 + 7$; Standard Form

7) $f(x) = x^2 + 4x - 17$; Vertex Form

8) $g(x) = x^2 - 7x + 12$; Intercept Form

9) What are the *x*-intercepts from #4?

10) What is the vertex from #7?

11) What is the y −intercept from #2?

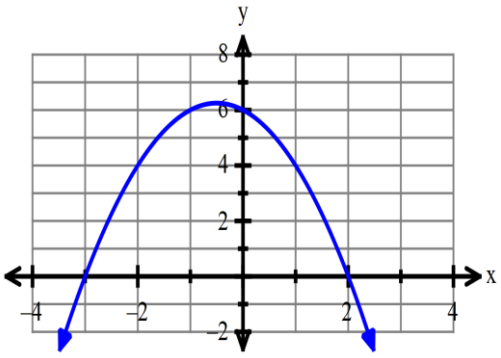
12) Which of the following equations matches the graph shown?

A) $y = (x-3)(x+2)$

B) $y = (x+3)(x-2)$

C) $y = -(x-3)(x+2)$

D) $y = -(x+3)(x-2)$



You Try #13!

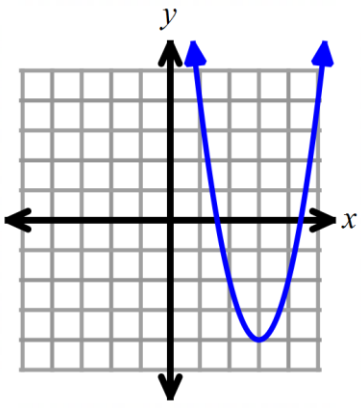
13) Which of the following equations matches the graph shown?

A) $y = -2(x+3)^2 + 4$

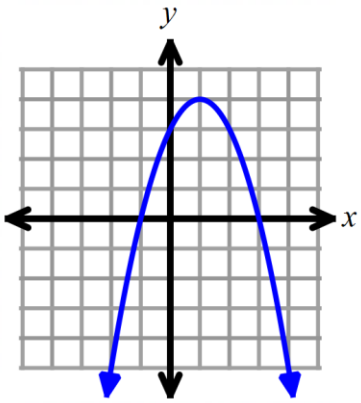
B) $y = 2(x+3)^2 + 4$

C) $y = -2(x-3)^2 + 4$

D) $y = 2(x-3)^2 + 4$



14) Which of the following functions model the graph shown? **Select all that apply.**



A) $y = -(x-1)^2 + 4$
B) $y = -x^2 + 2x + 3$
C) $y = -(x+3)(x-1)$
D) $y = -(x-3)(x+1)$
E) $y = x^2 - 2x - 3$

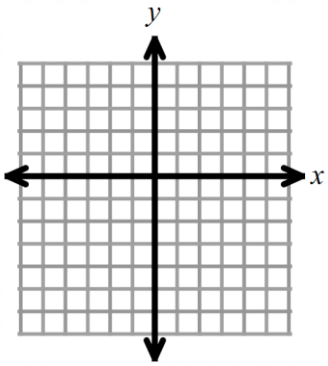
For Examples 15 – 18: Given the quadratic function $f(x) = -2x^2 + 8x - 6$.

15) What is the *y*-intercept?

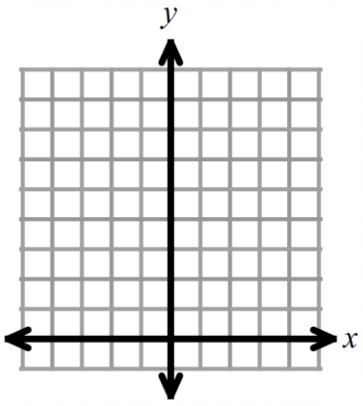
16) Find the *x*-intercepts. (Hint: factor)

17) Find the vertex. (Hint: complete the square.)

18) Use the coordinate system on the right to graph $f(x)$.



Example 19: Given the quadratic function $g(x) = x^2 + 4$, sketch a graph. Include any intercepts and the vertex.



What do you notice about the *x*-intercepts for this graph? Why does this happen?

11.5 Notes: Solving Problems with Quadratic Functions

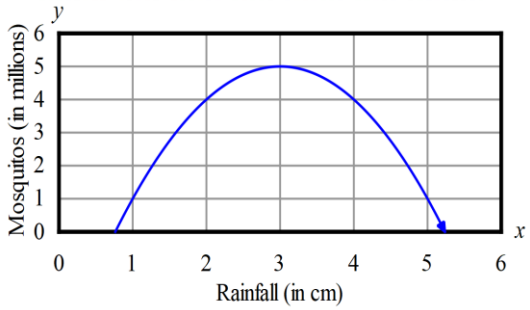
Lesson Objectives

- Use graphs of quadratic functions to solve problems.
- Analyze forms of quadratic functions to solve problems.

Exploration: Work with a partner or a group to answer the questions below.

The number of mosquitoes in Orange Walk, Belize (in millions of mosquitoes) is a function of rainfall (in cm) is modeled by $m(x) = -(x-3)^2 + 5$, as shown in the graph below.

- How many cm of rainfall would result in 4 million mosquitos?
- What is the maximum number of mosquitos?
- How many cm of rainfall would result in the maximum number of mosquitos?



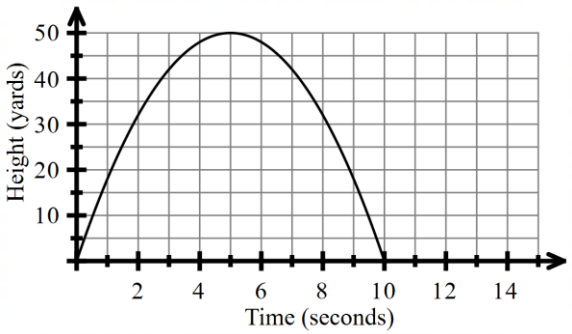
For Examples 1 – 5: A toy rocket is launched from the ground, and its height is shown at various distances from a house. **Use the graph** to answer the following questions, given that the height of the toy rocket can be modeled by $y = -3(x-5)^2 + 48$.

1) What is the maximum height achieved by the rocket?
2) What is the total horizontal distance traveled by the rocket?
3) How far away is the rocket from the house when it lands?

4) What is the *approximate* height of the rocket when it is 4ft from the house?
5) At what distance(s) from the house is the height of the rocket 20 feet?

Max/Min	Height on the ground	Starting height/value
Find the vertex. Use vertex form or complete the square.	Find the x-intercept(s), if any. Factor. solve by square rooting, or use the Quadratic Formula $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$	Find the y-intercept. Use Standard Form to find c, or plug in 0 for x and solve for y

You try Examples 6 – 9! A rocket was shot up into the air. The graph shows the height of its flight t seconds after it was shot. The equation $h(t) = -2t^2 + 20t$ models the height of the rocket (in yards) at t seconds.



6) At about what height was the rocket after 6 seconds?

7) What is the max height reached by the rocket?

8) At what approximate time(s) was the height of the rocket 30 yards?

9) At what time(s) was the rocket on the ground?

Example 10: The cross-section of a half-pipe at a skate park is shaped like a quadratic function that opens upward. The graph shows the ramp in terms of its height, *y*, measured in feet, and its horizontal distance, *x*, also measured in feet. Which of the following correctly model the relationship between *x* and *y*, given that $|a| = \frac{1}{6}$?

Choose all that apply!

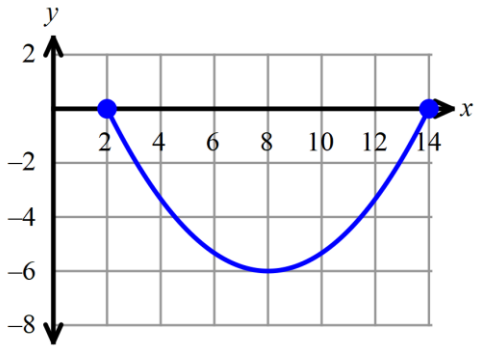
A. $y = -\frac{1}{6}(x-2)(x-14)$

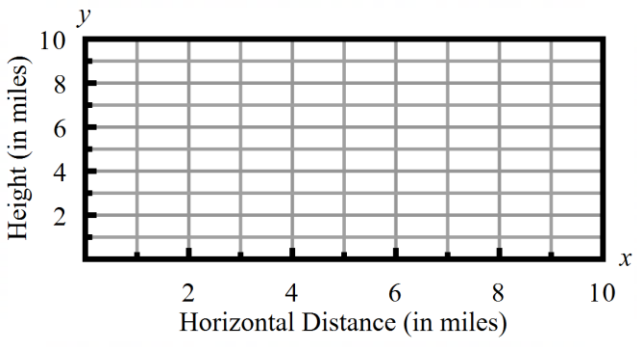
B. $y = \frac{1}{6}(x+2)(x+14)$

C. $y = \frac{1}{6}(x-2)(x-14)$

D. $y = \frac{1}{6}(x-8)^2 - 6$

E. $y = -\frac{1}{6}(x+8)^2 - 6$





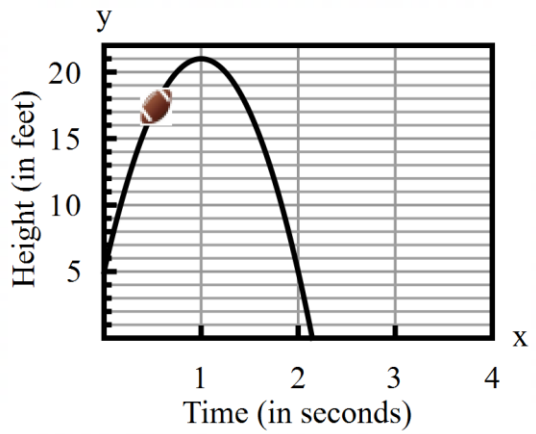
You Try #11! A rainbow can be modeled by $y = -\frac{1}{3}(x-5)^2 + 9$, where *x* is the horizontal distance in miles, and *y* is the height of the rainbow in miles. What is the maximum height of the rainbow?

Examples 12 – 14: A football is kicked in the air, and its path can be modeled by the equation $f(x) = -16x^2 + 32x + 5$, where x is the time (in seconds) and $f(x)$ is the height in feet.

12) What is the height of the football after 2 seconds?

13) What is the starting height of the football when it was first kicked?

14) At what time will the football hit the ground? Use the quadratic formula. Write your answer as a decimal, rounded to the nearest tenth.

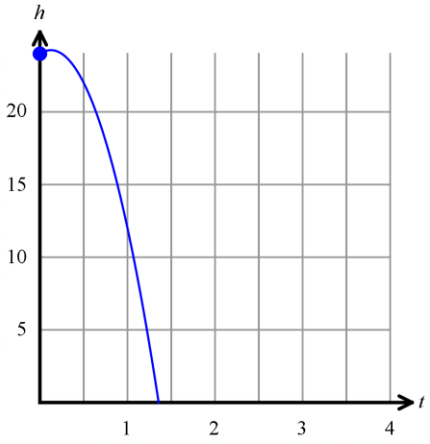


You try #15 – 17! The height (h), in feet, of Jon jumping off a rock into a lake can be modeled by the equation $h(t) = -16t^2 + 4t + 24$, where t represents the time in seconds after Jon has jumped off the rock.

15) What is Jon's height after 1 second?

16) What is the height of the rock?

17) After how many seconds does Jon enter the water? Use the quadratic formula. Round to the nearest tenth.



Ch 11 Study Guide

Graphing Quadratics

Form	What it tells us	Read about it in your notes!
Vertex Form $y = a(x-h)^2 + k$	• Vertex at (h, k) • Convert from Standard Form to Vertex Form by completing the square	Section 10.1 Section 10.2
Intercept Form $y = a(x-p)(x-q)$	• $x-$intercepts are at $(p, 0)$ and $(q, 0)$ • Convert from Standard Form to Intercept Form by factoring.	Section 10.3
Standard Form $y = ax^2 + bx + c$	• The $y-$intercept is at $(0, c)$. • Convert from Standard Form to Vertex Form by completing the square • Convert from Standard Form to Intercept Form by factoring.	Section 10.2 Section 10.2 Section 10.3
For all forms	• Domain is all real numbers • Opens upward if a is positive (range is $y > k$) • Opens down if a is negative (range is $y < k$) • Vertical stretch if $\|a\| > 1$ • Vertical compression of $0 < \|a\| < 1$	Section 10.1

Finding Information from Quadratic Functions

Max/Min	Height on the ground	Starting height/value
Find the vertex. Use vertex form or complete the square.	Find the x-intercept(s), if any. Factor to put into Intercept Form Or set the function = 0 and either • solve by square rooting or • use the Quadratic Formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	Find the y-intercept. Use Standard Form to find c or plug in 0 for x and solve for y