Chapter 3 Calendar

Day	Date	Assignment (Due the next class meeting)
		3.1 Worksheet
	100	Simplifying Radicals, real and imaginary
		3.2 Worksheet
		Dividing radicals
		3.3 Worksheet
		Operations with radicals and complex numbers
	34	3.4 Worksheet
		Division with binomials and complex numbers
3.1		3.5 Worksheet
		Solving equations with complex roots
11		Ch 3 Practice Test
		Ch 3 Test

* Be prepared for daily quizzes.

* Every student is expected to do every assignment for the entire unit.

* Students who complete *every assignment* for this semester are eligible for a 2% semester grade bonus and a pizza lunch paid by the math department if there are no late assignments.

* Try www.khanacademy.org or www.mathguy.us (Earl's website) if you need help.

3.1 Notes: Simplifying Radicals (Real and Imaginary)

Evaluate the following square roots:

7	Tollowing	square	10015
	1/49	1/	144

$$\sqrt{81}$$

$$\sqrt{25}$$

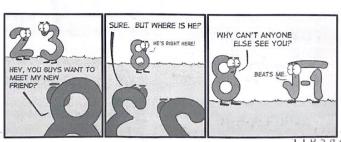
Explain in words how you knew the square roots of those values.

I found the number that when multiplied by itself is the number under the T.

Does $\sqrt{-36}$ have a solution? Why or why not?

Yes, it is imaginary

The imaginary number i:



Evaluate the following expressions:

$$\sqrt{-49}$$

$$\sqrt{-144}$$

$$\sqrt{-81}$$

$$\sqrt{-196}$$

$$\sqrt{-25}$$

Explore the powers of i below:

If $i = \sqrt{-1}$, then what are the values of the following expressions?

i J-V	i^5
	i^6
$i^2 \sqrt{-1} \cdot \sqrt{-1} = \sqrt{-1-1}$	
$i^3 \sqrt{1.1.1}$	i^7
-i	-i
i ⁴ √-1:-1:-1	i^8
500V 70	Incommand and only

What pattern exists with the powers of i? repeats every 4th power.

How would you find the value of i^{15} ?

What is the value of
$$i^{37}$$
?

 $4\sqrt{37}$
 $i'=1$

Find the value of i^{22} .

Simplifying Radicals

Examples: Simplify the following expressions. If needed, write your answer in terms of i.

1)
$$\sqrt{54}$$
 $\sqrt{2.21}$
 $\sqrt{2.3.9}$
 $\sqrt{2.3.3.3}$

3)
$$\sqrt{-75}$$
 $\sqrt{-1.75}$
 $\sqrt{-1.3.95}$
 $\sqrt{-1.3.5.5}$
 $\sqrt{-1.3.5.5}$

2)
$$-2\sqrt{48x^3}$$

 $-2\sqrt{4.12xx^2}$
 $-2\sqrt{4.4.3xx^2}$
 $-2.4.x\sqrt{3x}$

4)
$$\sqrt{-56}$$
 $\sqrt{-1.56}$
 $\sqrt{-1.2.28}$
 $\sqrt{-1.2.2.14}$
 $\sqrt{-1.2.2.2.7}$
 $\sqrt{2}$

Multiplying Radicals

Consider the following example to help you multiply radical expressions:

Example 7) $6\sqrt{7} \cdot 3\sqrt{21}$ $= 18\sqrt{147} \quad \text{don't mult,}$ $7 \quad 21 \quad \text{together.}$

What portions were multiplied?
Insides w/insides
Cutsides w/outsides
How were the factors of 147 found?

7.21 Hen 7.7.3

Multiply the following radical expressions:

 $= 18 \cdot 7\sqrt{3}$ $= 126\sqrt{3}$

5)
$$4\sqrt{12} \cdot 5\sqrt{8}$$

 $4.5\sqrt{12.8}$
 $20\sqrt{3.4.4.2}$

6)
$$2\sqrt{10} \cdot 3\sqrt{-5}$$

 $2 \cdot 3\sqrt{10 \cdot -1 \cdot 5}$
 $4\sqrt{2 \cdot 5 \cdot -1 \cdot 5}$
 $30 \cdot \sqrt{2}$

$$7) \sqrt{-4} \cdot \sqrt{-9}$$

$$2i \cdot 3i$$

$$6i^{2}$$

$$6(-1) = -6$$

Why isn't the answer positive, $since -4 \cdot -9 = 36$?

9)
$$7x\sqrt{15x} \cdot \sqrt{6x}$$

 $7x\sqrt{15x} \cdot 6x$
 $7x\sqrt{3} \cdot 5 \cdot x \cdot 2 \cdot 3 \cdot x$
 $7x \cdot 3 \cdot x\sqrt{10} = 2x^2\sqrt{10}$

8)
$$-4\sqrt{-6\cdot 2\sqrt{-18}}$$

 $-8\sqrt{-1\cdot 6\cdot -1\cdot 18}$
 $-8\sqrt{-1\cdot 6\cdot -1\cdot 6\cdot 3}$
 $-8\cdot -1\cdot 6\cdot \sqrt{3}$

Multiplying Imaginary Numbers

Use the example below of multiplying imaginary numbers, and then try the following problems on your own!

Multiply:
$$-5i \cdot 3i$$

= $-15i^2$
= $-15(-1)$
= 15

Multiply:

bly:

$$10) -2i \cdot 12i$$
 $11) -8i \cdot -7i$ $-24i^2$ $66i^2$ $-24(-1)$ -56

12)
$$6i \cdot 2i^2$$
 $12i^3$
 $12(-i)$

13) Create a multiplication problem involving 2 radicals that gives an answer of -20. -2-110·VIO

Review: Factor the following trinomials.

14)
$$3x^2 - 2x - 5$$

(3x-5) (x+1)

14)
$$3x^2 - 2x - 5$$
 15) $2x^2 + 11x + 14$ (3x+1) (x+2)

16)
$$4x^2 - 7x - 2$$

 $(4x + 1)(x - 2)$

3.2 Notes: Dividing Radicals

Simplify the following expressions:

Examples:

1)
$$\frac{\sqrt{49}}{\sqrt{25}} = \frac{1}{5}$$

2)
$$\frac{\sqrt{18}}{\sqrt{200}} = \sqrt{\frac{18}{200}} = \sqrt{\frac{9}{100}} = \sqrt{\frac{9}{100}} = \sqrt{\frac{3}{100}}$$

You try:

3)
$$\frac{\sqrt{64}}{\sqrt{169}} = \frac{8}{13}$$

4)
$$\frac{\sqrt{48}}{\sqrt{24}} = \sqrt{\frac{48}{34}} = \sqrt{2}$$

Examples:



5)
$$\frac{\sqrt{60}}{\sqrt{3}} = \sqrt{\frac{60}{3}} = \sqrt{\frac{20}{3}} = \sqrt{\frac{20}{3}}$$

6)
$$\frac{\sqrt{-36}}{\sqrt{-121}} = \sqrt{\frac{36}{121}} = \sqrt{\frac{36}{121}} = \sqrt{\frac{36}{121}} = \sqrt{\frac{70}{121}}$$

$$= \sqrt{\frac{36}{121}} = \sqrt{\frac{36}{121}} = \sqrt{\frac{36}{121}} = \sqrt{\frac{12}{121}}$$

$$= \sqrt{\frac{36}{121}} = \sqrt{\frac{36}{121}} = \sqrt{\frac{12}{121}} =$$

$$\frac{1}{\sqrt{18}} = \sqrt{\frac{3}{11}} = \sqrt{\frac{4}{3}} = \sqrt$$

You try:

8)
$$\frac{\sqrt{-180}}{\sqrt{-20}} = \sqrt{\frac{180}{-20}} = \sqrt{9} = \boxed{3}$$

9)
$$\frac{\sqrt{-98}}{\sqrt{8}} = \sqrt{-\frac{98}{5}} = \sqrt{\frac{-49}{4}} = \sqrt{\frac{-49}{4}}$$

8)
$$\frac{\sqrt{-180}}{\sqrt{-20}} = \sqrt{\frac{180}{20}} = \sqrt{\frac{9}{3}} = \sqrt{\frac{9}{8}} = \sqrt{\frac{9}{8}} = \sqrt{\frac{19}{3}} = \sqrt{\frac{19}{3}} = \sqrt{\frac{15}{3}} = \sqrt{\frac{1$$

11)
$$\frac{3\cancel{t}}{5\cancel{t}} = \boxed{\frac{3}{5}}$$

$$12)\frac{\sqrt[4]{2}}{6l} = \frac{\sqrt{2}}{6l}$$

Multiply each:

a)
$$\sqrt{6} \cdot \sqrt{6}$$

b)
$$\sqrt{3} \cdot \sqrt{3}$$

c)
$$\sqrt{5} \cdot \sqrt{5}$$

Rationalizing the denominator:

The following article is taken from West Texas AM University's website: http://www.wtamu.edu/academic/anns/mps/math/mathlab

When a radical contains an expression that is not a perfect root, for example, the square root of 3 or cube root of 5, it is called an <u>irrational number</u>. So, in order to <u>rationalize</u> the denominator, we need to get rid of all radicals that are in the denominator.

Step 1: Multiply numerator and denominator by a radical that will get rid of the radical in the denominator.

If the radical in the denominator is a square root, then you multiply by a square root that will give you a perfect square under the radical when multiplied by the denominator.

Note that the phrase "perfect square" means that you can take the square root of it.

Keep in mind that as long as you multiply the numerator and denominator by the exact same thing, the fractions will be equivalent.

Ex: Rationalize
$$\frac{1}{\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}} = \frac{\sqrt{10}}{10}$$

Step 2: Make sure all radicals are simplified.

Some radicals will already be in a simplified form, but make sure you simplify the ones that are not.

Ex: Simplify then rationalize
$$\frac{\sqrt{10}}{\sqrt{15}} \cdot \frac{\sqrt{15}}{\sqrt{15}} = \frac{\sqrt{10 \cdot 15}}{15} = \frac{\sqrt{2 \cdot 5 \cdot 5 \cdot 3}}{15} = \frac{\cancel{8} \cdot \cancel{10}}{\cancel{15}} = \frac{\cancel{10} \cdot \cancel{15}}{\cancel{15}} = \frac{\cancel{10} \cdot$$

Step 3: Simplify the fraction if needed.

Be careful. You cannot cancel out a factor that is on the outside of a radical with one that is on the inside of the radical. In order to cancel out common factors, they have to be both inside the same radical or be both outside the radical.

Ex: Rationalize and simplify
$$\frac{4}{\sqrt{6}} \cdot \frac{\sqrt{10}}{\sqrt{6}} = \frac{24\sqrt{50}}{\sqrt{50}} = \frac{2\sqrt{50}}{3}$$

Examples: Divide the following expressions. Rationalize, if needed.

1)
$$\frac{1}{\sqrt{5}}$$
 $\frac{1}{\sqrt{5}}$ $\frac{1}{\sqrt{5}}$ $\frac{1}{\sqrt{5}}$

2)
$$\frac{7}{\sqrt{-11}} \cdot \sqrt{\frac{-1}{1}} = \frac{7\sqrt{-11}}{-11} = \frac{7\sqrt{-11}}{11}$$

You try!

3)
$$\frac{5}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \boxed{5\sqrt{6}}$$

4)
$$\frac{1}{\sqrt{-7}} \cdot \frac{\sqrt{-7}}{\sqrt{-7}} = \frac{1\sqrt{7}}{-7} = \frac{1\sqrt{7}}{7}$$

Examples:



5)
$$\frac{\sqrt{6}}{\sqrt{-13}} = \frac{\sqrt{6} \cdot \sqrt{13}}{\sqrt{7}} = \frac{\sqrt{20} \cdot \sqrt{7}}{\sqrt{7}} = \frac{\sqrt{20} \cdot \sqrt{7}}{\sqrt{7}} = \frac{\sqrt{20} \cdot \sqrt{7}}{\sqrt{7}} = \frac{2}{9i} = \frac{2}{9i} = \frac{2}{9i-1} = \frac{2}{9i$$

6)
$$\frac{\sqrt{20}}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \frac{\sqrt{25.7}}{7}$$

$$= \frac{\sqrt{4.5.7}}{7}$$

$$= \frac{\sqrt{2.2.5.7}}{2.7}$$

$$= \frac{2.7.35}{7}$$

You try!

8)
$$\frac{\sqrt{8}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \sqrt{2 \cdot 2 \cdot 2 \cdot 3}$$

9)
$$\frac{\sqrt{5}}{\sqrt{-20}} \cdot \frac{\sqrt{50}}{\sqrt{-20}} = \frac{\sqrt{5.-1.4.5}}{-20} = \frac{\sqrt{5.-1.2.2.5}}{-20} = \frac{-10i}{20} = \frac{-1}{20}$$

$$10) \ \frac{3}{5i} \cdot i = \frac{3i}{5i^2} = \frac{3i}{5(4)} = \frac{-3i}{5}$$

11)
$$\frac{\sqrt{2}}{6i} \cdot \frac{1}{i} = \frac{i\sqrt{2}}{6i^2} = \frac{i\sqrt{2}}{6(-i)} = \frac{i\sqrt{2}}{6}$$

3.3 Notes: Operations with Radicals and Complex Numbers

Matching! Try to match each problem with its simplified expression.

Problems

1)
$$\left(\underline{5} + 3\sqrt{2}\right) + \left(\underline{-3} + 4\sqrt{2}\right)$$

2)
$$(5+3\sqrt{2})-(+3+4\sqrt{2})$$

8-12

3)
$$(5+3\sqrt{2})(-3+4\sqrt{2})$$

 $-1S + 20\sqrt{2} - 9\sqrt{2} + 12\sqrt{2} \cdot 2$ $-1S + 11\sqrt{2} + 24$ $\boxed{9 + 11\sqrt{2}}$ Perform the indicated operations:

1)
$$(\underline{14} + 7\sqrt{3}) + (-\underline{10} + 2\sqrt{3})$$

449-53

3)
$$(8 + \sqrt{11}) + (+12 + 7\sqrt{11})$$

Complex #:

Simplified Expression

A)
$$8 - \sqrt{2}$$

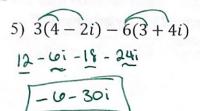
(B) $9 + 11\sqrt{2}$

C) $2 + 7\sqrt{2}$

2) (-27 + 9i) + (-13 - 6i)

-40 + 3i

4)
$$(-10+3i)+(\overline{15}-4i)$$



7)
$$(6-3\sqrt{5})(2+4\sqrt{5})$$
 $12+24\sqrt{6}-6\sqrt{5}-12\sqrt{5}\cdot 5$
 $12+18\sqrt{5}-60$
 $-48+18\sqrt{5}$

6)
$$3i + 2i(1-i) - (7i + 8)$$

 $3i + 2i - 2i^2 - 7i - 8$
 $-2i - 2(-1) - 8$
 $-2i + 2 - 8$
 $-2i - 0$
8) $(3 + 8i)(3 - 8i)$
 $9 - 24i + 24i - 64i^2$
 $9 + 64$
 $9 + 64$

11) If you square a complex number, a + bi, how many terms will you get in your answer? Create an example, and show the solution for squaring the binomial.

-2 +cms

(3+2)(4+21)

12+61+81+412 12+141+4(-1)=4 12) If you multiply two complex numbers, will you always get a binomial? Why or why not? Create two examples with complex numbers that justify your answer.

Challenge: Can you multiply two **binomials with radicals** and have a product with four terms? Try to create an example where this happens.

13)

$$4x^2 - 9 = (px + t)(px - t)$$

(2x+3)(2x-3)

In the equation above, p and t are constants. Which of the following could be the value of p?

- (A) 2
- B) 3
- C) 4
- D) 9

3.4 Notes: Division with Binomials and Complex Numbers

for each binomial below, multiply it by another binomial so that the final answer has only one term:

2)
$$(8+5i)(8-8i)$$

 $(4+40i-40i-25i^2)$
 $(4-28(-1))$
 $= 89$

The pairs of binomials multiplied above are called _____ Conjugate

Why could it be useful to have binomials that multiply out to a single term that is an integer? - We know the middle term goes away.

Examples: Simplify the following expressions. Rationalize if needed.

1)
$$\frac{1}{3-\sqrt{2}}$$
 (3 + $\sqrt{2}$)

2) $\frac{3}{4+5i}$ (4-5i) = $\frac{12-15i}{14-20i+20i-25i^2}$

$$\frac{3+\sqrt{2}}{9+3\sqrt{2}-3\sqrt{2}-\sqrt{2}\cdot 2}$$

$$\frac{3+\sqrt{2}}{9-2} = \frac{3+\sqrt{2}}{3+\sqrt{2}}$$

$$= \frac{12 - 18i}{10 + 28} = \boxed{\frac{12 - 18i}{41}}$$

You try!
$$|-\sqrt{3}|$$

3) $\frac{3}{1+\sqrt{3}} \cdot |-\sqrt{3}|$

4)
$$\frac{5}{2-4i} \cdot (2+4i) = \frac{10+3}{4+8i}$$

$$= \frac{10 + 20i}{4 + 10}$$

$$= \frac{10 + 20i}{20}$$

$$= \frac{1 + 2i}{20}$$

Examples:

$$5) \frac{5i}{2-9i} \cdot 2 + 9i$$

$$\frac{10i - 45}{4 + 81} = \frac{10i - 45}{85} = \boxed{2i - 9}$$

6)
$$\frac{3+\sqrt{5}}{4-\sqrt{2}}$$
. $4+\sqrt{2}$ $\frac{12+4\sqrt{5}+3\sqrt{2}+\sqrt{60}}{16+4\sqrt{2}-4\sqrt{2}-\sqrt{2}}$

12+3-12+4-13+10 16-2 12+3-52+4-5+50

You try!
7)
$$\frac{\sqrt{7}}{8+\sqrt{3}}$$
, 8- $\sqrt{3}$

8)
$$\frac{(1+2i)}{5-4i}$$
 (5 +4i)

Review:

A motor powers a model car so that after starting from rest, the car travels s inches in t seconds, where $s = 16t\sqrt{t}$. Which of the following gives the average speed of the car, in inches per second, over the first t seconds after it starts?

$$(B)$$
 $6\sqrt{t}$

C)
$$\frac{16}{\sqrt{t}}$$

3.5 Notes: Solving equations with complex roots
Review: For each equation below, work with a partner to solve for x using square roots.

1)
$$\frac{5x^2}{S} = \frac{45}{S}$$

2)
$$2x^2 - 10 = 88$$

$$3) -3x^2 + 5 = 5$$

$$-3x^{2} = 0$$
 -3

$$X = 0$$

With your partner, list the steps to solve for x:

- Opposite of PEMDAS (SADMEP)

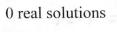
How many solutions does each example have above?

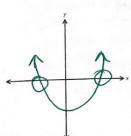


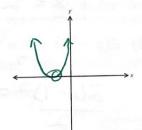
Draw a quadratic that has:

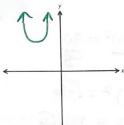
2 real solutions

1 real solution









A quadratic that has no real solutions has 2 complex solutions.

Examples: solve for x to find the solutions (real or complex).

4)
$$x^{2} + 50 = 34$$

 $-50 - 50$
 $\sqrt{x^{2}} = \sqrt{-16}$
 $x = \pm 4$

You try!
6)
$$\frac{10(x-3)^2}{10} = \frac{-90}{10}$$

 $(x-3)^2 = \sqrt{-9}$
 $x-3 = \pm 3i$
 $+3$
 $x = 3 \pm 8i$

Examples:
8)
$$2(x-1)^2 - 9 = -7$$

 $+9$ $+9$
 $2(x-1)^2 = 2$
 $\sqrt{(x-1)^2} = \sqrt{1}$
 $x - 1 = \pm 1$
 $x = 1 \pm 1$

You try!
10)
$$-5(x+6)^2 = 30$$

 -5
 $\sqrt{(x+6)^2} = \sqrt{-6}$
 $x+6 = \pm i\sqrt{6}$
 $x = -6 \pm i\sqrt{6}$

5)
$$(x+5)^2 + 8 = 4$$

 -8
 $\sqrt{(x+5)^2} = \sqrt{-4}$
 $x+5 = \frac{1}{2} = 2\pi$
 -5
 $\sqrt{x} = -5 \pm 2\pi$

7)
$$-2x^{2} + 5 = 13$$

 $-5 - 5$
 $-2x^{2} = 8$
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9)
$$-(x-2)^2 - 2 = 6$$
 $+2 + 2$
 $-(x-2)^2 = 8$
 -1
 $\sqrt{(x-2)^2} = 8$
 $\sqrt{-1}$
 $\sqrt{(x-2)^2} = -8$
 $\sqrt{-2} = 2 = 2i\sqrt{2}$
 $\sqrt{2}$
 $\sqrt{2}$
 $\sqrt{2}$

11)
$$2(x-7)^2 - 7 = 13$$
 $+7 + 7$

$$2(x-7)^2 = 20$$

$$\sqrt{(x-7)^2} = 10$$

$$x-7 = \pm \sqrt{10}$$

$$x = 7 \pm \sqrt{10}$$