Chapter 6 Calendar

Name: ____________________

Day	Date	Assignment (Due the next class meeting)
Friday Monday	11/18/22 (A) 11/21/22 (B)	6.1 Worksheet Adding/subtracting & Multiplying polynomial functions
Tuesday Monday	11/22/22 (A) 11/28/22 (B)	6.2 Worksheet Factoring polynomials
Tuesday Wednesday	11/29/22 (A) 11/30/22 (B)	6.3 Worksheet Polynomial division, remainder and factor theorems
Thursday Friday	12/1/22 (A) 12/2/22 (B)	6.4 Worksheet Rational Roots Theorem
Monday Tuesday	12/5/22 (A) 12/6/22 (B)	6.5 Worksheet Graphing polynomial functions day 1
Wednesday Thursday	12/7/22 (A) 12/8/22 (B)	Chapter 6 Practice Test
Friday Monday	12/9/22 (A) 12/12/22 (B)	**Ch 6 TEST** 1st Semester Review Packet
Tuesday Wednesday	12/13/22 (A) 12/14/22 (B)	**Review Day**
Thursday Friday	12/15/22 (A) 12/16/22 (B)	**PRACTICE FINAL**
Monday	12/19/22 (C)	**Review Day**
Tuesday Wednesday Thursday	12/20/22 12/21/22 12/22/22	**FINALS**

- Be prepared for daily quizzes.
- Every student is expected to do every assignment for the entire unit.
- Students who complete *every assignment* for this semester are eligible for a 2% semester grade bonus and a pizza lunch paid by the math department.
- Try www.khanacademy.org or www.mathguy.us (Earl's website) if you need help.

6.1 Notes: Adding, Subtracting, & Multiplying Polynomials

Work with a partner to perform the indicated operation. Be prepared to share with the class.

1) Add: $2x^3 - 5x^2 + 3x - 9$ and $x^3 + 6x^2 + 11$

$3x^3 + x^2 + 3x + 2$

2) $(5x^5 - 3x^4 + 2x) + (-x^5 + 3x^4 - x)$

$4x^5 + x$

3) Subtract: $5z^2 - z + 3$ from $4z^2 + 9z - 12$

$$4z^2 + 9z - 12$$
$$-(5z^2 + z + 3)$$
$$\rightarrow -z^2 + 10z - 15$$

4) $-(t^2 - 6t + 2) - (5t^2 - t - 8)$

$-t^2 + 6t - 2 - 5t^2 + t + 8$

$-6t^2 + 7t + 6$

5) $-2 + 3(t^2 - 6t + 2) + 4(5t^2 - t - 8) - (6t + 1)$

$-2 + 3t^2 - 18t + 6 + 20t^2 - 4t - 32 - 6t - 1$

$23t^2 - 28t - 29$

6) $20 - \frac{1}{2}(-6t^2 + 8t - 4) - 3(t^2 - 6t + 5) + (t - 3)$

$20 + 3t^2 - 4t + 2 - 3t^2 + 18t - 15 + t - 3$

$15t + 4$

7) According to data from the U.S. Census Bureau for the period 2000-2007, the number of male students enrolled in high school in the United States can be approximated by the function $M(x) = -0.004x^3 + 0.037x^2 + .049x + 8.11$ where x is the number of years since 2000 and $M(x)$ is the number of male students in the millions. The number of female students enrolled in high school in the United States can be approximated by the function $F(x) = -0.006x^3 + 0.029x^2 + 0.165x + 7.67$ where x is the number of years since 2000 and $F(x)$ is the number of female students in millions. Estimate the total number of students enrolled in high school in the United States in 2007.

$F(x) + M(x) = T(x)$

$$F(x) = -.006x^3 + .029x^2 + .165x + 7.67$$
$$+\ M(x) = -.004x^3 + .037x^2 + .049x + 8.11$$
$$T(x) = .01x^3 + .066x^2 + .214x + 15.78$$

→ Functions showing trend
→ useful for making later predictions

plug in 7.

$T(7) = 17.1$ million

Multiplying Polynomials

8) If $f(x) = 2x + 1$ and $g(x) = x - 6$, find $f(x) \cdot g(x)$.

$(2x+1)(x-6)$

$2x^2 - 11x - 6$

9) If $h(x) = x^2 - 5$ and $g(x) = x - 1$, find $h(x) \cdot g(x)$.

$(x^2-5)(x-1)$

$x^3 - x^2 - 5x + 5$

10) Multiply: $-2y^2 + 3y - 6$ *and* $y - 2$

$(-2y^2+3y-6)(y-2)$

$-2y^3 + 3y^2 - 6y$

$+4y^2 - 6y + 12$

$-2y^3 + 7y^2 - 12y + 12$

11) $-(2x - 3y)(x^2 + 3y - 2)$

$(-2x+3y)(x^2+3y-2)$

$-2x^3 - 6xy + 4x + 3x^2 + 9y^2 - 6y$

12) $(x^2 + 3x)(3x^2 - 2x + 4)$

$3x^4 - 2x^3 + 4x^2$

$+9x^3 - 6x^2 + 12x$

$3x^4 + 7x^3 - 2x^2 + 12x$

13) $-(x - 5)(x + 2)(3x - 1)$

$(-x+5)(3x^2+5x-2)$

$-3x^3 - 5x^2 + 2x$

$+15x^2 + 25x - 10$

$-3x^3 + 10x^2 + 27x - 10$

14) What is the degree of the function, $f(x) = (x^2 + 4x - 3)(3x^5 + 6x^3)$?

$3x^7 + \ldots\ldots$

degree is 7, no need to foil

15) Write the area of a triangle if its height is $2x - 3$ and its base is $5x^2 + 1$.

$A = \frac{1}{2} b \cdot h$

$A = \frac{1}{2}(2x-3)(5x^2+1) \rightarrow = \frac{1}{2}(10x^3 + 2x - 15^2 - 3$

$5x^3 - \frac{15}{2}x^2 + x - \frac{3}{2}$

16) You want to build a raised rectangular garden bed with a certain height h. You want the width to be the height plus 10 feet, and the length to be 5 times the height. Write a polynomial to describe the volume of the garden bed, in feet.

$V = lwh$

$V = (h+10)(5h)(h)$

$V = 5h^3 + 50h^2$

6.2 Notes: Factoring Polynomials

In boxes 1 – 4, Factor:

1) Greatest Common Factor (GCF) $3x + 6$ $3(x+2)$	2) Difference of Perfect Squares $x^2 - 9$ $(x+3)(x-3)$	3) Trinomials $x^2 - x - 2$ $(x-2)(x+1)$
4) Trinomials with coefficient $5x^2 - 7x - 6$ $(5x+3)(x-2)$	5) Grouping $\underbrace{x^3 + 6x^2}\ \underbrace{- 3x - 18}$ $x^2(x+6) - 3(x+6)$ $(x^2-3)(x+6)$	6) Sum/Difference of Cubes $x^3 + 27$ $a = x \quad b = 3$ $(x+3)(x^2-3x+9)$

Sum of two cubes

$a^3 + b^3 = (a + b)(a^2 – ab + b^2)$

Difference of two cubes

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Look at these two formulas and describe the similarities between them.

both end with b^2 a^2, ab, b^2

What are the differences?

2^{nd} term: ~~one~~, one -

One +

Factor the polynomials completely: ****Remember to find the GCF first*****

1) $x^3 + 64$

$a = x$

$b = 4$

$(x+4)(x^2-4x+16)$

2) $x^3 - 8$

$a = x$

$b = 2$

$(x-2)(x^2+2x+4)$

3) $27x^3 - 125$

$a = 3x$

$b = 5$

$(3x-5)(9x^2+15x+25)$

4) $-2d^5 - 250d^2$

$-2d^2(d^3+125)$

$a = d \quad b = 5$

$-2d^2(d+5)(d^2-5d+25)$

5) $16b^6 - 686b^3$

$2b^3(8b^3-343)$

$a = 2 \quad b = 7$

$2b^3(2b-7)(4b^2+b+4)$

For some polynomials, you can factor by graphing pairs of terms that have a common monomial factor.

$$ra + rb + sa + sb = r(a + b) + s(a + b) \quad \text{*or use the box method}$$
$$= (r+s)(a+b)$$

Factor the polynomials completely:

6) $\underbrace{x^3 - 3x^2}\,\underbrace{-\,16x + 48}$

$x^2(x-3) - 16(x-3)$

$(x^2-16)(x-3)$

$(x+4)(x-4)(x-3)$

7) $\underbrace{8t^2 + 28ts}\,\underbrace{-\,6ts - 21s^2}$

$4t(2t+7s) - 3s(2t+7s)$

$(4t-3s)(2t+7s)$

8) $\underbrace{2x^3 - 6x^2}\,+\underbrace{x - 3}$

$2x^2(x-3) + 1(x-3)$

$(2x^2+1)(x-3)$

*Go back to the chart and complete the examples in boxes 5 and 6.

6.3 Notes: Dividing Polynomials

Long Division

Divide using long division:
$473 \div 12$

$39 \quad 39\, 5/12$

$12 \overline{)473}$
-36
113
-108
5

What steps did you do?

1. decide goes into
2. multiply
3. subtract
4. bring down repeat
5. Find r, write as fraction

Divide $f(x) = x^3 + 3x^2 - 7$ by $x^2 - x - 2$

$x + 4 + \frac{6x+1}{x^2-x-2}$

$x^2 - x - 2 \overline{)\cancel{x^3} + 3x^2 + 0x - 7}$
$-\cancel{x^3} + x^2 + 2x$
$\cancel{4x^2} + 2x - 7$
$-\cancel{4x^2} + 4x + 8$
$6x + 1$

$\boxed{x + 4 + 6x+1/x^2-x-2}$

1) Divide $f(x) = 3x^4 - 5x^3 + 4x - 6$ by $x^2 - 3x + 5$

$3x^2 + 4x - 3 \quad + -25x + 4/x^2 - 3x + 5$

$x^2 - 3x + 5 \overline{)\cancel{3x^4} - 5x^3 + 0x^2 + 4x - 6}$
$-\cancel{3x^4} + 9x^3 + 15x^2$
$\cancel{4x^3} + 15x^2 + 4x$
$-\cancel{4x^3} + 12x^2 - 20x$
$-\cancel{3x^2} - 16x - 6$
$+\cancel{3x^2} - 9x + 15$
$-25x + 9$

2) $(2x^3 + 10x^2 + 6x - 18) \div (2x + 6)$

$x^2 + 2x - 3$

$2x + 6 \overline{)\cancel{2x^3} + 10x^2 + 6x - 18}$
$-\cancel{2x^3} - 6x^2$
$\cancel{4x^2} + 6x$
$-\cancel{4x^2} - 12x$
$-6x - 18$
$+6x + 18$
0

Write about it: Imagine that you had to get in front of the class and explain how to do long division with polynomials. Write a paragraph describing what you would say to the class to help them understand the process.

List the steps

Synthetic Division

A shorthand method for dividing a polynomial by $x - a$ is called synthetic division. It is similar to long division, but you use only the coefficients.

3) Divide $(2x^3 + x^2 - 8x + 5)$ by $(x + 3)$

$x+3=0$

$x=-3$ root

$$\begin{array}{r|rrrr} -3 & 2 & 1 & -8 & 5 \\ & & -6 & 15 & -21 \\ \hline & 2 & -5 & 7 & \boxed{-16} \end{array}$$

coefficients

divide by power of one so each exponent goes down by one.

$2x^2-5x+7+{-16}/{x+3}$

4) Divide $(4x^3 - 3x + 7)$ by $(x - 1)$

$$\begin{array}{r|rrrr} 1 & 4 & 0 & -3 & 7 \\ & & 4 & 4 & 1 \\ \hline & 4 & 4 & 1 & \boxed{8} \end{array}$$

$4x^2+4x+1+8/x-1$

Factor Theorem: a polynomial $f(x)$ has a factor $x - a$ if and only if $f(a)=0$
(REMAINDER = 0)

5) Factor $f(x) = 3x^3 - 4x^2 - 28x - 16$ completely given that $x + 2$ is a factor.

$$\begin{array}{r|rrrr} -2 & 3 & -4 & -28 & -16 \\ & & -6 & 20 & 16 \\ \hline & 3 & -10 & -8 & \end{array}$$

$3x^2-10x-8$

$(3x+2)(x-4)(x+2)$

**Because you know x+2 is a factor, you know that f(-2) = 0 or that x = -2 is a root. Use synthetic division to find the other factors.*

no remaineder means factor or root

6) Factor the polynomial completely given that $f(x) = x^3 - 6x^2 + 5x + 12$ and that $x - 4$ is a factor.

$$\begin{array}{r|rrrr} 4 & 1 & -6 & 5 & 12 \\ & & 4 & -8 & -12 \\ \hline & 1 & -2 & -3 & 0 \end{array}$$

$(x-3)(x+1)(x-4)$

7) Find the other zeros of f given that $f(2) = 0$ and $f(x) = x^3 - x^2 - 22x + 40$

2 zero
must factor

$$\begin{array}{r|rrrr} 2 & 1 & -1 & -22 & 40 \\ & & 2 & 2 & -40 \\ \hline & 1 & 1 & -20 & 0 \end{array}$$

-5 & 4

$(x+5)(x-4)$

8) Find the other solutions of f given that $x = -7$ is a root and $f(x) = x^3 + 8x^2 + 5x - 14$

$(x+7)$ is the factor

$$\begin{array}{r|rrrr} -7 & 1 & 8 & 5 & -14 \\ & & -7 & -7 & 14 \\ \hline & 1 & 1 & -2 & 0 \end{array}$$

$(x+2)(x-1)(x+7)$
-2 1

6.4 Notes: Rational Root Theorem

The Fundamental Theorem of Algebra: Any polynomial of degree **n** has at most **n** roots, both real and complex.

1) How many x-intercepts does the following function have? $f(x) = 7x^5 - 4x^2 + 1$

Degrees: 5 or less

ex: $(x+3)^3(x-1)(x+2)$
or imaginary roots also

2) With a partner try to solve (find the roots of) the polynomial, $x^3 + 7x^2 + 15x + 9 = 0$.

$(x^3+7x^2)(+15x+9)=0$

$\underbrace{x^2(x+7)+3(5x+3)}=0$

Not the same !!

Can it be factored? Any other ways to solve it?

No, I hope so!

If we know <u>possible roots</u> we can use synthetic division to tell if they are <u>actual roots</u>! So how do we find a possible root?

The Rational Zeros Theorem: If $f(x)$ is a polynomial with integer coefficients and if $\frac{p}{q}$ is a zero of $f(x)$, then p is a factor of the constant term of $f(x)$ and q is a factor of the leading coefficient of $f(x)$.

3) Make a list of all possible rational zeros of $f(x)$ given below.

$f(x) = 2x^4 + x^3 - 19x^2 - 9x + 9$

$\frac{\pm 1, 2, 3, 4, 6, 12}{\pm 1}$

$\to \pm 1, 2, 3, 4, 6, 12$

Steps for finding possible roots:
1. Write down all the factors of the constant term (p)
2. Write down all the factors of the leading coefficient (q)
3. Write down all the possible value of $\frac{p}{q}$. Remember to include both positive and negative factors.
4. Remove any duplicate values.

switch

4) Make a list of all possible roots for the function, $f(x) = x^3 + x^2 - 8x - 12$.

$\frac{\pm 1, 3, 9}{\pm 1, 2} \to \pm 1, \frac{1}{2}, 3, \frac{3}{2}, 9, \frac{9}{2}$

Now use synthetic division to see which possible roots are actual solutions to *f(x)*. If it is a solution the remainder needs to be equal to <u>0</u>.

Try: 1 $\quad 1+1-8-12 \neq 0$

Try: -1 $\quad -1+1+8-12 \neq 0$

Try: 2 $\quad 8+4-16-12 \neq 0$

Try: -2 $\quad -8+4+16-12 = 0$ ✓

so ~~-2~~ -2 is a root

$$\begin{array}{r|rrrr} -2 & 1 & 1 & -8 & -12 \\ & & -2 & 2 & 12 \\ \hline & 1 & -1 & -6 & 0 \end{array}$$

$(x-3)(x+2)$

$\downarrow \quad \downarrow$

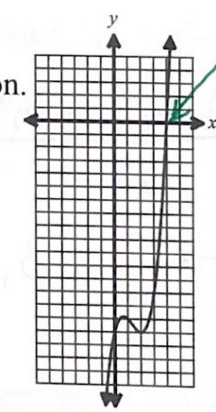
$3 \quad -2$

5) Which of the following are <u>not possible solutions</u> to the function below. Choose all that apply! $\quad f(x) = 2x^4 + 5x^3 - 5x^2 - 5x + 3$

$\pm 1, 3 / \pm 1, 2$

A. 2 $\quad$ B. $-\frac{3}{2}$ $\quad$ C. 1 $\quad$ D. $\frac{1}{2}$ $\quad$ E. $-\frac{2}{3}$ $\quad$ F. 3 $\quad$ G. $\frac{1}{3}$

6) The equation $x^3 - 4x^2 + 4x - 16 = 0$ is graphed to the right.
Use the graph to help solve the equation and find all the roots of the function.



$$\begin{array}{r|rrrr} 4 & 1 & -4 & 4 & -16 \\ & & 4 & 0 & 16 \\ \hline & 1 & 0 & 4 & 0 \end{array}$$

$x^2 + 4$ — imaginary roots

$(x+2i)(x-2i) \quad \pm 2i$

7) a. Find all possible roots of the function, $g(x) = 2x^4 - 3x^3 + 7x^2 + 12x$.

$x(2x^3 - 3x^2 + 7x + 12)$

$\downarrow$

$0 \quad \pm 1, 2, 3, 4, 6, 12 / \pm 1, 2$

$\pm 1, \frac{1}{2}, 2, 3, \frac{3}{2}, 4, 6, 12$

b. Use the possible roots and synthetic division to find the solutions. *on final how can calculator help you?

$$\begin{array}{r|rrrr} -1 & 2 & -3 & 7 & 12 \\ & \downarrow & -2 & 5 & -12 \\ \hline & 2 & -5 & 12 & 0 \end{array}$$

$g(x) = x(x+1)(2x^2 - 5x + 12)$

$\quad\quad\quad\; \downarrow \quad \downarrow$

$\quad\quad\quad\; 0 \quad -1$

use quadratic

$\frac{5 \pm \sqrt{25 - 4(2)(12)}}{4} = \frac{5 \pm \sqrt{-71}}{4} = \frac{5 \pm i\sqrt{71}}{4}$

*4 solutions
-2 real
-2 imaginary

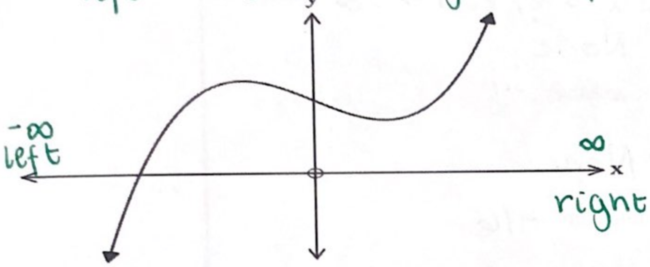
6.5 Notes: Graphing Polynomials day 1

End Behavior:

The end behavior of a function describes what happens to the range as the domain approaches ___∞___ and ___$-\infty$___.

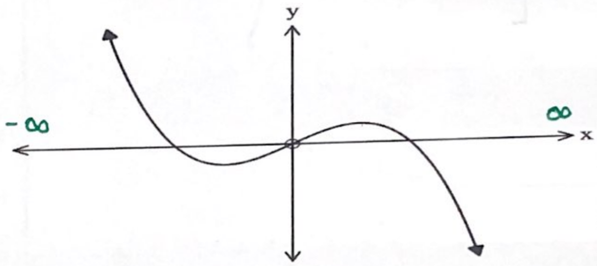
Example 1:

as $x \to -\infty, y \to -\infty$ as $x \to \infty, y \to \infty$
left down right up



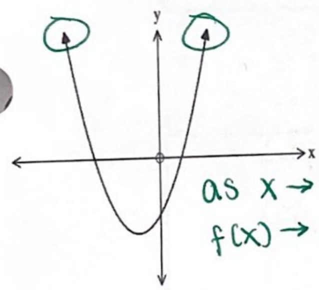
Example 2:

as $x \to -\infty, y \to \infty$ as $x \to \infty, y \to -\infty$
left up right down



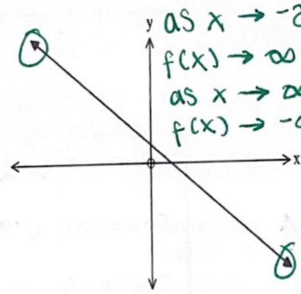
For #3-5, describe the end behavior:

Example 3:



as $x \to \pm\infty$
$f(x) \to \infty$

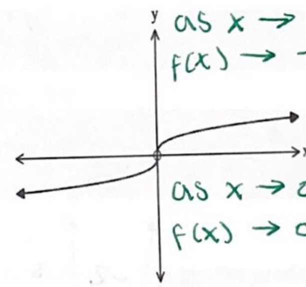
Example 4:



as $x \to -\infty$
$f(x) \to \infty$
as $x \to \infty$
$f(x) \to -\infty$

Example 5:

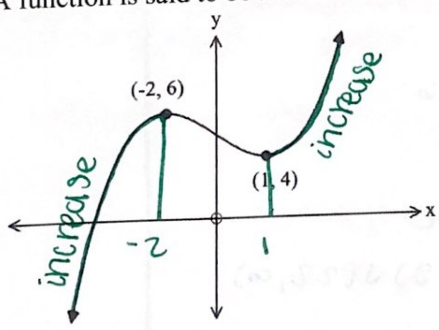
as $x \to -\infty$
$f(x) \to -\infty$



as $x \to \infty$
$f(x) \to \infty$

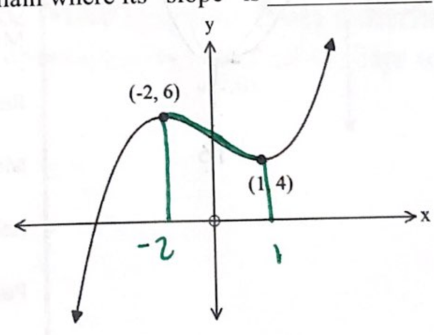
Increasing and Decreasing

A function is said to be *increasing* across a certain domain where its "slope" is ___+___.
A function is said to be *decreasing* across a certain domain where its "slope" is ___−___.



Highlight where the function is increasing.
Write in interval notation:

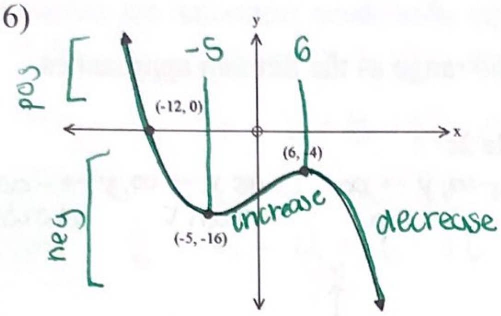
$(-\infty, -2) \cup (1, \infty)$



Highlight where it's decreasing.
Write in interval notation:

$(-2, 1)$

6)



Increasing: (-5, 6)

Decreasing: $(-\infty, -5) \cup (6, \infty)$

Degree: 3

End behavior: as $x \to -\infty, f(x) \to \infty$
as $x \to \infty, f(x) \to -\infty$

Max: None

Relative Max: -4

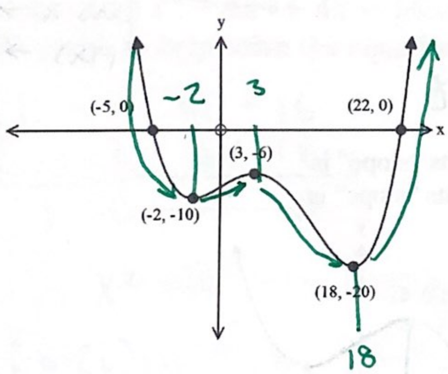
Min: None

Relative Min: -16

Positive: above x axis $(-\infty, -12)$

Negative: below x axis $(-12, \infty)$

7)



Increasing: $(-2, 3) \cup (18, \infty)$

Decreasing: $(-\infty, -2) \cup (3, 18)$

Degree: 3

End behavior: as $x \to \pm\infty, f(x) \to \infty$

Max: None

Relative Max: -6

Min: -20

Relative Min: -10

Positive: $(-\infty, -5) \cup (22, \infty)$

Negative: (-5, 22)