Chapter 11 Calendar **Name:** Key

Day	Date	Assignment (Due the next class meeting)
Monday Tuesday	4/25/22 4/26/22	**Exponential Review**
Wednesday Thursday	4/27/22 4/28/22	**11.1 Worksheet** Introduction to Logarithmic Expressions
Friday Monday	4/29/22 5/2/22	**11.2 Worksheet** Graphing Logarithmic Expressions
Tuesday Wednesday	5/3/22 5/4/22	**11.3 Worksheet** Properties of Logarithmic Expressions
Thursday Friday	5/5/22 5/6/22	**11.4 Worksheet** Solving Logarithmic Equations
Monday Tuesday	5/9/22 5/10/22	**11.5 Worksheet** Solving Exponential Equations
Wednesday Thursday	5/11/22 5/12/22	**Unit 11 Practice Test**
Friday Monday	5/13/22 5/16/22	**Unit 11 Review**
Tuesday Wednesday	5/17/22 5/18/22	**Unit 11 Test**

- Be prepared for daily quizzes.
- Every student is expected to do every assignment for the entire unit.
- Try www.khanacademy.org if you need help outside of school hours.
- Student who complete 100% of their homework second semester will receive a pizza party and 2% bonus to their grade!

11.1 Notes: Introduction to Logarithmic Expressions

Can you find the inverse of $y = 2^x$?

Can't solve for y after switching x & y.
How can this be done.

What can we say about Exponentials and Logarithms?

$y = \log_b x$

read "y equals log base b of x"

* Think b to the power of y is x

$b^y = x$

log form		exponential form
$y = \log_b x$	$\Longleftrightarrow$	$b^y = x$

Log Form:	**Exponential Form:**
Logarithm with Base b: $y = \log_b x$	$b^y = x$
Logarithm with Base *e*: (Natural Log) [*ln*]: $y = \log_e x = \ln x$	$e^y = x$
Logarithm with Base 10: (Common Log) $y = \log_{10} x$	$10^y = x$

Examples: Rewrite the following equations in logarithm form or exponential form.

1) $\log_3 81 = 4$

$3^4 = 81$

2) $\log_{10} 1 = 0$

$10^0 = 1$

3) $2^3 = 8$

$\log_2 8 = 3$

4) $(1/4)^{-1} = 4$

$\log_{1/4} 4 = -1$

You try! Rewrite the following equations in logarithm form or exponential form.

a) $4^3 = 64$

$\log_4 64 = 3$

b) $\log_2 32 = 5$

$2^5 = 32$

c) $(1/2)^{-2} = 4$

$\log_{1/2} 4 = -2$

Examples: Evaluate the following *without* a calculator.

5) $\log_4 64 = x$ $4^x = 64$ $\log_4 4 = 3$

6) $\log 0.001$ $\log_{10} 10^{-3}$ $10^? = .001$ -3

7) $\log_{1/4} 256$ $\log_{1/4}(\frac{1}{4})^{-4}$ $\frac{1}{4}^? = 256$ -4

8) $\log_{64} 2$ $\log_{64} 64^{\frac{1}{6}}$ $64^? = 2$ $\frac{1}{6}$

You try! Evaluate the following *without* a calculator.

a) $\log 100$ 2

b) $\log_{27} 3$ $\frac{1}{3}$

c) $\log_{1/2} 8$ -3

*__Inverse functions:__ Logarithms and exponentials are inverses if they have the same base.

Properties of Logs	$\log_b b^m =$	$\log_b 1 =$	$\log_b b =$	$b^{\log_b m} =$
	m	0	1	m

Examples: Simplify the following expressions.

9) $10^{\log 4}$ 4

10) $e^{\ln 9}$ $e^{\log_e 9}$ 9

11) $\log_5 5^3$ 3

12) $\log_3 27^x$ $\log_3 3^{3x}$ $3x$

13) $\log_5 25^{-2x}$ $\log_5 5^{2(-2x)}$ $-4x$

14) $\ln e^{-2x}$

$-2x$

15) $\underbrace{\log_2 16}_{4} + \underbrace{\log_5 625}_{4}$

$4 + 4$

8

16) $\underbrace{\log_3 27}_{3} - \underbrace{\ln e}_{1} + \underbrace{\log_2 32}_{5}$

$3 - 1 + 5$

7

You try! Simplify the following expressions.

a) $3^{\log_3 81}$

81

b) $e^{\ln 12}$

12

c) $\log_4 64^{3x} + \ln e^{2x}$

$\log_4 4^{3(3x)} + 2x$

$9x + 2x$

$11x$

d) $\log_2 64 - \log_{1/5} 25$

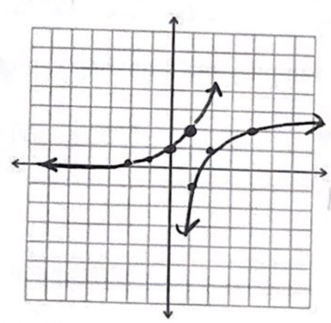
$6 - (-2)$

8

11.2 Notes: Graphs of Logarithmic Equations

a) Make a table and graph $y = 2^x$.

x	-2	-1	0	1	2
y	1/4	1/2	1	2	4



What is the Domain? $(-\infty, \infty)$

What is the Range? $\{y \mid y > 0\}$ $(0, \infty)$

b) Now switch the input and output values (x's and y's) and graph them on the same grid. What are you graphing? inverse

x	1/4	1/2	1	2	4
y	-2	-1	0	1	2

c) What is the domain and range for the function graphed in part b?

D: $\{x \mid x > 0\}$ R: $\{y \mid y < \mathbb{R}\}$

d) Fill in the table and graph $y = \log_2 x$. What do you notice?

x	y
¼	-2
½	-1
1	0
2	1
4	2

$\log_2 \frac{1}{4} = -2$
$\log_2 \frac{1}{2} = -1$
$\log_2 1 = 0$
$\log_2 2 = 1$
$\log_2 4 = 2$

So, $y = 2^x$ and $y = \log_2 x$ are inverse.

Summarize the transformations for the graph of a logarithmic equation:

y: neg reflect

$$f(x) = a\log_b(x - h) + k$$ → ↓↑ keep

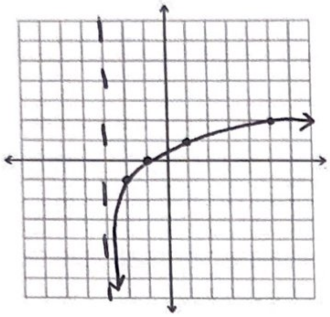
stretch/compress

left or right opp

Steps to Graph Logarithmic Functions: $y = a \cdot \log_b(x - h) + k$
Hint: Identify all transformations first
1.) List your a, b, h, k
2.) Graph your vertical Asymptote (x=h)
3.) Jump off VA → 1, ↑ or ↓ k (1st point)
4.) Back to VA, ↑↓ a, → b (2nd point)

Examples: Graph the following functions and state the domain and range in set notation.

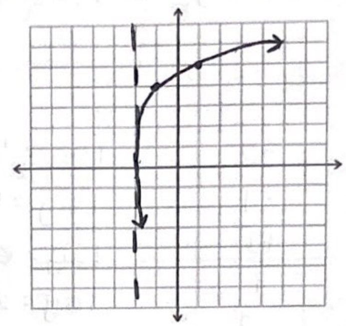
1) $f(x) = \log_2(x + 3) - 1$



Domain: $(-3, \infty)$ $\{x \mid x > -3\}$

Range: $(-\infty, \infty)$ $\{y \mid y < \mathbb{R}\}$

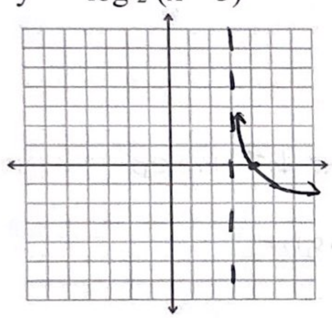
2) $f(x) = \log_3 (x + 2) + 4$



Domain: $(-2, \infty)$ $\{x \mid x > -2\}$

Range: $(-\infty, \infty)$ $\{y \mid y < \mathbb{R}\}$

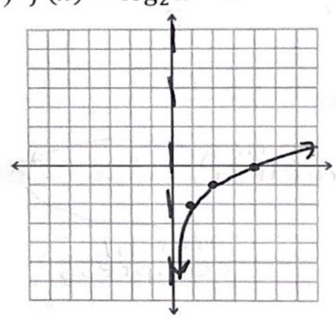
3) $y = -\log_2 (x - 3)$



Domain: $(3, \infty)$

Range: $(-\infty, \infty)$ $\{y \mid y < \mathbb{R}\}$

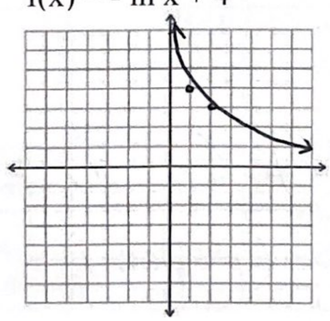
4) $f(x) = \log_2 x - 2$



Domain: $(0, \infty)$

Range: $\{y \mid y < \mathbb{R}\}$

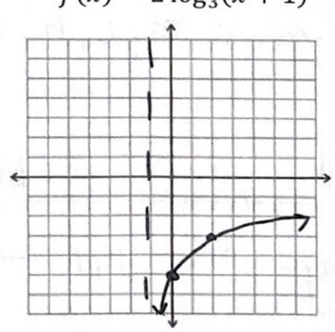
5) $f(x) = -\ln x + 4$



Domain: $(0, \infty)$

Range: $\{y \mid y < \mathbb{R}\}$

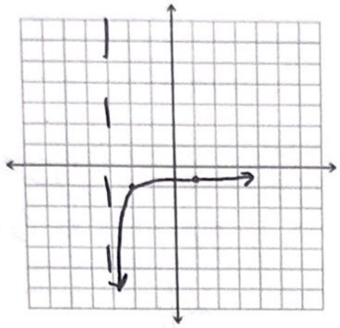
6) $f(x) = 2\log_3(x + 1) - 5$



Domain: $(-1, \infty)$

Range: $\{y \mid y + \mathbb{R}\}$

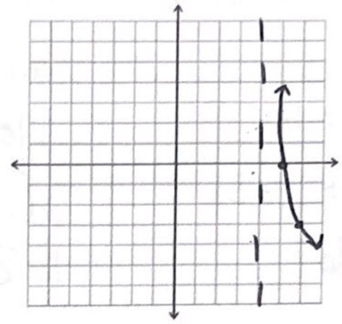
7) $f(x) = \frac{1}{3}\log_2(x+3) - 1$



Domain: $\{x \mid x > -3\}$

Range: $\{y \mid y \in \mathbb{R}\}$

8) $f(x) = -3\log_2(x-4)$



Domain: $\{x \mid x > 4\}$

Range: $\{y \mid y \in \mathbb{R}\}$

11.3 Notes: Properties of Logarithmic Expressions

Product Property: $\log_b mn = \log_b m + \log_b n$

Quotient Property: $\log_b \frac{m}{n} = \log_b m - \log_b n$

Power Property: $\log_b m^p = p \cdot \log_b m$

Examples: Use properties of logarithms to evaluate the following
Use $\log_6 5 \approx 0.898$ and $\log_6 8 \approx 1.161$ to evaluate the logarithm

1) $\log_6\left(\frac{5}{8}\right)$

$\log_6 5 - \log_6 8$

$(.898) - (1.161)$

2) $\log_6 40$

$\log_6 5 + \log_6 8$

$.898 + 1.161$

2.059

3) $\log_6 125$

$3\log_6 5$

$3(.898)$

2.694

You try! Use properties of logarithms to evaluate the following
Use $\log_3 6 \approx 1.631$ and $\log_3 2 \approx 0.631$ to evaluate the logarithm

a) $\log_3 12$

$\log_3 6 + \log_3 2$

$1.631 + .631$

2.262

b) $\log_3 8$

$3\log_3 2$

$3(.631)$

1.893

c) $\log_3\left(\frac{2}{6}\right)$

$\log_3 2 - \log_3 6$

$.631 - 1.631$

-1

Examples: Condense each logarithmic expression.

4) $2\log x + 3\log y - 2\log z$

$\log \frac{x^2y^3}{z^2}$

5) $\ln 4 + 3\ln 3 - \ln 12 + 6$

$\ln \frac{4(27)}{12} = \ln 9 + 6$

You try! Condense each logarithmic expression.

a) $\log_5 3 - 4\log_5 a + 5\log_5 b$

$\log_5 \frac{3b^5}{a^4}$

b) $3\ln 2 - \ln 6 - \ln 4$

$\ln \frac{8}{6(4)} = \ln \frac{1}{3}$

Examples: Expand each logarithmic expression.

6) $log_7 \frac{3x}{5y}$

$\log_7 3 + \log_7 x - \log_7 5 - \log_7 y$

7) $\ln \frac{2x^2}{5y}$

$\ln 2 + 2\ln x - \ln 5 - \ln y$

You try! Expand each logarithmic expression.

a) $\log \frac{2ab^5}{3c^3}$

$\log 2 + \log a + 5\log b - \log 3 - 3\log c$

b) $\ln \frac{5x}{yz^4}$

$\ln 5 + \ln x - \ln y - 4\ln z$

Change-of-base Formula

$\log_a x = \frac{\log_b x}{\log_b a}$

Examples: Use the change-of-base formula to evaluate each logarithm. Give an exact answer and an approximate solution to three decimal places.

8) $\log_5 8$

$\frac{\log 8}{\log 5} = 1.292$

9) $\log_8 14$

$\frac{\log 14}{\log 8} = 1.269$

10) $\log_3 12$

$\frac{\log 12}{\log 3} = 2.262$

You try! Use the change-of-base formula to evaluate each logarithm. Give an exact answer and an approximate solution to three decimal places.

a) $\log_4 15$

$\frac{\log 15}{\log 4} = 1.953$

b) $\log_3 2$

$\frac{\log 2}{\log 3} = .631$

c) $\log_{100} 1$

$\frac{\log 1}{\log 100} = 0$

11.4 Notes: Solving Logarithmic Equations

Property of Equality for Logarithmic Equations

$\log_a x = \log_a y \iff x = y$

Examples: Solve each logarithmic equation and check for extraneous solutions.

1) $\log_5 (4x - 7) = \log_5 (x + 5)$

$4x - 7 = x + 5$

$3x = 12$

$x = 4$

2) $\log_4 (5x - 1) = 3$

$4^3 = 5x - 1$

$64 = 5x - 1$

$65 = 5x$

$x = 13$

3) $\ln (7x - 4) = \ln (2x + 11)$

$7x - 4 = 2x + 11$

$5x = 15$

$x = 3$

4) $\log_3(x - 1) + \log_3 x = \log_3 20$

$\log_3 (x^2 - x) = \log_3 20$

$x^2 - 5 = 20$

$x^2 - x - 20 = 0$

$(x - 5)(x + 4) = 0$

$\downarrow \quad \downarrow$

$5 \quad 4$

5) $\log_4 (x + 12) + \log_4 x = 3$

$\log_4(x^2+12x)=3$

$x^2+12x=4^3$

$x^2+12x-64=0$

$(x+16)(x-4)=0$

$-16 \quad 4$

6) $10 = \log_2(x-3) + 6$

$\log_2(x-3)=4$

$x-3=16$

$x=19$

You try! Solve each logarithmic equation and check for extraneous solutions.

a) $\log (5x - 21) = \log (3x - 4)$

$5x-21=3x-4$

$2x=17$

$x=17/2$

b) $\ln (x - 2) + \ln x = \ln 8$

$\ln(x^2-2x)-\ln 8$

$x^2-2x-8=0$

$(x-4)(x+2)=0$

$4 \quad \cancel{-2}$

c) $3\log_2(x-7) = 24$

$\log_2(x-7)=8$

$x-7=256$

$x=263$

d) $\log x + \log(x + 15) = 2$

$\log(x^2+15x)=2$

$x^2+15x=10^2$

$x^2+15x-100=0$

$(x+20)(x-5)=0$

$\cancel{-20} \quad 5$

8) The population of deer in a forest preserve can be modeled by the equation $P = 50 + 200 \ln (t + 1)$, where t is the time in years from the present. In how many years will be deer population reach 500?

$500=50+200\ln(t+1)$

$e^{\frac{450}{200}} = e^{\ln(t+1)}$

$e^{2.25}=t+1$

$t=e^{2.25}-1 \approx 8.5$ years

9) One of the strongest earthquakes in recent history occurred in Mexico City in 1985 and measured 8.1 on the Richter scale. Find the amount of energy, E, released by this earthquake with a magnitude of M.

Use the formula: $M = \frac{2}{3}\log\frac{E}{10^{11.8}}$

$8.1 = \frac{2}{3}\log\frac{E}{10^{11.8}}$

$12.15 = \log\frac{E}{10^{11.8}}$

$\approx 8.9 \times 10^{23}$

11.5 Notes: Solving Exponential Equations

Examples: Solve the following equations by taking the logarithm of both sides or by re-writing it in logarithm form. Give an exact solution and an approximate solution rounded to three decimal places.

1) $4^x = 11$

$\log_4 11 = x$

$x = \frac{\log 11}{\log 4}$

≈ 1.730

2) $7^{9x} = 15$

$\log_7 15 = 9x$

$\frac{\log 15}{\log 7} = 9x$

$x = \frac{\log 15}{9\log 7}$

≈ 0.155

3) $4e^{-0.3x} - 7 = 13$

$e^{-.3x} = 5$

$-.3x = \ln 5$

$x = \frac{\ln 5}{-0.3}$

≈ -5.365

You try! Solve the following equations by taking the logarithm of both sides or by re-writing it in logarithm form. Give an exact solution and an approximate solution rounded to three decimal places.

a) $3e^{2x} + 5 = 20$

$e^{2x} = 5$

$2x = \ln 5$

$x = \frac{\ln 5}{2}$

b) $3^{2x} = 18$

$\log_3 18 = 2x$

$x = \frac{\log 18}{2\log 3}$

Work with a partner to answer this question. Be prepared to explain your process to the class.

You want to buy a car that is going to cost $4,000. Right now you have saved $3,000 but you want to invest your money to help is grow a little faster. The bank will offer you a savings account that earns 5% interest compounded annually. How long will it take for you to have enough money for your car?

$$4000 = 3000(1.05)^t$$

$$t \ln 1.05 = \frac{4}{3}$$

$$t = \frac{4}{3 \ln 1.05} = 5.89 \text{ years}$$

Example 4: If $5,000 is deposited in an account at the bank and earns 7.5% annual interest, compounded annually, how long does it need to stay in the account in order to double? Use the formula $A = P(1 + \frac{r}{n})^{nt}$

$$10000 = 5000\left(1 + \frac{0.075}{1}\right)^{1t} \quad \text{or} \quad 2 = (1 + 0.075)^t$$

$$\frac{\ln 2}{\ln 1.075} = t \qquad t = 9.58 \text{ yrs.}$$

Example 5: If $2,500 is invested at a rate of 3% compounded continuously, find the amount of time for the account to have $3,300 in it. Use the formula $A = Pe^{rt}$

$$3300 = 2500e^{0.03t}$$

$$\ln \frac{33}{25} = \ln e^{0.03t}$$

$$t = \frac{\ln \frac{33}{25}}{0.03} = 9.25 \text{ yrs}$$

Example 6: Your freshman year of high school you were given some money to invest in a savings account. You wanted to have $3,000 when you graduated in four years. The bank gave you an interest rate of 8% compounded continuously. How much money did you invest? Use the formula $A = Pe^{rt}$

$$3000 = Pe^{4(0.08)} \qquad P = \frac{3000}{e^{4(0.08)}}$$

$$P = \$2178.45$$

You try!

a) Samantha and Ryan are having a contest to see who can double their investment first. Both have \$250 to deposit. Samantha puts her money into an account with an 8.5% interest rate, compounded continuously. Ryan puts his money into an account with a 9% interest rate, compounded quarterly. Who will double their investment first? Explain your answer.

$500 = 250e^{0.085t}$

$\frac{\ln 2}{0.085} = t = 8.15$ yrs

Sam

$500 = 250\left(1+\frac{0.09}{4}\right)^{4t}$

$2 = 1.0225^{4t}$

$\frac{2}{4\ln 1.0225} = \frac{4t\ln 1.0225}{4\ln 1.0225}$

$t = 7.79$ yrs

$\boxed{\text{Ryan}}$ will double first.

Example 7:

Write an exponential function in the form $y = ab^x$ whose graph passes through the points $(2, 12.5)$ and $(4, 312.5)$.

A. $y = \frac{1}{5}(2)^x$

C. $y = 2(5)^x$

B. $y = \frac{1}{2}(5)^x$

D. $y = 5(2)^x$

$\frac{312.5 = ab^4}{12.5 = ab^2}$

$\sqrt{25} = \sqrt{b^2}$

$5 = b$

$\frac{12.5}{25} = \frac{a \cdot 5^2}{25}$

$a = \frac{1}{2}$

$y = \frac{1}{2} \cdot 5^x$