Name________________________ **Period** _______

Ch 7 Calendar: Polynomials and Factoring

Day	Date	Assignment (Due the next class meeting)
Friday Monday	1/31/20 2/3/20	7.1 Worksheet Adding, Subtracting Polynomials, Multiplying by a Monomial
Tuesday Wednesday	2/4/20 2/5/20	7.2 Worksheet Multiplying Polynomials
Thursday Friday	2/6/20 2/7/20	7.3 Worksheet Factoring by GCF
Monday Tuesday	2/10/20 2/11/20	7.4 Worksheet Intro to Factoring Trinomials and Binomials
Wednesday Thursday	2/12/20 2/13/20	7.5 Worksheet More Factoring Trinomials and Binomials
Friday Tuesday	2/14/20 ❤ 2/18/20 President's Day 2/17/20	7.6 Worksheet Factoring Completely
Wednesday Thursday	2/19/20 2/20/20	**Topic 7 Practice Test**
Friday Monday	2/21/20 2/24/20	**Topic 7 Test** HW: Topic 7 Spiral Review Wk

NOTE: Be prepared for daily quizzes.

Students with 100% homework completion AND no late/missing homework for the semester will be rewarded with a pizza party. Students with 100% homework completion and no missing homework for the semester will get a 2% grade increase.

Do you need a worksheet or a copy of the teacher notes?
Go to www.washoeschools.net/DRHSmath

Online textbook information: Go to www.washoeschools.net
Click on Student and Parent
Click on Envision
Click on Sign In – sign in using washoe\studentID#, and then your school computer password. (Note: use a back slash not a forward slash.)

7.1 Notes: Adding, Subtracting Polynomials, Multiplying by a Monomial

Lesson Objectives

1) **Define terms used for polynomials**
2) **Add and subtract polynomial**
3) **Multiply a monomial and a polynomial**

Monomial 1 term $3xy^2$	Binomial 2 terms $5x-1$
Trinomial 3 terms $3x+5y^2-3$	Polynomial many terms Note: All the exponents must be whole (positive) numbers!
Degree of a polynomial The highest degree $x^5+3x^4+2x^3+x^2+x+3$ Degree is 5	Leading Coefficient Number in front of polynomials $\underline{5}x^2$
Descending order highest Degree to lowest $x^4+x^3+x^5+x+2+x^2 \longrightarrow x^5+x^4+x^3+x^2+x+2$	

Adding polynomials

To add polynomials, add like terms (same degree) by adding leading coefficients and leaving the degree the same.

Example 1: $(4x^3 + x^2 - 5) + (7x + x^3 - 3x^2)$

$5x^3 - 2x^2 + 7x - 5$

Example 2: Find the sum: $(x^2 + x + 8) + (x^2 - x - 1)$

$2x^2 - 7$

Subtracting polynomials

To subtract polynomials, distribute the negative to the second polynomial, then add the polynomials.

Example 3: Find the difference: $(4z^2 - 3) - (-2z^2 + 5z - 1)$

$4z^2 - 3 + 2z^2 - 5z - 1$

$\boxed{6z^2 - 5z - 4}$

Remember to multiply each term in the polynomial by - 1 when you write the subtraction as addition.

Example 4: Find the difference of $(3x^2 + 6x - 4) - (x^2 - x - 7)$

$3x^2 + 6x - 4 - x^2 + x + 7$

$\boxed{2x^2 + 7x + 3}$

Example 5: You try! Simplify the expression: $(3x^2 + 5) - (x^2 + 2) + (-3x + 1)$

$3x^2 + 5 - x^2 - 2 - 3x + 1$

$\boxed{2x^2 - 3x + 4}$

Multiplying polynomials by monomials

Recall rules of exponents: $x^m \cdot x^n = x^{(m+n)}$ (to multiply like bases and different exponents you ADD the exponents)

To multiply a monomial to a polynomial, multiply the leading coefficients and CHANGE the exponent according to the above rule for each term.

Example 6: Find the product $3x^3(2x^3 - x^2 - 7x - 3)$

$6x^6 - 3x^5 - 21x^4 - 9x^3$

Example 7: Multiply: $-x^2(x - 6)$

$-x^3 + 6x^2$

Example 8: Simplify: $\frac{1}{2}y^3(6xy^2 + 8xy - 4)$

$3xy^5 + 4xy^4 - 2y^3$

Example 9: An online store purchases boxes to ship their products. The large box has a volume of $4x^3 + x^2 + 5$ units. The medium box has a volume of $2x^3 + 3x - 4$ units. The store purchases one large box and two medium boxes. What polynomial expression represents the total volume of the purchased boxes?

add

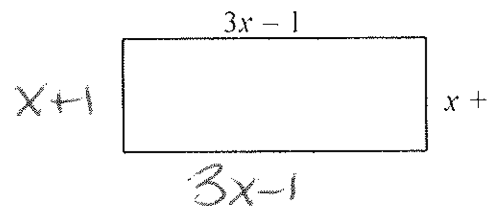
$4x^3 + x^2 + 5 + 2x^3 + 3x - 4$

$\boxed{6x^3 + x^2 + 3x + 1 \text{ units}}$

Example 10: Angela, Christie, and Mark each did the problem below. Who did the problem correctly, if anyone? Describe the mistake made, if any, by each student.

	Student work	Describe the mistake, if any.
Angela	$(5x^2 - 3x + 7) - (3x - 4x^2) + (2x^2 + 9 - 5x^3)$ $= 5x^2 - 3x + 7 - 3x - 4x^2 + 2x^2 + 9 - 5x^3$ $= -5x^3 + 3x^2 - 6x + 16$	Didn't distribute the neg
Christie	$(5x^2 - 3x + 7) - (3x - 4x^2) + (2x^2 + 9 - 5x^3)$ $= 5x^2 - 3x + 7 - 3x + 4x^2 + 2x^2 + 9 - 5x^3$ $= 11x^4 - 5x^3 - 6x^2 + 16$	Changed the exp.
Mark	$(5x^2 - 3x + 7) - (3x - 4x^2) + (2x^2 + 9 - 5x^3)$ $= 5x^2 - 3x + 7 - 3x + 4x^2 + 2x^2 + 9 - 5x^3$ $= -5x^3 + 11x^2 - 6x + 16$	Correct

Example 11: Find the perimeter of the rectangle shown.
Perimeter = sum of all four sides



$2(3x-1) + 2(x+1)$

$6x - 2 + 2x + 2 = \boxed{8x}$

Example 12: Find $h(x) = f(x) + g(x)$ if $f(x) = (7x^2 - 3x + 2)$ and $g(x) = (5x - 2)$

$h(x) = 7x^2 - 3x + 2 + 5x - 2$

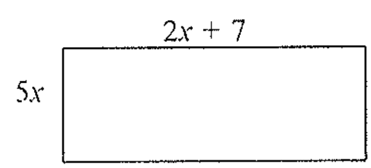
$\boxed{h(x) = 7x^2 + 2x}$

Example 13: Find $h(x) = f(x) - g(x)$ if $f(x) = (-2x^3 - 4x + 2)$ and $g(x) = (5x^3 + 5x^2 - 2x)$

$h(x) = -2x^3 - 4x + 2 - 5x^3 - 5x^2 + 2x$

$\boxed{h(x) = -7x^3 - 5x^2 - 2x + 2}$

Example 14: Write a polynomial expression to represent the area of the rectangle shown.
Area = (length)(width)



$5x(2x + 7)$

$\boxed{10x^2 + 35x}$

7.2: Multiplying Polynomials

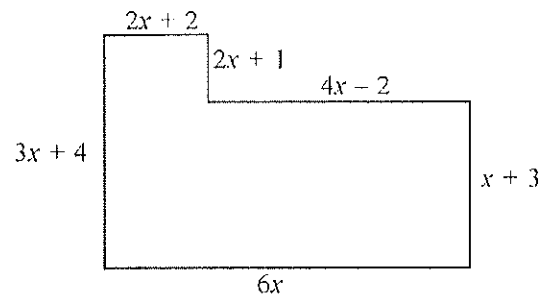
Lesson Objectives

1) **Multiply two binomials**
2) **Expand a squared binomial**
3) **Multiply a binomial and a polynomial**

Warm-Up: For #1 – 2, molding was installed around the edge of the ceiling of the room shown below.

1) Write an expression to represent the amount of molding needed.

$18x+8$



2) If there is 80 feet of molding around the room, then find the length of the longest side of the room.

$18x+8=80$

$18x=72$

$x=4$

$6(4)$

$\boxed{24\text{ft}}$

Multiplying binomials	**1) Distribute each term in the first binomial into the in the second binomial.** **2) Combine like terms.**

Example 1: Multiply the binomials (continued on the next page).

a) $(x+3)(x+4)$

$x^2+4x+3x+12$

$\boxed{x^2+7x+12}$

b) $(x+3)(x-2)$

x^2-2x

$3x-6$

$\boxed{x^2+x-6}$

c) Multiply: $(3x+7)(x-8)$

$3x^2-24x$

$7x-56$

$\boxed{3x^2-17x-56}$

d) Multiply: $(x^2-4)(x-x^2)$

$x^3-x^4-4x+4x^2$

$\boxed{-x^4+x^3+4x^2-4x}$

Example 2: Find $h(x) = f(x) \cdot g(x)$ if $f(x) = (2x + 7)$ and $g(x) = (x - 9)$.

$h(x) = (2x+7)(x-9)$

$2x^2 - 18x + 7x - 63$

$\boxed{h(x) = 2x^2 - 11x - 63}$

Example 3: Multiply each expression.

a) $(x + 4)^2$

$(x+4)(x+4)$

$x^2 + 4x + 4x + 16$

$\boxed{x^2 + 8x + 16}$

b) $(x - 7)^2$

$(x-7)(x-7)$

$x^2 - 7x - 7x + 49$

$\boxed{x^2 - 14x + 49}$

c) $(3x + 4)^2$

$(3x+4)(3x+4)$

$9x^2 + 12x + 12x + 16$

$\boxed{9x^2 + 24x + 16}$

d) $(5x - 1)^2$

$(5x-1)(5x-1)$

$25x^2 - 5x - 5x + 1$

$\boxed{25x^2 - 10x + 1}$

Example 4: Simplify the expression: $(m + 7)(m - 3) + (m - 4)(m + 5)$

$m^2 - 3m + 7m - 21 + m^2 + 5m - 4m - 20$

$\boxed{2m^2 + 5m - 41}$

Example 5: Find each product.

a) $(x - 5)(x + 5)$

$x^2 + 5x - 5x - 25$

$\boxed{x^2 - 25}$

b) $(y - 3)(y + 3)$

$y^2 + 3y - 3y - 9$

$\boxed{y^2 - 9}$

c) $(2a - 7)(2a + 7)$

$4a^2 + 14a - 14a - 49$

$\boxed{4a^2 - 49}$

What do you notice about the products for Example 5?

Middle term cancles out

Conjugates: formed by changing the sign between two terms in binomial.

Conjugate of: $(x+a)$ is $(x-a)$ or Conjugate of $(x-a)$ is $(x+a)$

What happens when you multiply two conjugates (see example 5)?

Middle term cancles.

Example 6: Write two binomial expressions which are conjugates and whose product equals $x^2 - 4$.

$(x+2)(x-2)$

Example 7: Multiply the polynomials.

a) $(2a-5)(a^2-6a-3)$

$2a^3 - 12a^2 - 6a$
$- 5a^2 + 30a + 15$

$\boxed{2a^3 - 17a^2 + 24a + 15}$

b) $(5p-2)(3p^2-2p+1)$

$15p^3 - 10p^2 + 5p$
$- 6p^2 + 4p - 2$

$\boxed{15p^3 - 16p^2 + 9p - 2}$

Example 8: You try! Find each product.

a) $(3x-2)(4x^2-5x+1)$

$12x^3 - 15x^2 + 3x$
$- 8x^2 + 10x - 2$

$\boxed{12x^3 - 23x^2 + 13x - 2}$

b) $(2a^2+3a-2)(a-4)$

$2a^3 + 3a^2 - 2a$
$- 8a^2 - 12a + 8$

$\boxed{2a^3 - 5a^2 - 14a + 8}$

Challenge! Example 9: Find $a(x) \cdot b(x)$ if $a(x) = \frac{1}{4}x - \frac{2}{3}$ and $b(x) = \frac{1}{4}x + \frac{2}{3}$.

$\left(\frac{1}{4}x - \frac{2}{3}\right)\left(\frac{1}{4}x + \frac{2}{3}\right)$

$\boxed{\frac{1}{16}x^2 - \frac{4}{9}}$

Example 10: Simplify: $2(-4a+9)^2+5$
Be sure to do $(-4a+9)^2$ first, THEN distribute the 2!

$(-4a+9)(-4a+9)$
$16a^2-36a$
$-36a+81$
$16a^2-72a+81$

$2(16a^2-72a+81)+5$
$32a^2-144a+162+5$
$\boxed{32a^2-144a+167}$

Example 11: If $f(x)=g(x)$ and $f(x)=-3(x+1)^2+4$, then what polynomial represents $g(x)$?

$(x+1)(x+1)$
x^2+x
$+x+1$
x^2+2x+1

$-3(x^2+2x+1)+4$
$-3x^2-6x-3+4$
$\boxed{-3x^2-6x+1}$

Example 12: Which option below has a product of x^2-4x+4? Choose all that apply.

A) $(x+2)(x-2) \rightarrow x^2-4$
B) $(x-2)(x-2) \rightarrow x^2-4x+4$
C) $(x-2)^2$
D) $2(x^2-x) \rightarrow 2x^2-2x$

Challenge! Example 13: What would you have to multiple $(x-3)$ by to have a product of x^2-9?

A) $(x-3)$
B) $(x-9)$
C) (x^2-3)
D) $(x+3)$

Example 14: Which of the following expressions are equivalent to $-x^2+3x+28$? Select all that apply.

A) $(x+4)(x-7) \rightarrow x^2-3x-28$
B) $-(x+4)(x-7) \rightarrow -(x^2-3x-28) = -x^2+3x+28$
C) $(x+4)(7-x) \rightarrow 7x-x^2+28-4x = -x^2+3x+28$
D) $(-x-4)(x-7) \rightarrow -x^2+7x-4x+28$
E) $(-x-4)(7-x) \rightarrow -7x+x^2-28+4x$

7.3: Factoring Out the Greatest Common Factor (GCF)

Lesson Objectives

1) Recognize GCF and factor them out of a polynomial
2) Discover a pattern for multiplying (x + a)(x + b)
3) Find the perimeter and area of a rectangle given two expressions for the side lengths.

Exploration: What are the factors of each number below?

6: 1 2 3 6

15: 1 3 5 15

18: 1 2 3 6 9 18

What is the greatest common factor of all three numbers?

3

Expand each expression to show all factors. Then find the greatest common factor for all three of the expressions.

$8x^3$: $2 \cdot 2 \cdot 2 \cdot x \cdot x \cdot x$

$-4x^2$: $-1 \cdot 2 \cdot 2 \cdot x \cdot x$

$6x^3$: $3 \cdot 2 \cdot x \cdot x \cdot x$

$\boxed{2x^2}$

The GCF (Greatest Common Factor) is the largest common factor of two or more terms.

~~$2x^2$~~

$3x^2+6x$

GCF: $3x$

Factoring an expression by using the GCF:

Divide your GCF

$3x^2+6x$

$3x(x+2)$

If you distribute you should get your original function

Examples 1 – 6: Factor each expression by taking out the GCF.

1) $5x + 20$

$5(x+4)$

2) $8x - 4x^2$

$4x(2-x)$

3) $16x^2y + 40xy + 8xy^2$

$8xy(2x+5+y)$

4) $-6m^2 - 30m^3$

$-6m^2(1+5m)$

5) $-12ab + 32b$

$4b(-3a+8)$

6) $-8ax^3 + ax^2 - 3ax$

$\boxed{-ax(8x^2+x+3)}$

You Try! Examples 7 – 10: Factor each expression by taking out the GCF.

7) $-4nm - 2n^2$

$-2n(2m+n)$

8) $-5wx^3 + 10wx^2$

$-5wx^2(x-2)$

9) $6y - 15y^3$

$3y(2-5y^2)$

10) $-9dm^3 + dm^2 - 2d^4m$

$-dm(9m^2 - m + 2d^3)$

11) One factor of $-7x^3y - 21x^2y^2$ is $(-7x^2y)$. What is the other factor?

$\frac{-7x^3y}{-7x^2y} \quad \frac{-21x^2y^2}{-7x^2y}$

$x+3y$

12) Factor: $15x^3 - 7y^4 - 2z$

For #13 – 15: Find each product.

13) $(x+2)(x+3)$

x^2+5x+6

14) $(y+4)(y+7)$

$y^2+11y+28$

15) $(h-3)(h+5)$

$h^2+2h-15$

Can you find the secret pattern (shortcut) for the answers above? Hint: Look at the constants (numbers).

Challenge! What are the factors of each trinomial? (Try to work backwards to figure this out!)

16) $x^2 + 6x + 8$

$\underline{4} + \underline{2} = 6$

$\underline{4} \times \underline{2} = 8$

$(x+4)(x+2)$

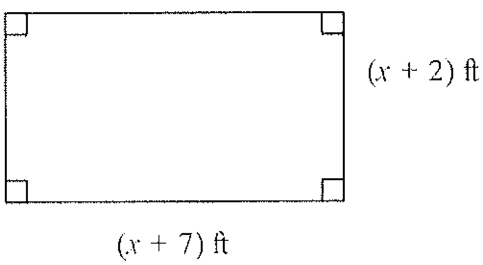
17) $x^2 + 7x + 10$

$\underline{5} + \underline{2} = 7$

$\underline{5} \times \underline{2} = 10$

$(x+5)(x+2)$

Example 18: Write a polynomial expression to represent both the perimeter and area of the rectangle shown below, if $A = bh$.



PERIMETER

$2(x+2)+2(x+7)$

$2x+4+2x+14$

$4x+18$

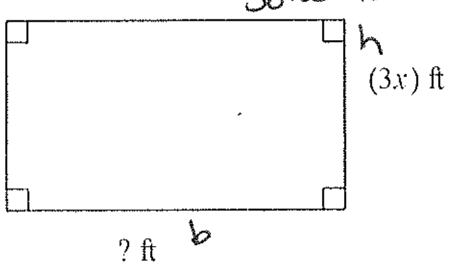
AREA

$(x+7)(x+2)$

$x^2+9x+14$

Example 19: A rectangle has an area that can be represented by $(3x^3 + 9x^2 + 6x)\ ft^2$. If the height of the rectangle is $3x\ ft$, then what expression can represent the base?

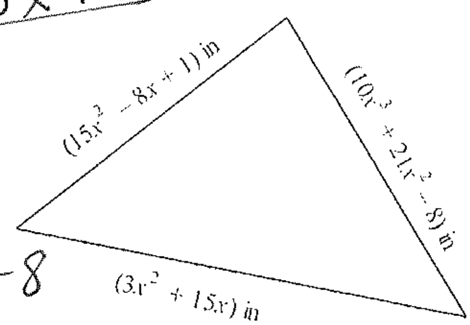
Solve for b



$$\frac{3x\,b}{3x} = \frac{3x^3}{3x} + \frac{9x^2}{3x} + \frac{6x}{3x}$$

$$\boxed{b = x^2 + 3x + 2}$$

Example 20: Given the triangle shown to the right, write a polynomial expression to represent the perimeter of the triangle.



$15x^2 \boxed{-8x} + 1 + 3x^2 \boxed{+15x} + \underline{10x^3} + 21x^2 - 8$

$$\boxed{10x^3 + 39x^2 + 7x - 7}$$

7.4 Notes: Intro to Factoring Trinomials and Binomials

Lesson Objectives

1) **Factor a trinomial into two binomials (a = 1 or GCF)**
2) **Factor a difference of two squares $a^2 - b^2 = (a+b)(a-b)$**
3) **Determine if a polynomial is prime (unable to be factored).**

Warm-Up.

1) $(x^2y - 3y^2 + 5xy^2) - (-x^2y + 3xy^2 - 3y^2)$

Which of the following is equivalent to the expression above?

A) $4x^2y^2$

B) $8xy^2 - 6y^2$

C) $2x^2y + 2xy^2$

D) $2x^2y + 8xy^2 - 6y^2$

$x^2y - 3y^2 + 5xy^2$
$+x^2y + 3y^2 - 3xy^2$

$2x^2y + 2xy^2$

2) If $3x - y = 12$, what is the value of $\frac{8^x}{2^y}$?

A) 2^{12}

B) 4^4

C) 8^2

D) The value cannot be determined from the information given.

Work in groups to multiply (expand) the following expressions: Challenge: Try this without work!

$(x+5)(x-3)$ $\quad$ $(x+2)(x+8)$ $\quad$ $(x+4)(x-4)$

$x^2 + 2x - 15$ $\quad$ $x^2 + 10x + 16$ $\quad$ $x^2 - 16$

Factoring a trinomial is the inverse (opposite) of multiplying binomials.

if $a = 1$ $\quad$ $ax^2 + bx + c$ $\quad$ $\underline{A} + \underline{B} = b$

$(x+A)(x+B)$ $\quad$ $\underline{A} \times \underline{B} = c$

Example 1: Factor $x^2 + 10x + 16$

$\boxed{(x+8)(x+2)}$ $\quad$ $\underline{8} + \underline{2} = 10$ $\quad$ $\underline{8} \times \underline{2} = 16$

Check by multiplying your answer:

$(x+8)(x+2)$

$x^2 + 2x$
$+8x + 16$

$\boxed{x^2 + 10x + 16}$

1 30
2 15
3 10

Examples 2 – 4: Factor each expression.

2) $x^2 + 10x + 9$

$9 + 1 = 10$
$9 \times 1 = 9$
$\boxed{(x+9)(x+1)}$

3) $a^2 - 6a + 9$

$-3 + -3 = -6$
$-3 \times -3 = 9$
$\boxed{(x-3)(x-3)}$

4) $x^2 + 7x - 30$

$+10 + -3 = 7$
$+10 \times -3 = -30$
$\boxed{(x+10)(x-3)}$

You try! Examples 5 – 7: Factor each expression.

6 1
2 3

5) $x^2 + 5x + 6$

$3 + 2 = 5$
$3 \times 2 = 6$
$\boxed{(x+3)(x+2)}$

6) $y^2 - y - 12$

$-4 + 3 = -1$
$-4 \times 3 = -12$
$\boxed{(x-4)(x+3)}$

7) $x^2 - 2x + 1$

$\_\_ + \_\_ = -2$
$\_\_ \times \_\_ = 1$

Some expressions have a GCF that need to be factored out BEFORE you factor the trinomial.

Example 8: Factor the expression below completely: $4y^2 + 12y - 40$

Step 1: Factor out the GCF:

$4(y^2 + 3y - 10)$

$+5 + -2 = 3$
$+5 \times -2 = -10$

Step 2: Factor the remaining trinomial.
(Make sure to leave the GCF as part of your answer.)

$\boxed{4(x+5)(x-2)}$

Examples 9 – 12: Factor each expression completely.

9) $-x^2 + 4x + 12$

$-(x^2 - 4x - 12)$

$-6 + 2 = -4$
$-6 \times 2 = -12$
$\boxed{-(x-6)(x+2)}$

10) $w^3 - 10w^2 + 25w$

$w(w^2 - 10w + 25)$

$-5 + -5 = -10$
$-5 \times -5 = 25$
$\boxed{w(w-5)(w-5)}$

You try! Factor completely.

11) $-2x^2 + 14x - 24$

$-2(x^2 - 7x + 12)$

$-4 + -3 = -7$
$-4 \times -3 = 12$
$\boxed{-2(x-4)(x-3)}$

12) $2a^2b - 10ab + 8b$

$2b(a^2 - 5a + 4)$

$-4 + -1 = -5$
$-4 \times -1 = 4$
$\boxed{2b(a-4)(a-1)}$

Do you remember how to multiply conjugates? Multiply each expression below. (Try to do this without work!)

$(x-5)(x+5)$

x^2-25

$(x+11)(x-11)$

x^2-121

Factoring Difference of Two Perfect Squares:

a^2-b^2

$(a-b)(a+b)$

For #13 – 16: Factor each expression.

13) x^2-25

x^2-5^2

$(x+5)(x-5)$

14) a^2-49b^2

$a^2-(7b)^2$

$(a+7b)(a-7b)$

15) $36-y^2$

6^2-y^2

$(6+y)(6-y)$

16) x^6-100

$(x^3)^2-10^2$

$(x^3-10)(x^3+10)$

You try! For #17 – 20: Factor each expression.

17) g^2-4

g^2-2^2

$(g+2)(g-2)$

18) $1-b^2$

1^2-b^2

$(1-b)(1+b)$

19) k^2-81j^2

$k^2-(9j)^2$

$(k+9j)(k-9j)$

20) $n^{10}-9$

$(n^5)^2-3^2$

$(n^5-3)(n^5+3)$

Sometimes we need to factor out the GCF before we factor the difference of two perfect squares. Also, not all expressions factor. If an expression does not factor at all, then it is Irreducible

Examples 21 – 29: Factor each expression completely. Write "prime" if no factoring can be done.

21) $5x^3-20x$

$5x(x^2-4)$

$5x(x^2-2^2)$

$\boxed{5x(x+2)(x-2)}$

22) $5x^2+10x-15$

$3+-1=2$

$3\times-1=-3$

$5(x^2+2x-3)$

$\boxed{5(x+3)(x-1)}$

23) g^2+16

g^2+4^2

can't be factored

You try #24 – 25!

24) $-x^5 + 9x^3$

$-x^3(x^2-9)$

$-x^3(x^2-3^2)$

$\boxed{-x^3(x+3)(x-3)}$

25) $3x^2 - 24x + 12$

$\boxed{3(x^2-8x+4)}$

__ + __ = −8

__ × __ = 4

4 1

2 2

you can pull out the GCF but can't factor further

26) $-a^5 - 3a^4$

$\boxed{-a^4(a+3)}$

27) $-x^3 + 6x^2 + 16x$

$-x(x^2-6x-16)$

$\underline{-8} + \underline{2} = -6$

$\underline{-8} \times \underline{2} = -16$

16 1

8 2

4 4

$\boxed{-x(x-8)(x+2)}$

You try #28 – 29!

28) $-2x^3 - 20x^2 - 42x$

$\underline{7} + \underline{3} = 10$

$\underline{7} \times \underline{3} = 21$

$-2x(x^2+10x+21)$

$\boxed{-2x(x+7)(x+3)}$

29) $4a^5 - 4a^3$

$4a^3(a^2-1)$

$4a^3(a^2-1^2)$

$\boxed{4a^3(a-1)(a+1)}$

7.5: More Factoring Trinomials and Binomials

Lesson Objectives

1) **Factor a trinomial into two binomials ($a \neq 1$)**
2) **Review other factoring techniques.**

Warm-Up: Simplify each expression. Try to do these without showing work!

1) $(3x - 1)(x + 4)$

$3x^2 + 12x - x - 4$

$\boxed{3x^2 + 11x - 4}$

2) $(5x + 2)(3x - 7)$

$15x^2 - 35x + 6x - 14$

$\boxed{15x^2 - 29x - 14}$

Factoring Trinomials with a leading coefficient different than one:

$ax^2 + bx + c$ $\quad \underline{A} + \underline{B} = b$ Our two values then devide by a

$\underline{A} \times \underline{B} = c \cdot a$ $\quad \frac{A}{a}$ and $\frac{B}{a}$ Simplify

$(ax + A)(ax + B)$

Example 1: Factor $2x^2 - 11x + 5$

$\underline{-10} + \underline{-1} = -11$

$\underline{-10} \times \underline{-1} = 10$

$\boxed{(x-5)(2x-1)}$

$\frac{-10}{2} = -5$ and $\frac{-1}{2}$

Check your solution by using multiplication:

$(x-5)(2x-1)$

$2x^2 - x - 10x + 5$

$2x^2 - 11x + 5$ ✓

Examples 2 – 3: Factor each expression.

24 1
12 2
8 3
6 4

2) $3n^2 + 2n - 8$

$\underline{6} + \underline{-4} = 2$

$\underline{6} \times \underline{-4} = -24$

$\frac{6}{3} = 2$ and $\frac{-4}{3}$

$\boxed{(3n-4)(n+2)}$

3) $2y^2 - 13y - 7$

$\underline{-14} + \underline{1} = -13$

$\underline{-14} \times \underline{1} = -14$

$\frac{1}{2}$ and $\frac{-14}{2} = -7$

$\boxed{(2y+1)(y-7)}$

Examples 4 – 6: Factor each expression.

4) $9y^2 + 6y + 1$

$\underline{3} + \underline{3} = 6$

$\underline{3} \times \underline{3} = 9$

$\frac{3}{9} = \frac{1}{3}$

$\boxed{(3x+1)(3x+1)}$

5) $6x^2 + 5xy - 6y^2$

$\underline{9} + \underline{-4} = 5$

$\underline{9} \times \underline{-4} = -36$

$\frac{9}{6} = \frac{3}{2}$ and $\frac{-4}{6} = \frac{-2}{3}$

$\boxed{(2x+3y)(3x-2y)}$

6) $15x^2 - x - 6$

$\underline{-10} + \underline{9} = -1$

$\underline{-10} \times \underline{9} = 90$

$\frac{-10}{15} = \frac{-2}{3}$ and $\frac{9}{15} = \frac{3}{5}$

$\boxed{(3x-2)(5x+3)}$

You Try! **Examples 7 – 9: Factor each expression.**

7) $3x^2 - 5x + 2$

$\underline{-3} + \underline{-2} = -5$

$\underline{-3} \times \underline{-2} = 6$

$\frac{-3}{3} = -1$

and

$\frac{-2}{3}$

$\boxed{(3x-2)(x-1)}$

8) $8x^2 + 14x - 15$

$\underline{20} + \underline{-6} = 14$

$\underline{20} \times \underline{-6} = 120$

$\frac{20}{8} = \frac{5}{2}$

and

$\frac{-6}{8} = \frac{-3}{4}$

$\boxed{(2x+5)(4x-3)}$

9) $2m^2 + mn - 21n^2$

$\underline{7} + \underline{-6} = 1$

$\underline{7} \times \underline{-6} = -42$

$\frac{7}{2}$ and $\frac{-6}{2} = -3$

$\boxed{(2x+7)(x-3)}$

Factoring Binomials with a leading coefficient different than one:

Examples 10 – 12: Factor each expression.

10) $25x^2 - 4$

$(5x)^2 - 2^2$

$\boxed{(5x-2)(5x+2)}$

11) $49b^4 - 9d^2$

$(7b^2)^2 - (3d)^2$

$\boxed{(7b^2+3d)(7b^2-3d)}$

12) $36a^2 - b^6$

$(6a)^2 - (b^3)^2$

$\boxed{(6a-b^3)(6a+b^3)}$

You try! **For examples 13 – 15, factor each expression.**

13) $121h^2 - 4g^8$

$(11h)^2 - (2g^4)^2$

$\boxed{(11b - 2g^4)(11b + 2g^4)}$

14) $25 - 16k^2$

$5^2 - (4k)^2$

$\boxed{(5-4k)(5+4k)}$

15) $169x^2 - 49y^{12}$

$(13x)^2 - (7y^6)^2$

$\boxed{(13x + 7y^6)(13x - 7y^6)}$

If you are able to factor out a GCF from an expression, always do that first!

Examples 16 – 18: Factor each expression completely. (Hint: look for a GCF first!)

16) $6x^2 - 2x - 4$ $\underline{-3} + \underline{2} = -1$

$2(3x^2 - x - 2)$ $\underline{-3} \times \underline{2} = -6$

$\frac{-3}{3} = -1$ and $\frac{2}{3}$

$\boxed{2(x-1)(3x+2)}$

17) $36a^5 - 9a^3$

$9a^3(4a^2 - 1)$

$9a^3((2a)^2 - 1^2)$

$\boxed{9a^3(2a-1)(2a+1)}$

18) $-4x^3 + 4x^2y + 3xy^2$

$\underline{-6} + \underline{2} = -4$

$\underline{-6} \times \underline{2} = -12$

$-x(4x^2 - 4xy - 3y^2)$

$\boxed{-x(2x-3y)(2x+1y)}$

$\frac{-6}{4} = \frac{3}{2}$ and $\frac{2}{4} = \frac{1}{2}$

You Try! **For examples 19 – 21, factor each expression completely.**

19) $28x^2 + 38xw - 6w^2$

$21 + -2 = 19$

$21 \times -2 = -42$

$2(14x^2 + 19xw - 3w^2)$

$\frac{21}{14} = \frac{3}{2}$ and $\frac{-2}{14} = \frac{-1}{7}$

$\boxed{2(2x+3w)(7x-1w)}$

20) $-6x^2 + 12x + 90$ $\underline{-5} + \underline{3} = -2$

$-6(x^2 - 2x - 15)$ $\underline{-5} \times \underline{3} = -15$

$\boxed{-6(x-5)(x+3)}$

21) $25x^4 - 100x^2$

$25x^2(x^2 - 4)$

$25x^2(x^2 - 2^2)$

$\boxed{25x^2(x-2)(x+2)}$

Example 22: A rectangular patio has an area of $2x^2 + 13x - 24$ ft^2. Use factoring to find expressions that might represent the dimensions of the patio.

1 48
2 24
3 16

$\underline{16} + \underline{-3} = 13$

$\underline{16} \times \underline{-3} = -48$

$\frac{16}{2} = 8$ and $\frac{-3}{2}$

$\boxed{(2x-3)(x+8)}$

7.6 Notes: Factoring Completely

Lesson Objectives

1) **Completely factor all polynomials (or state they are prime).**

Warm-Up

1) Factor: $x^2 + 5x + 6$

$(x+3)(x+2)$

2) Factor: $x^2 - 64$

$x^2 - 8^2$

$\boxed{(x+8)(x-8)}$

3) Simplify: $(x+7)^2$

$(x+7)(x+7)$

$\boxed{x^2+14x+49}$

4) Simplify: $(4x^2 - 3x + 7) - (7x^2 - 6x + 2)$

$4x^2-3x+7-7x^2+6x-2$

$\boxed{-3x^2+3x+5}$

Factoring Completely	Step 1: If possible, factor out a Greatest Common Factor. Step 2: Can you factor the binomial or trinomial any further? Step 3: Keep factoring until each portion of your answer is fully factored.

Examples 1 – 4: Factor each polynomial completely.

1) $5a^4 - 405$

$5(a^4 - 81)$

$5((a^2)^2 - 9^2)$

$5(a^2+9)(a^2-9)$

$\boxed{5(a^2+9)(a+3)(a-3)}$

2) $2x^2 - 8x - 10$

$2(x^2-4x-5)$

$\boxed{2(x-5)(x+1)}$

You try #3 – 4!

3) $x^4 - 625$

$(x^2)^2 - 25^2$

$(x^2-25)(x^2+25)$

$\boxed{(x+5)(x-5)(x^2+25)}$

4) $-x^3 - x^2 + 12x$

$-x(x^2+x-12)$

$\boxed{-x(x+4)(x-3)}$

Examples 5 – 10: Factor completely.

5) $3r^3 - 21r^2 + 30r$

$3r(r^2-7r+10)$

$\boxed{3r(r-5)(r-2)}$

6) $81d^5 - d$

$d(81d^4-1)$

$d((9d^2)^2-1^2)$

$d(9d^2+1)(9d^2-1)$

$\boxed{d(9d^2+1)(3d+1)(3d-1)}$

7) $2x^2 + 5xy + 2y^2$

$\boxed{(2x+y)(x+2y)}$

$4 + 1 = 5$

$4 \times 1 = 4$

$\frac{4}{2} = 2$

$\frac{1}{2}$

You try!

8) $2y^4 - 32$

$2(y^4-16)$

$2((y^2)^2-4^2)$

$2(y^2-4)(y^2+4)$

$\boxed{2(y+2)(y-2)(y^2+4)}$

9) $49y^2 - 25w^6$

$(7y)^2-(5w^3)^2$

$\boxed{(7y-5w^3)(7y+5w^3)}$

10) $-x^3 - 2x^2 + 15x$

$5 + -3 = 2$

$5 \times -3 = -15$

$-x(x^2+2x-15)$

$\boxed{-x(x+5)(x-3)}$

11) **Challenge:** Given $(x + 4)$ is a factor of $2x^2 + 11x + 2m$, determine the value of m.

$(2x+?)(x+4)$

$2x^2 \boxed{+8x+?x} + 4\cdot ?$

$8x + ?x = 11x$

$? = 3$

$4\cdot ? = 2m$

$4\cdot 3 = 2m$

$12 = 2m$

$\boxed{6 = m}$

12) Which of the following expressions are equivalent to $-x^2 + 4x + 21$? Select all that apply.

A) $(x + 3)(x - 7) \longrightarrow x^2 - 4x - 21$
B) $-(x + 3)(x - 7) \longrightarrow -(x^2 - 7x + 3x - 21) \longrightarrow -x^2 + 4x + 21$
C) $(x + 3)(7 - x) \longrightarrow 7x - x^2 + 21 - 3x \longrightarrow -x^2 + 4x + 21$
D) $(-x - 3)(x - 7) \longrightarrow -x^2 + 7x - 3x + 21$
E) $(-x - 3)(7 - x)$

$\hookrightarrow -7x + x^2$

Activity: Work with your group to match each polynomial to its factors. Each numbered problem will have 2 factors and no factors will be used twice.

1 648
2 324
3 216
4 162
6 108
8 81
9 72

$\underline{-72} + \underline{9} = -63$
$\underline{-72} \cdot \underline{9} = -648$

Polynomials

1. $8x^2 - 63x - 81$

$\frac{-72}{8} = 9$ $(8x+9)(x-9)$
$\frac{9}{8}$ E G

2. $49x^2 - 25$

$(7x)^2 - 5^2$ $(7x+5)(7x-5)$
F B

3. $8x^2 - 2x - 45$

D L

24 1
12 2
8 3

4. $2x^2 + 11x + 12$ $\underline{8} + \underline{3} = 11$ $\underline{8} \times \underline{3} = 24$

$(2x+3)(x+4)$
H P

5. $x^2 - 7x - 8$ O N

$(x-8)(x+1)$

6. $x^2 + 6x - 27$ A K

$(x+9)(x-3)$

7. $x^2 - 4$ C M

$x^2 - 2^2$
$(x+2)(x-2)$

8. $7x^2 + 32x - 60$ $\underline{42} + \underline{-10} = 32$ $\underline{42} \times \underline{-10} = -420$

$\frac{42}{7} = 6$

$\frac{-10}{7}$

$(7x-10)(x+6)$
I J

1 420
2 210
3 140
4 105
5 84
6 70
7 60
10 42

Factors

- ~~A. $(x+9)$~~
- ~~B. $(7x-5)$~~
- ~~C. $(x-2)$~~
- ~~D. $(4x+9)$~~
- ~~E. $(8x+9)$~~
- ~~F. $(7x+5)$~~
- ~~G. $(x-9)$~~
- ~~H. $(2x+3)$~~
- ~~I. $(7x-10)$~~
- ~~J. $(x+6)$~~
- ~~K. $(x-3)$~~
- ~~L. $(2x-5)$~~
- ~~M. $(x+2)$~~
- ~~N. $(x-8)$~~
- ~~O. $(x+1)$~~
- ~~P. $(x+4)$~~